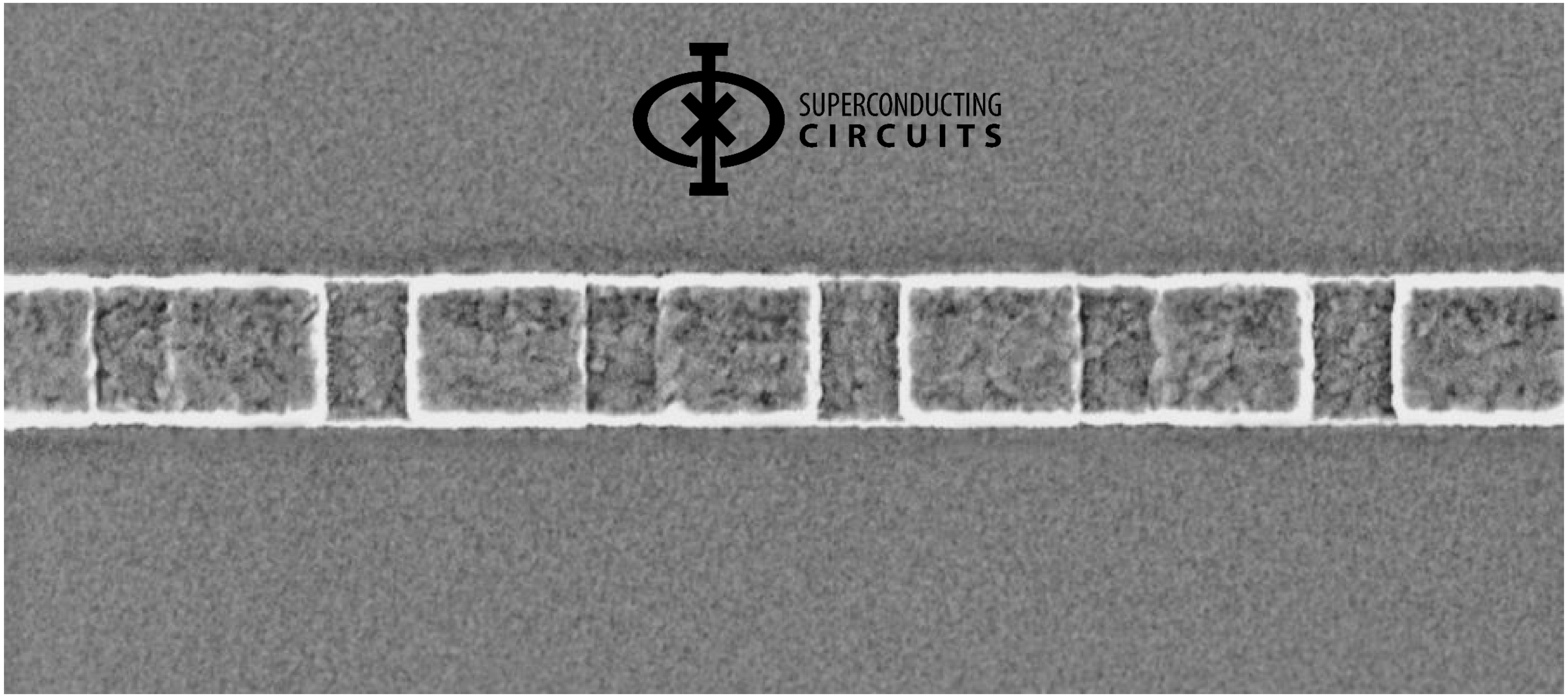
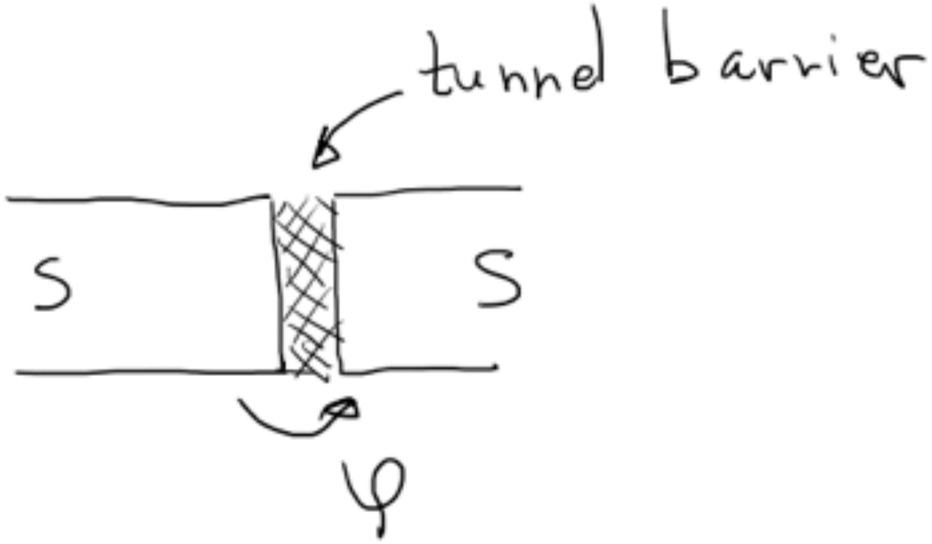
# Towards Analog Simulations of Quantum Impurity Physics

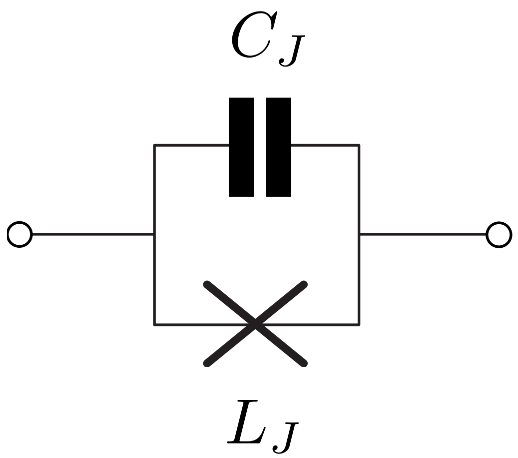
Vladimir Manucharyan (U of Maryland)



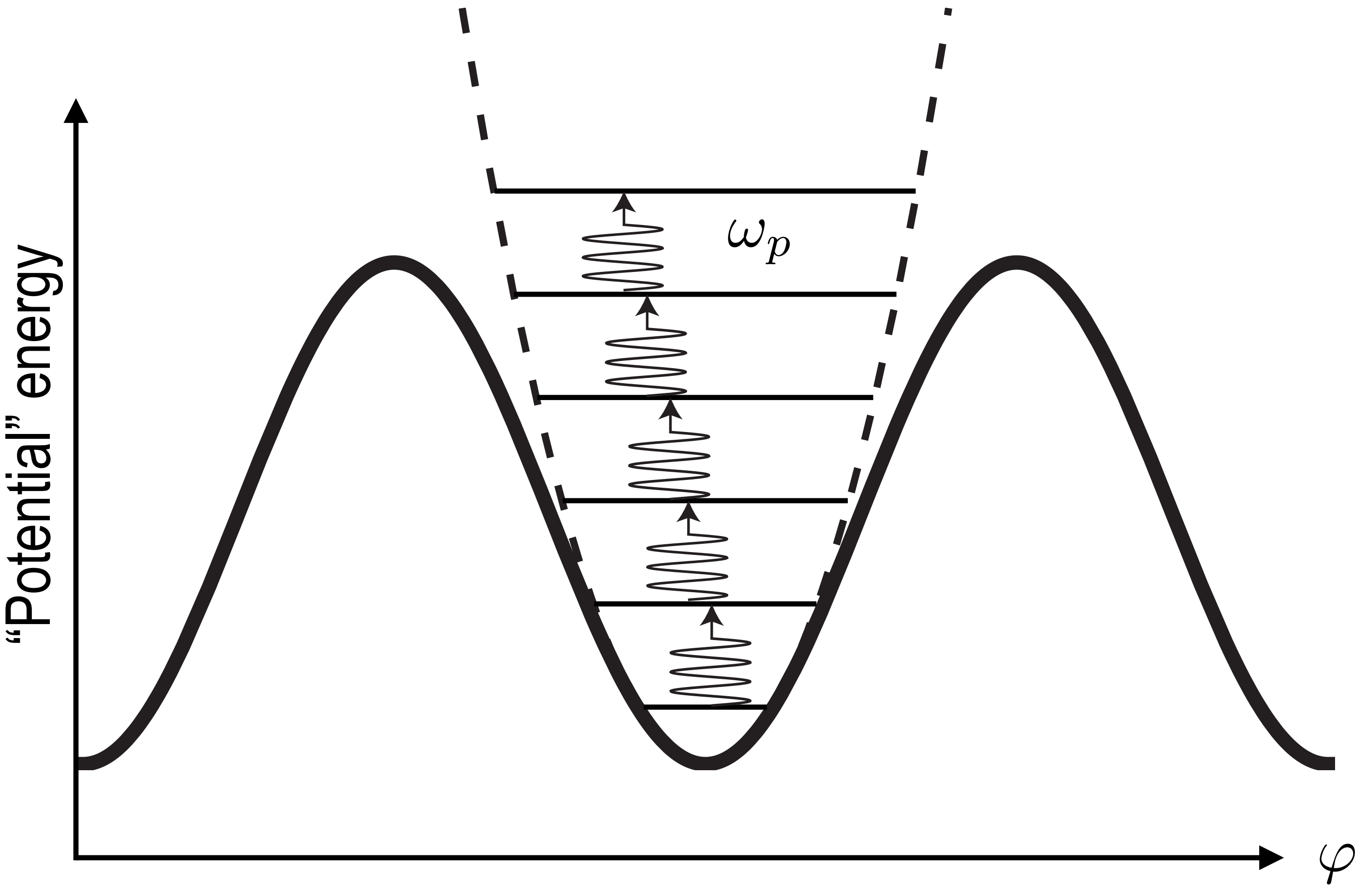
physics University of Maryland
jqi PHYSICS FRONTIER CENTER JOINT QUANTUM INSTITUTE
NSF
SCIENCE TO SHAPE THE FUTURE OF THE ARMY ARMY RESEARCH OFFICE 1951
ALFRED P. SLOAN FOUNDATION 1934
Center for Nanophysics and Advanced Materials
BSF United States - Israel Binational Science Foundation
DARPA
This can be your agency

# The magic of Al/AlO$_x$/Al tunnel junction





Non-dissipative non-linearity



"ultraviolet cut-off"

$$\omega_p \equiv 1/\sqrt{L_J C_J}$$

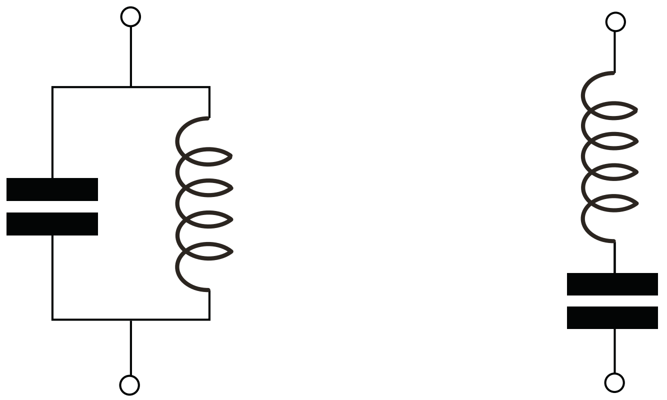
$$\omega_p/2\pi \approx 20\ \mathrm{GHz} \approx 1\ \mathrm{K}$$

Enormous kinetic inductance

$$L_J/\sqrt{A} > 10^4 \mu_0$$

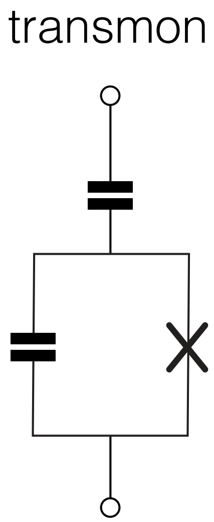
# Common circuits

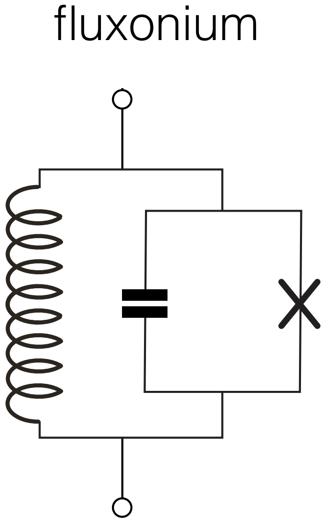
photons



$$H = \hbar\omega_r a^\dagger a$$

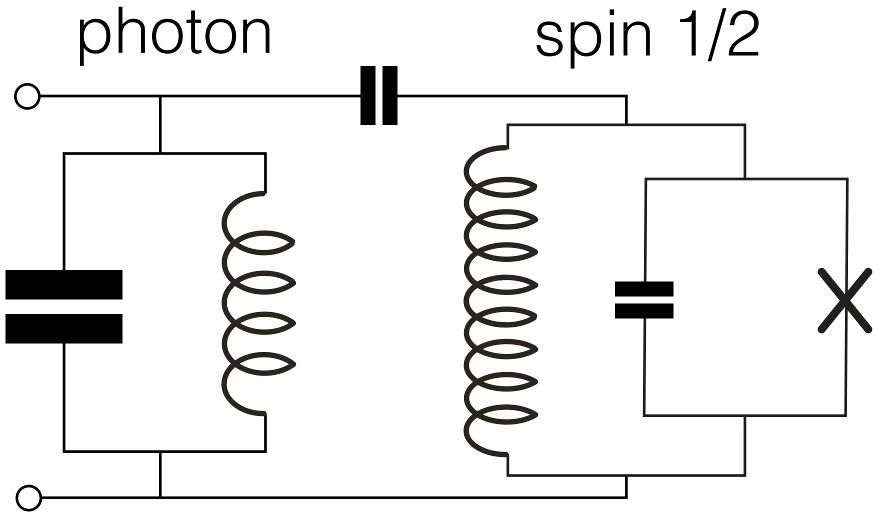
spins (artificial atoms)





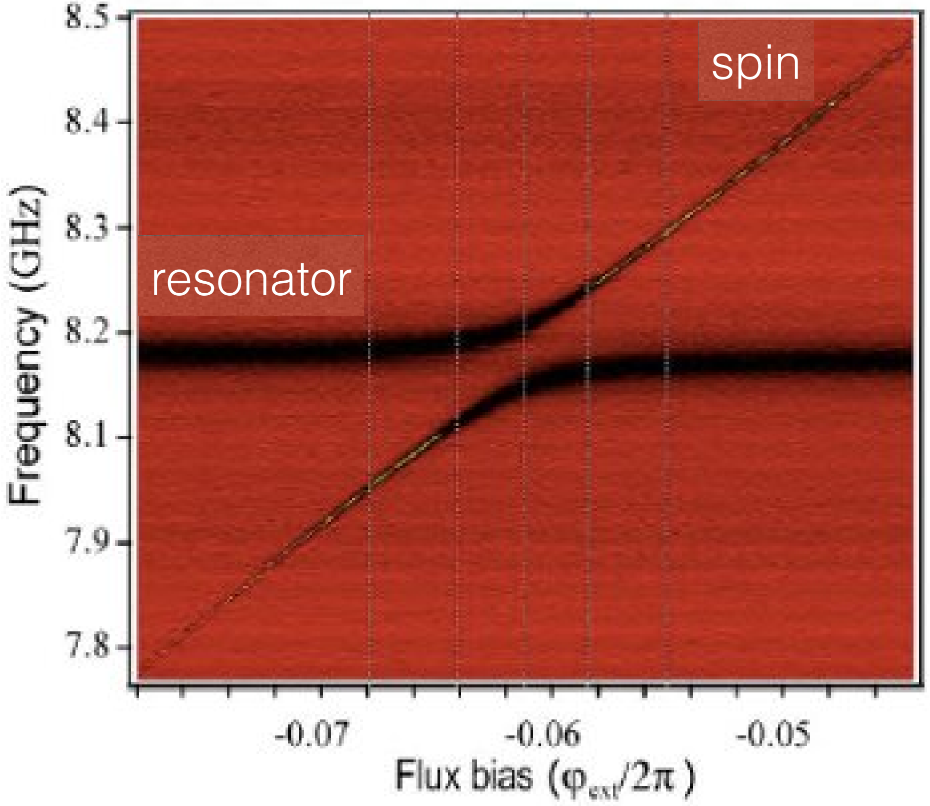
$$H = \hbar\omega_a \sigma_z/2$$

circuit quantum electrodynamics



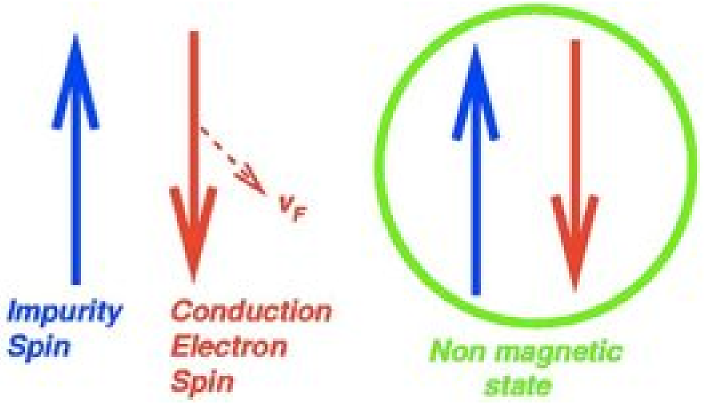
$$H = \hbar\omega_r a^\dagger a + \hbar\omega_a \sigma_z/2 + g\sigma_x(a + a^\dagger)$$

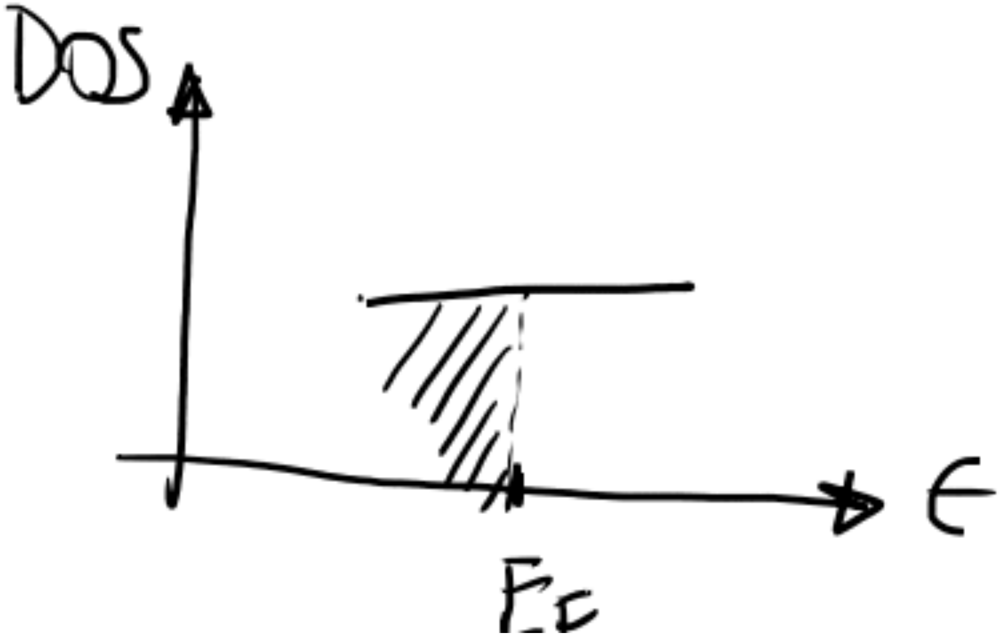
A. Wallraff et al. (2004)

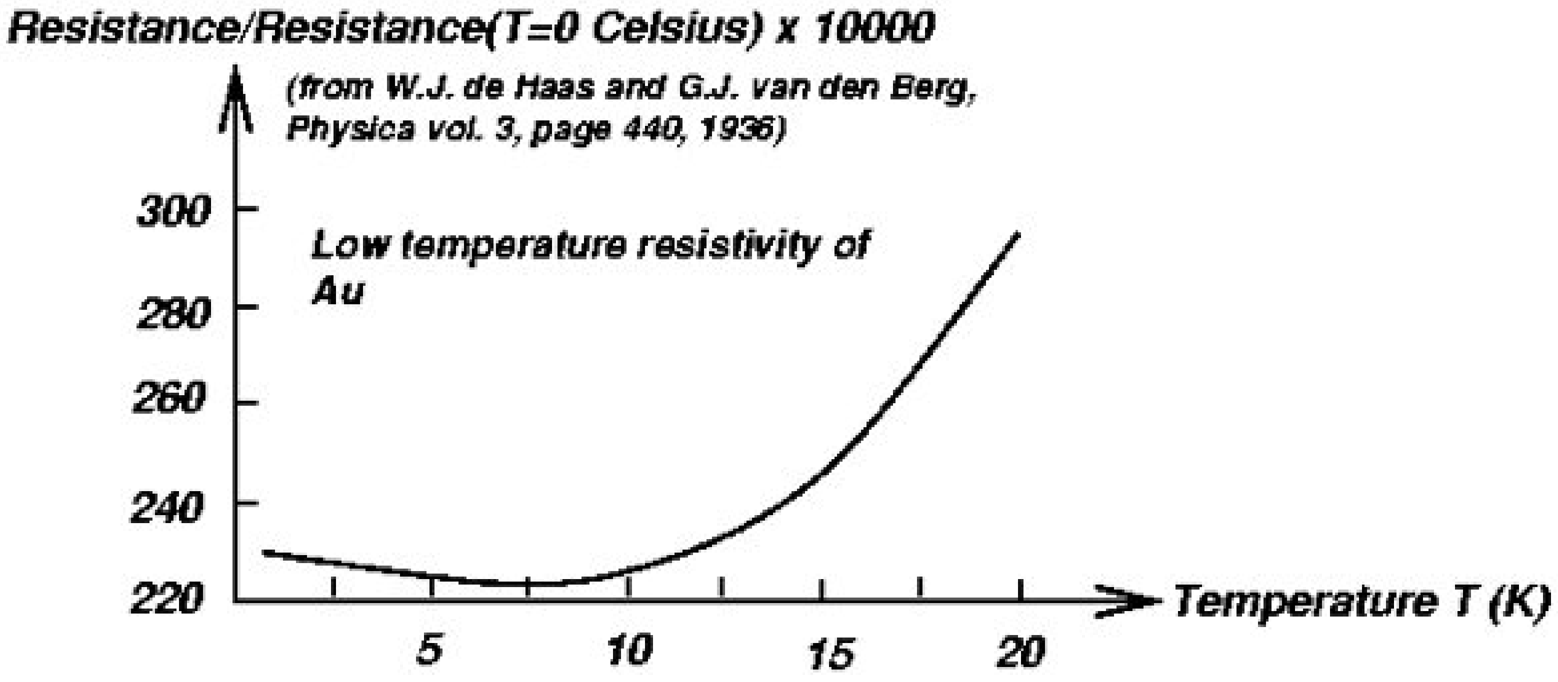


# Kondo effect:
# an example of quantum impurity physics

Many **free** electrons (modes) interact via a **single** spin ½ (impurity)

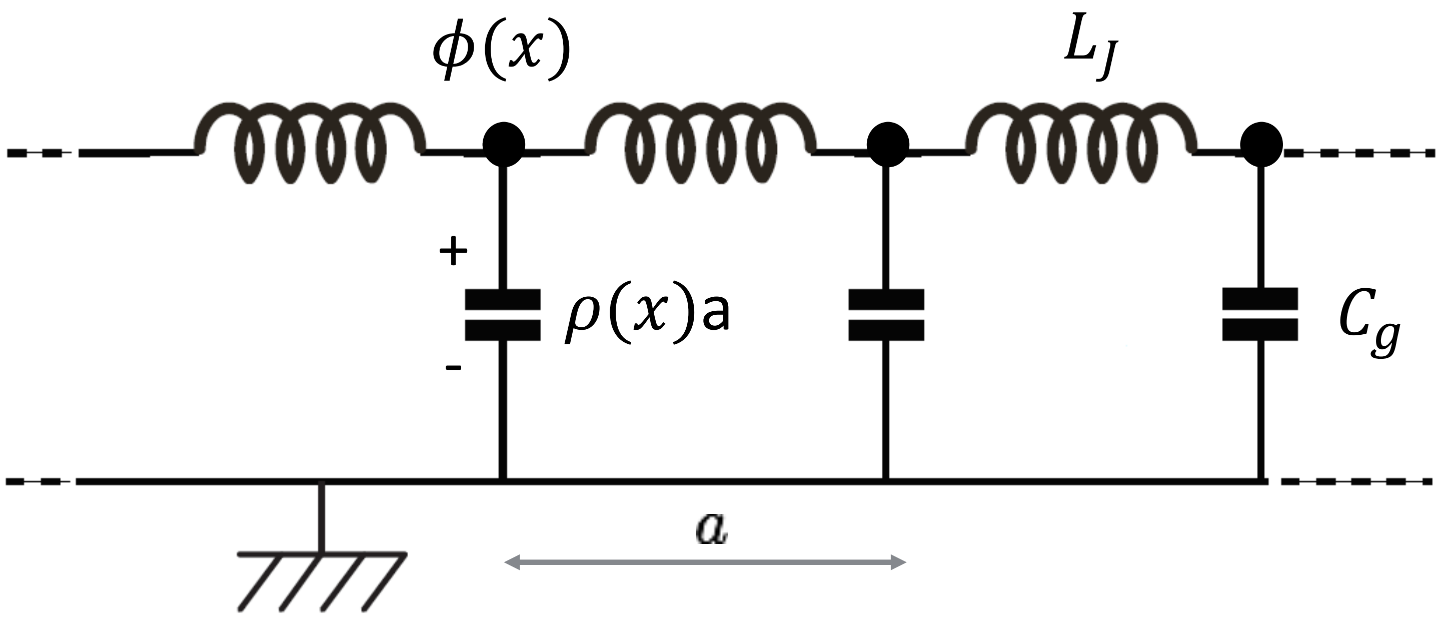






Source: Kondo Wiki

# Luttinger liquid physics



$$v = \frac{a}{\sqrt{L_J C_g}}$$

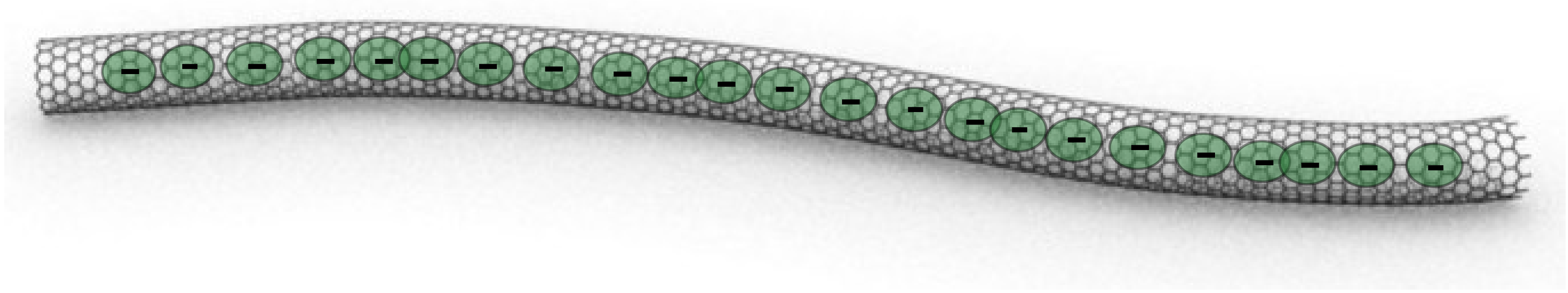
$$Z = \sqrt{\frac{L_J}{C_g}}$$

$$R_Q = \frac{h}{(2e)^2}$$

$$H_0 = \frac{1}{2} vh \int \left[ \frac{Z}{R_Q} \rho(x)^2 + \frac{R_Q}{Z} \left( \frac{\nabla\phi(x)}{2\pi} \right)^2 \right] dx$$

A TEM waveguide is a **spinless Luttinger liquid**

$$[\phi(x), \rho(x)] = i\delta(x - x')$$



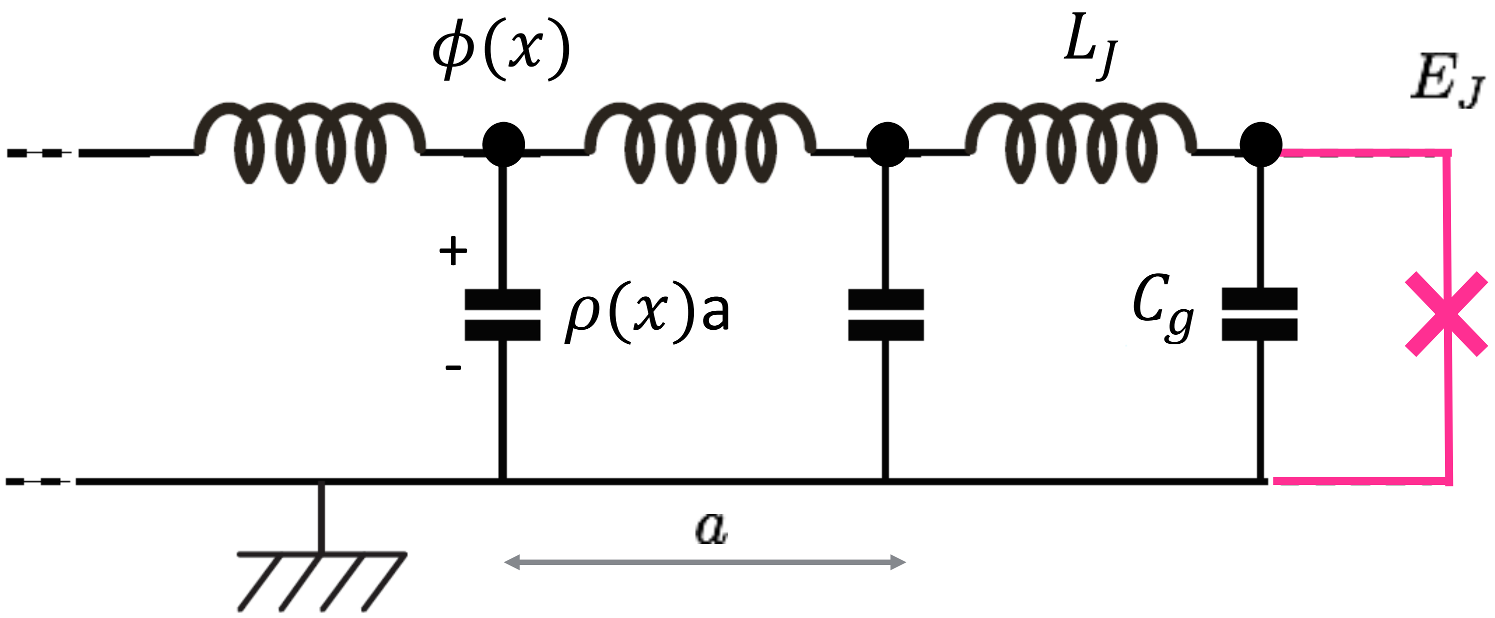
$Z/R_Q > 1$ repulsion

$Z/R_Q < 1$ attraction

Fazio, R., et al. (1996); Glazman, L. I., Larkin, A. I. (1997)

# The boundary sine-Gordon quantum impurity model

Gogolin, Nersesyan, Tsvelik, “Bosonisation and strongly correlated systems"



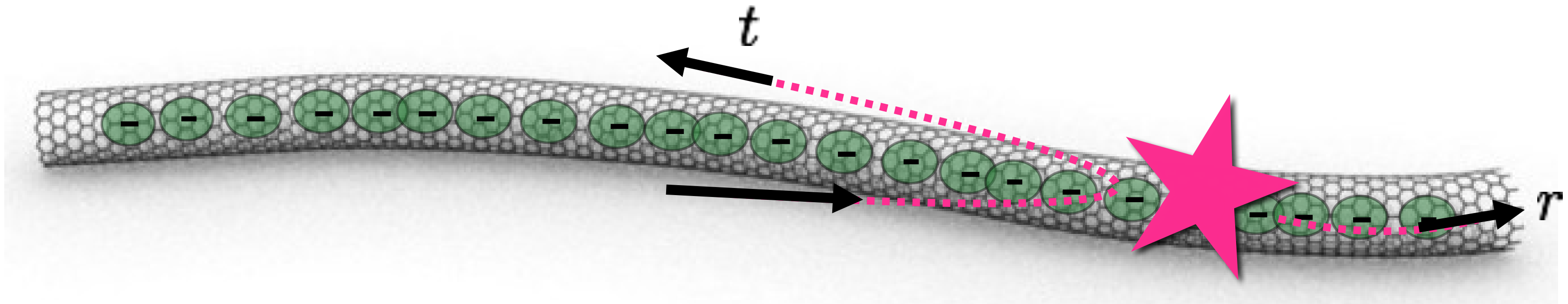
$$v = \frac{a}{\sqrt{L_J C_g}}$$

$$Z = \sqrt{\frac{L_J}{C_g}}$$

$$R_Q = \frac{h}{(2e)^2}$$

$$H = \frac{1}{2} v h \int \left[ \frac{Z}{R_Q} \rho(x)^2 + \frac{R_Q}{Z} \left( \frac{\nabla\phi(x)}{2\pi} \right)^2 \right] dx - E_J cos\phi(x = 0)$$

$$[\phi(x), \rho(x)] = i\delta(x - x')$$

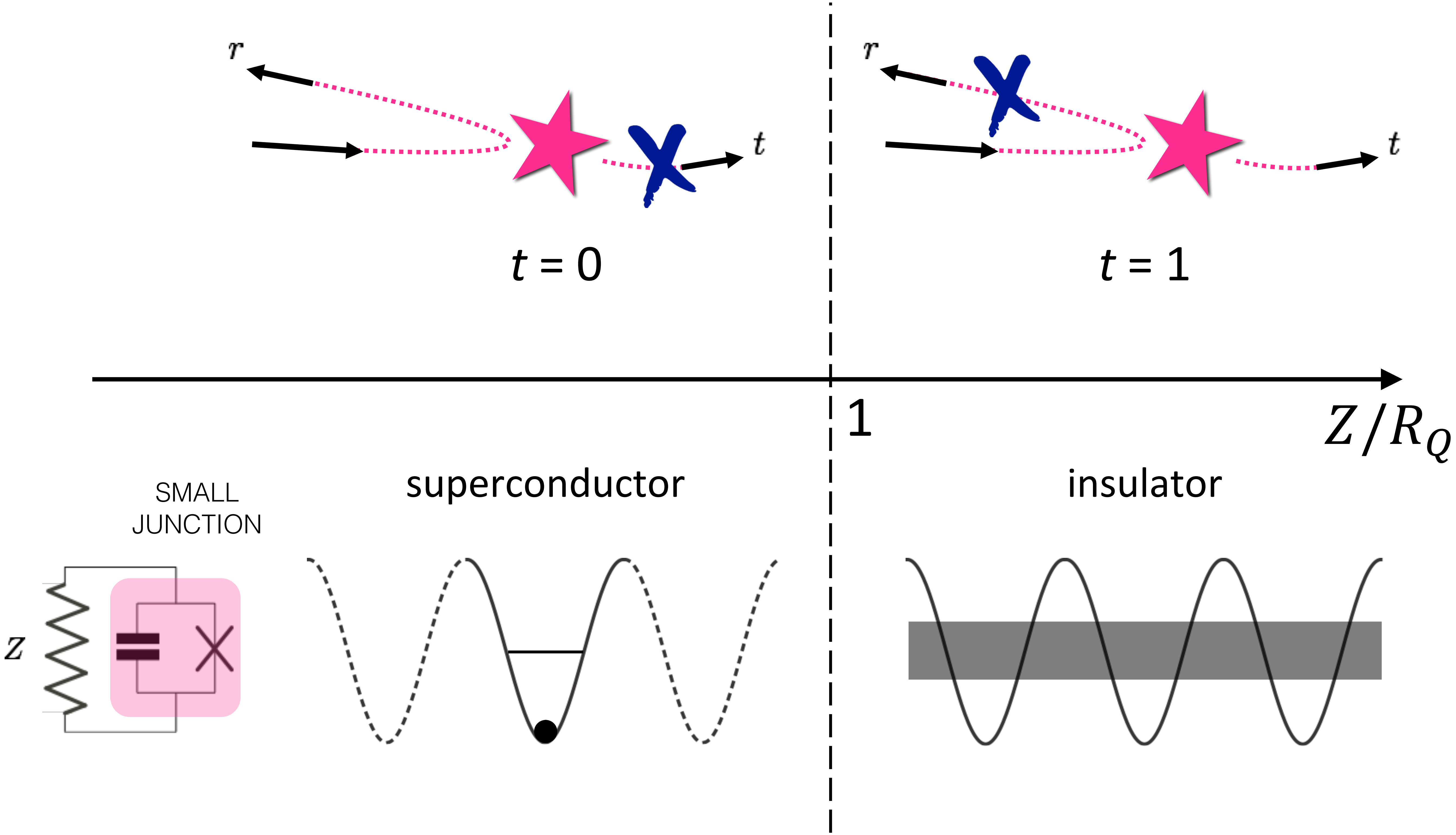


Backscattering ⇔ Josephson tunneling

$Z/R_Q > 1$ repulsion

$Z/R_Q < 1$ attraction

# A zero-energy picture of the critical point



$t = 0$

$t = 1$

1

$Z/R_Q$

SMALL
JUNCTION

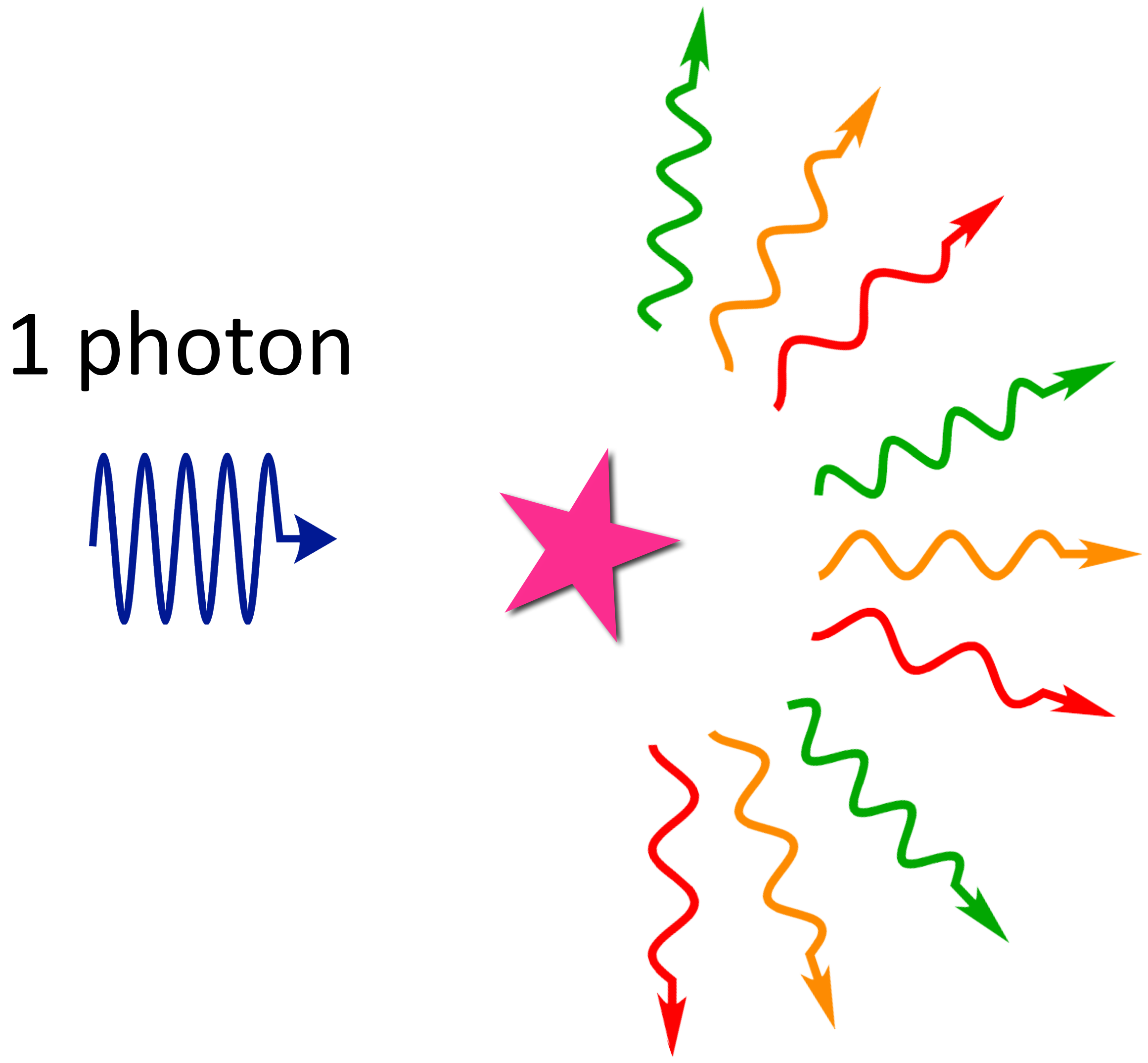
superconductor

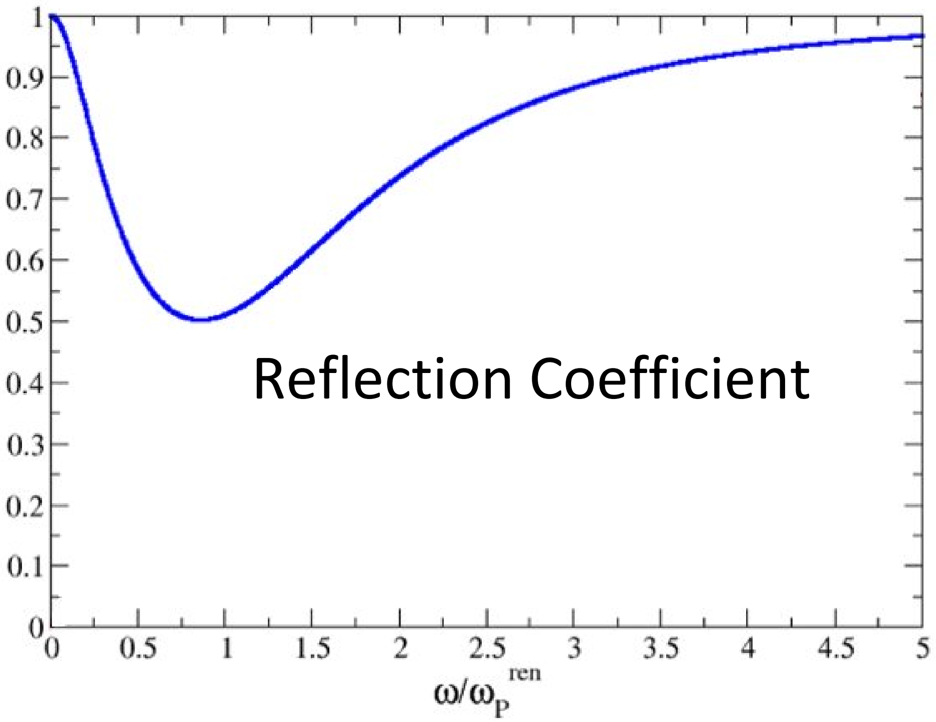
insulator

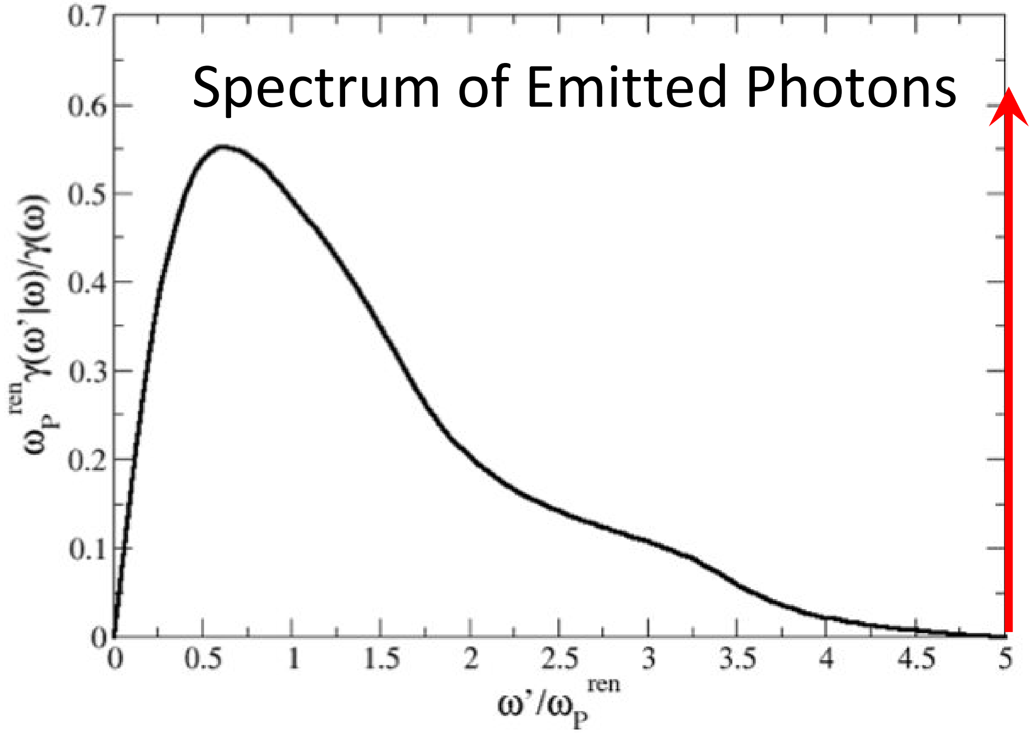
A. Schmid (1983); S. Bulgadaev (1984); C. Kane & M. Fisher (1992)

# The finite frequencies picture: inelastic scattering of single photons

$\omega_p^{ren} \to 0$ as $Z/R_Q \to 1^-$

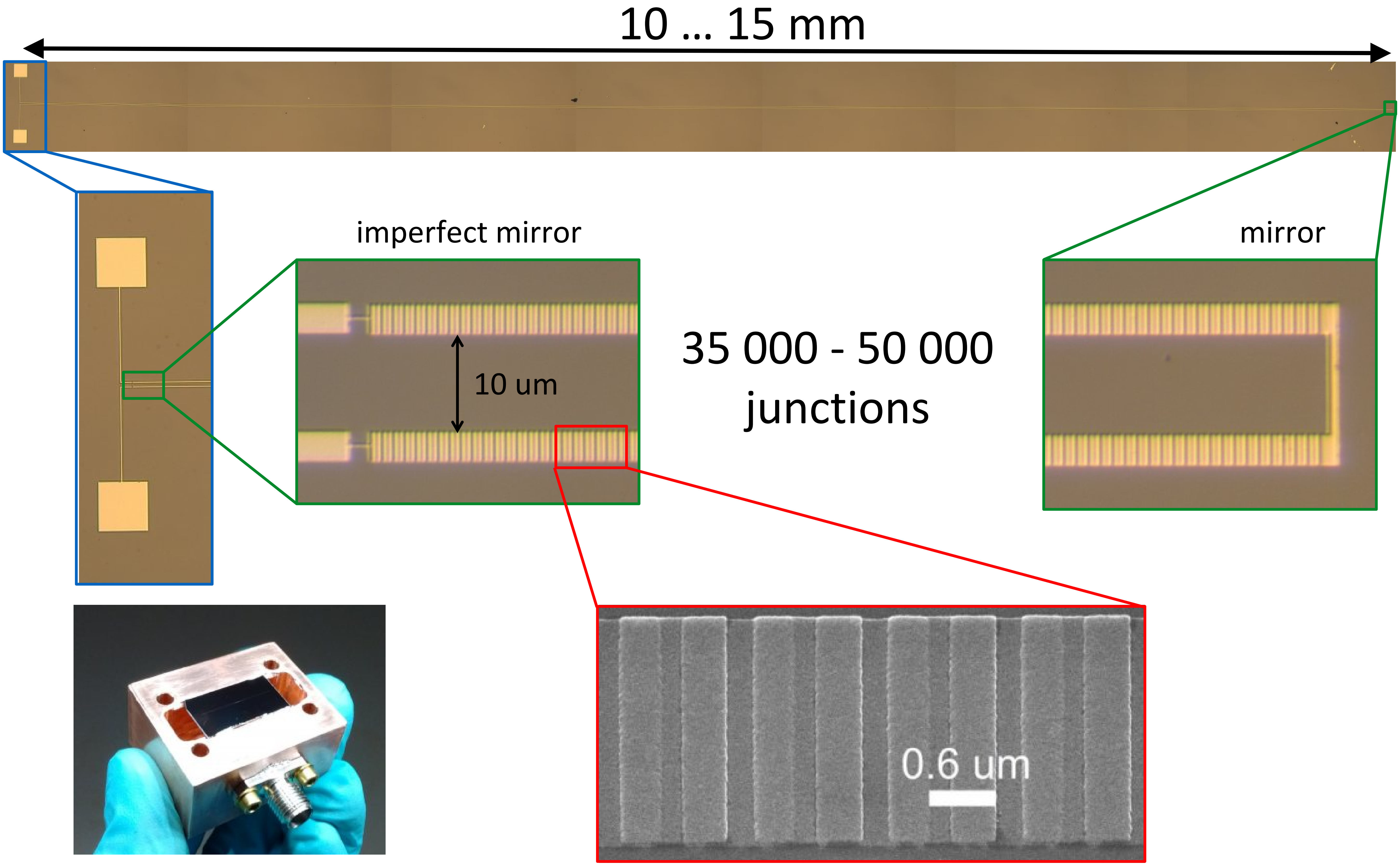



Reflection Coefficient


Spectrum of Emitted Photons

M. Goldstein et al. (2013); M. Goldstein and V. Manucharyan, (in progress 2017)

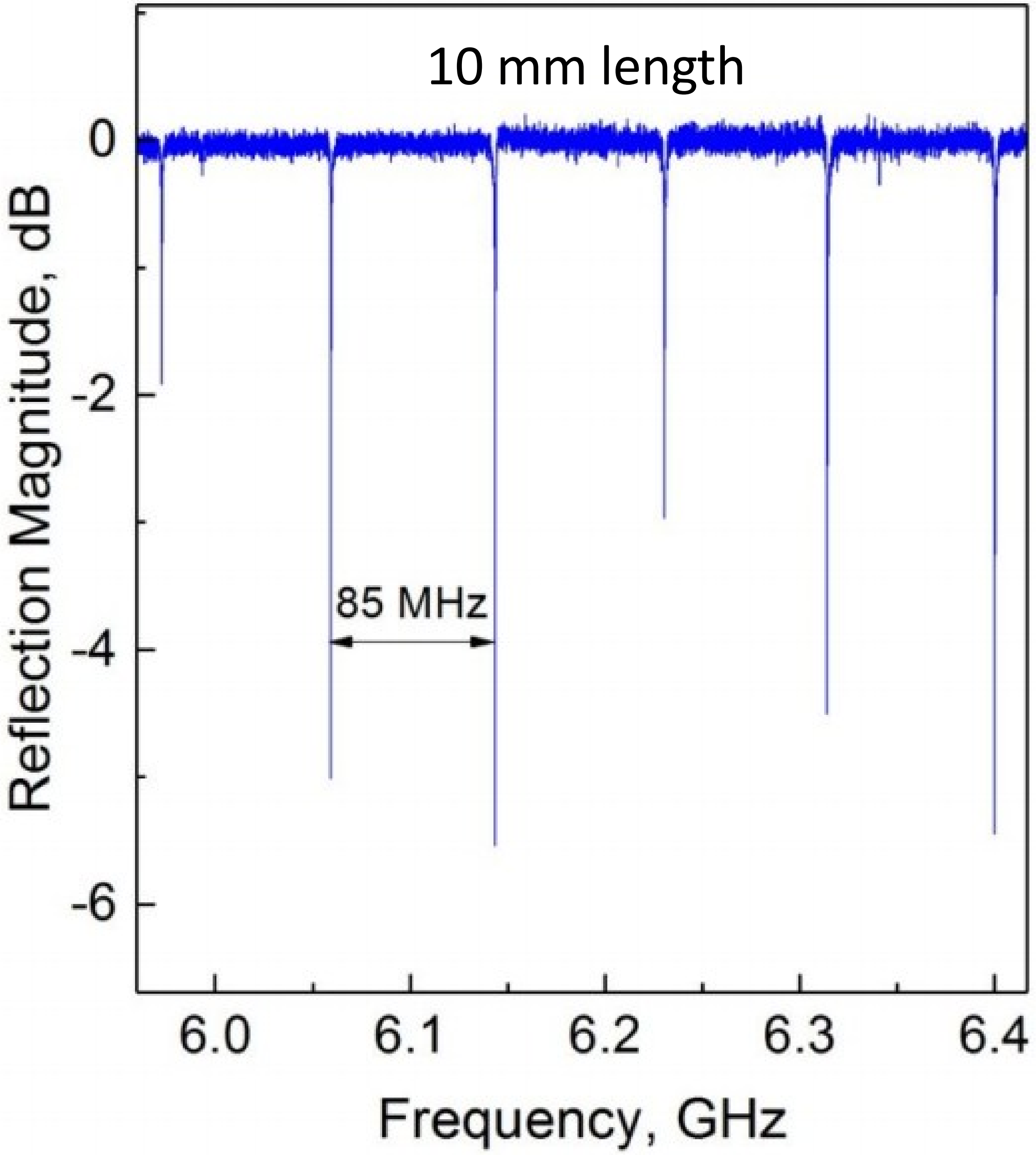
# High-impedance Josephson transmission line



35 000 - 50 000 junctions

# Ultra-slow microwave "light"

Measured speed of light $c = 2.1 \cdot 10^6 \frac{m}{sec}$



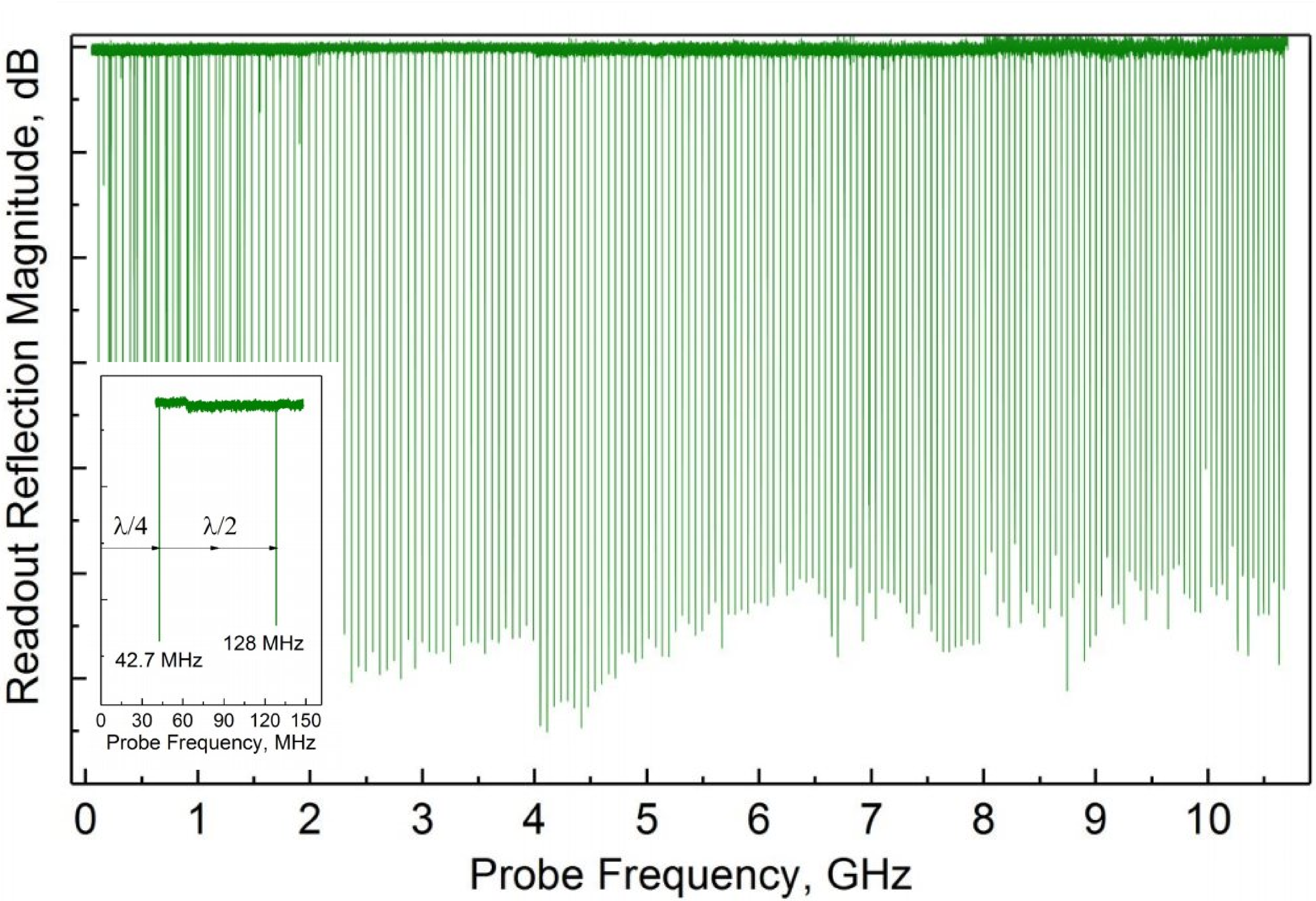
$$c \rightarrow c/150$$

$$\alpha \rightarrow \alpha \times 150$$

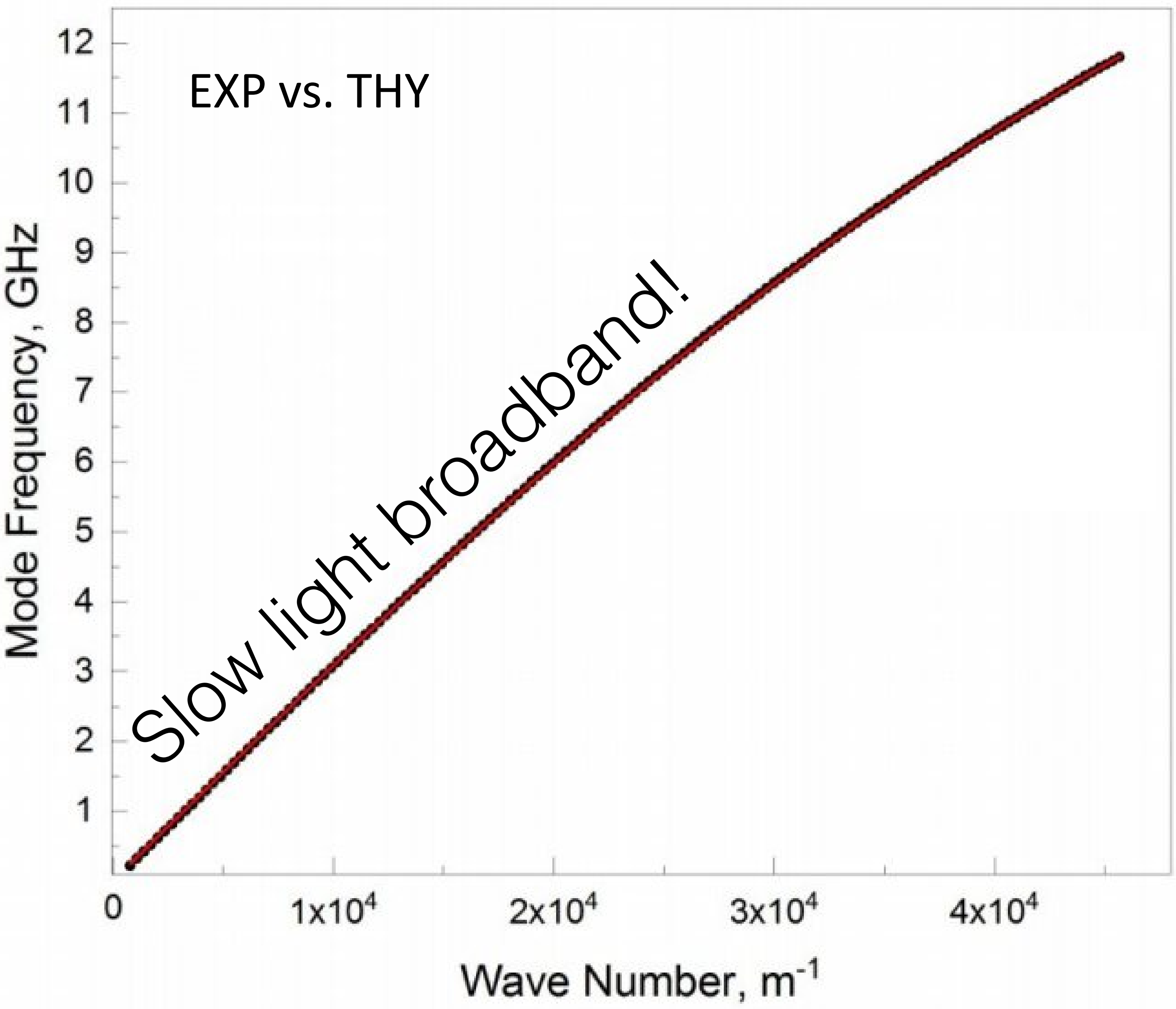
$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

**g ~ 1**

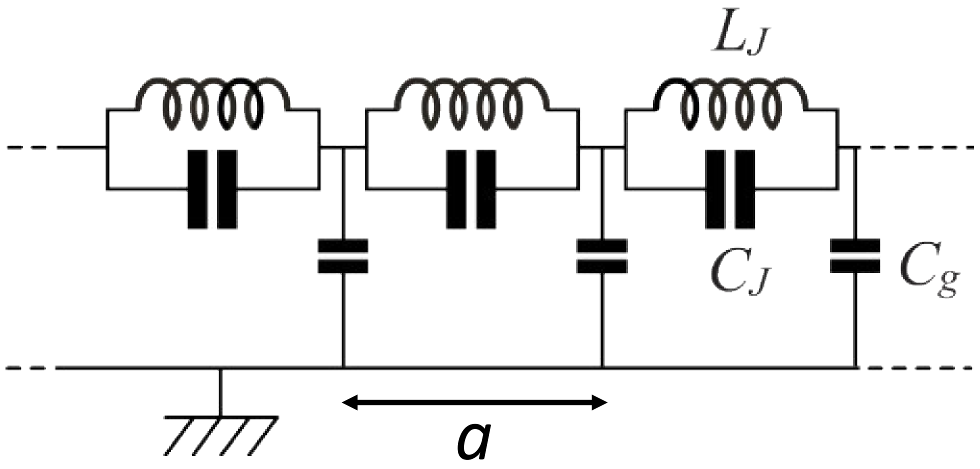
# Reservoir where every mode is individually available!



# Dispersion relation: exp vs thy



Slow light broadband!

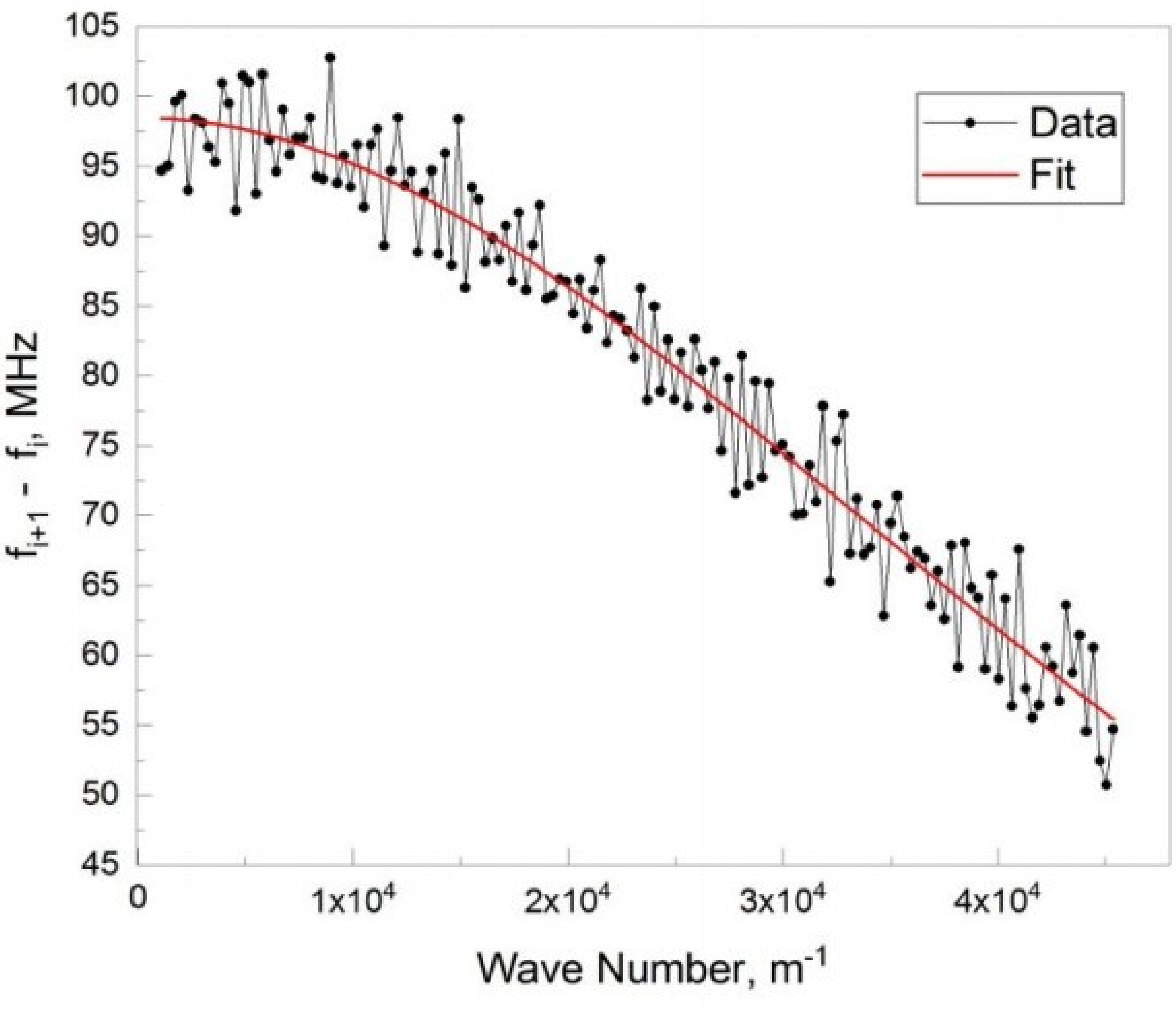


$$c = \frac{a}{\sqrt{L_J C_g}}, \quad \omega_p = \frac{1}{\sqrt{L_J C_J}}, Z = \sqrt{\frac{L_J}{C_g}}$$

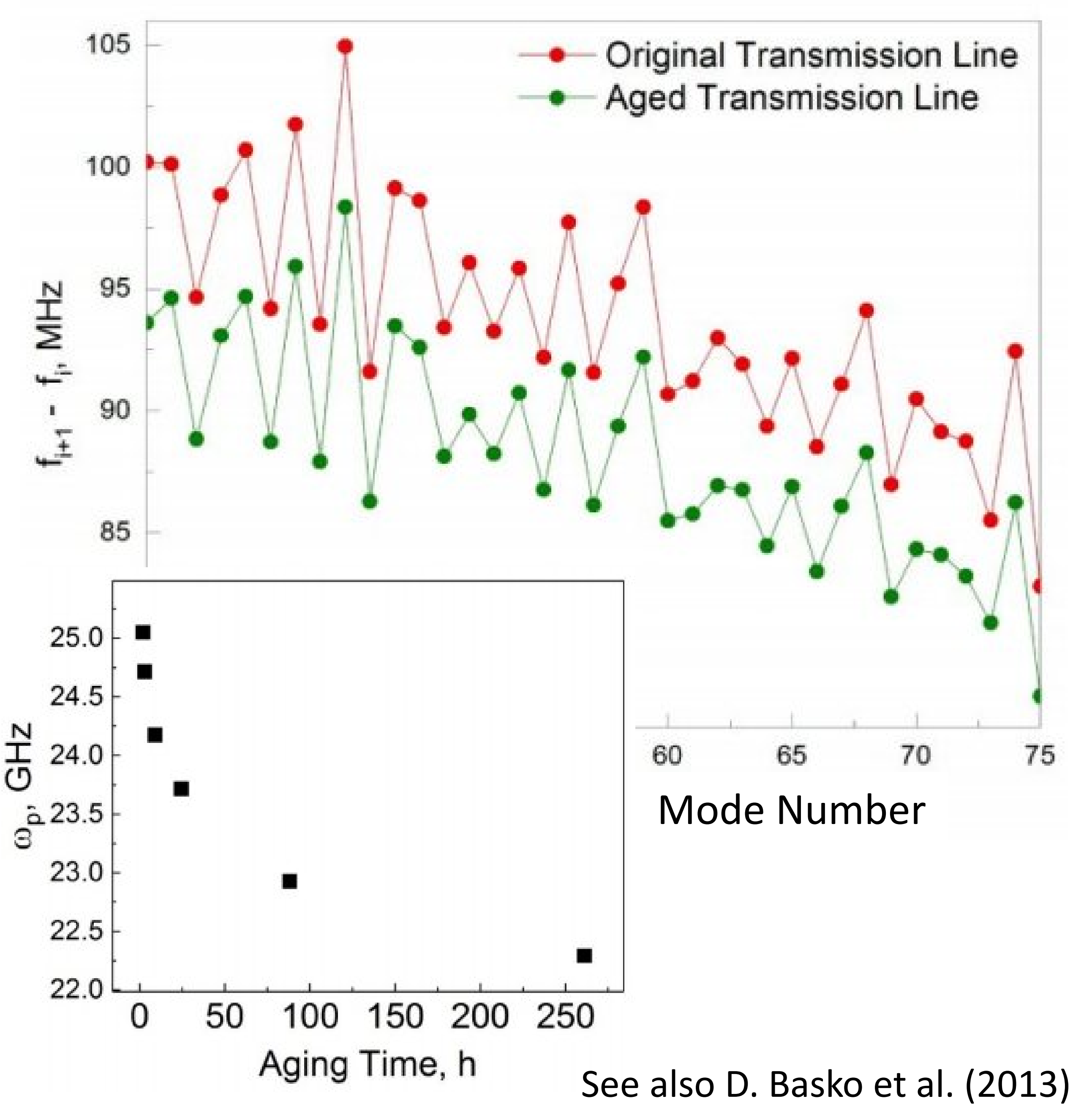
$$\omega(k) = \frac{ck}{\sqrt{1 + \left(\frac{ck}{\omega_p}\right)^2}}$$

# Long-range disorder in the junction parameters

Modes Spacing vs Mode Number

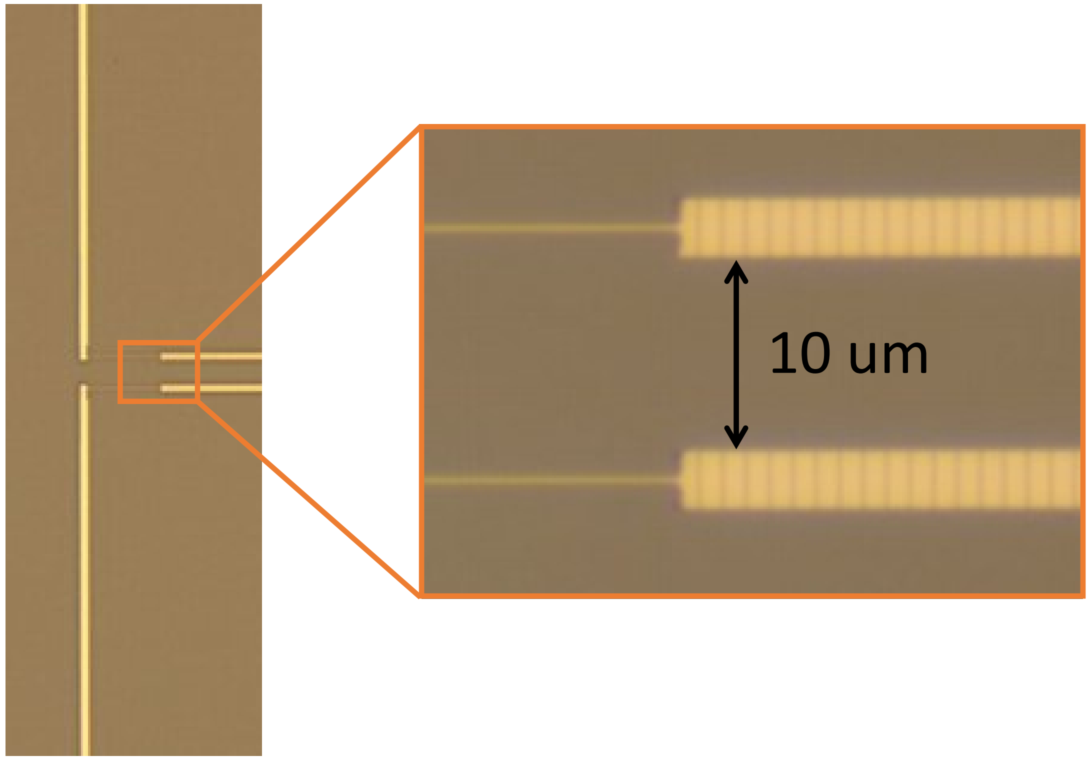


$$\widetilde{L_J} = L_J(1+\xi)$$



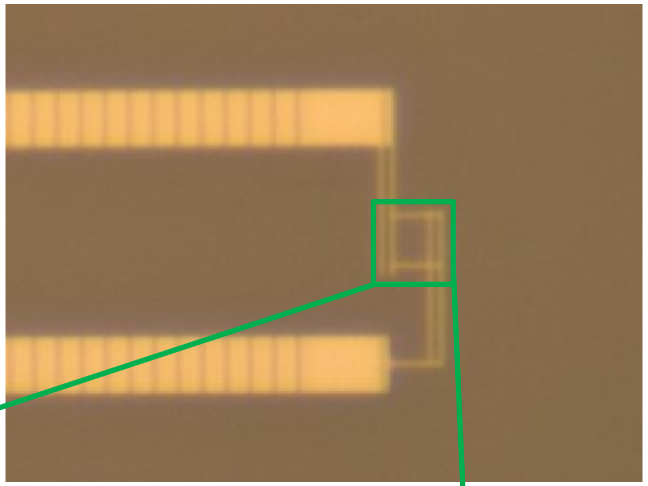
See also D. Basko et al. (2013)

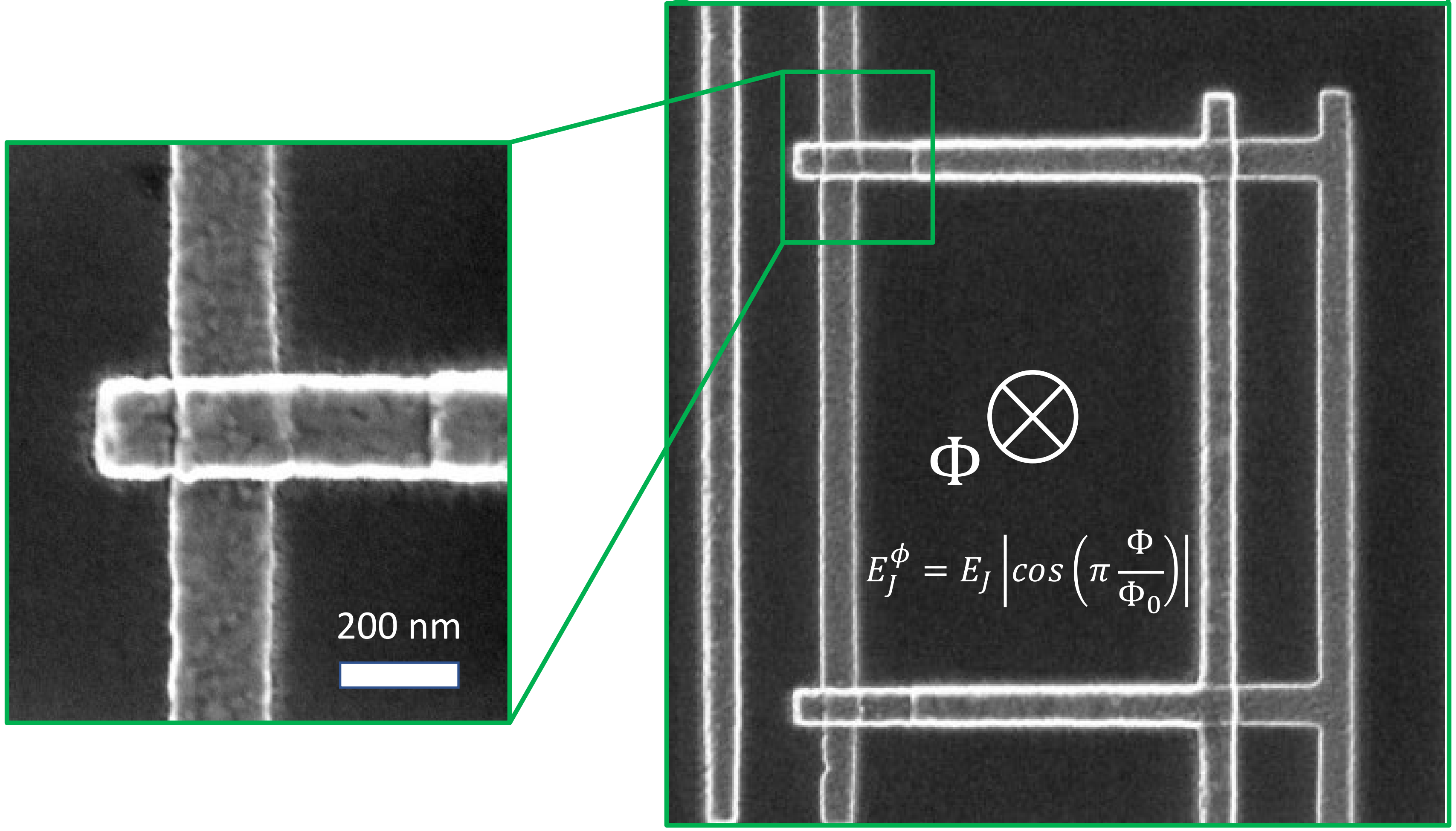
# The boundary sine-Gordon quantum impurity



| $\omega_p/2\pi$ | 28.5 GHz |
|---|---|
| $Z$ | **5.8** $k\Omega$ |
| $E_J/E_c$ | **817** |

Quantum Impurity





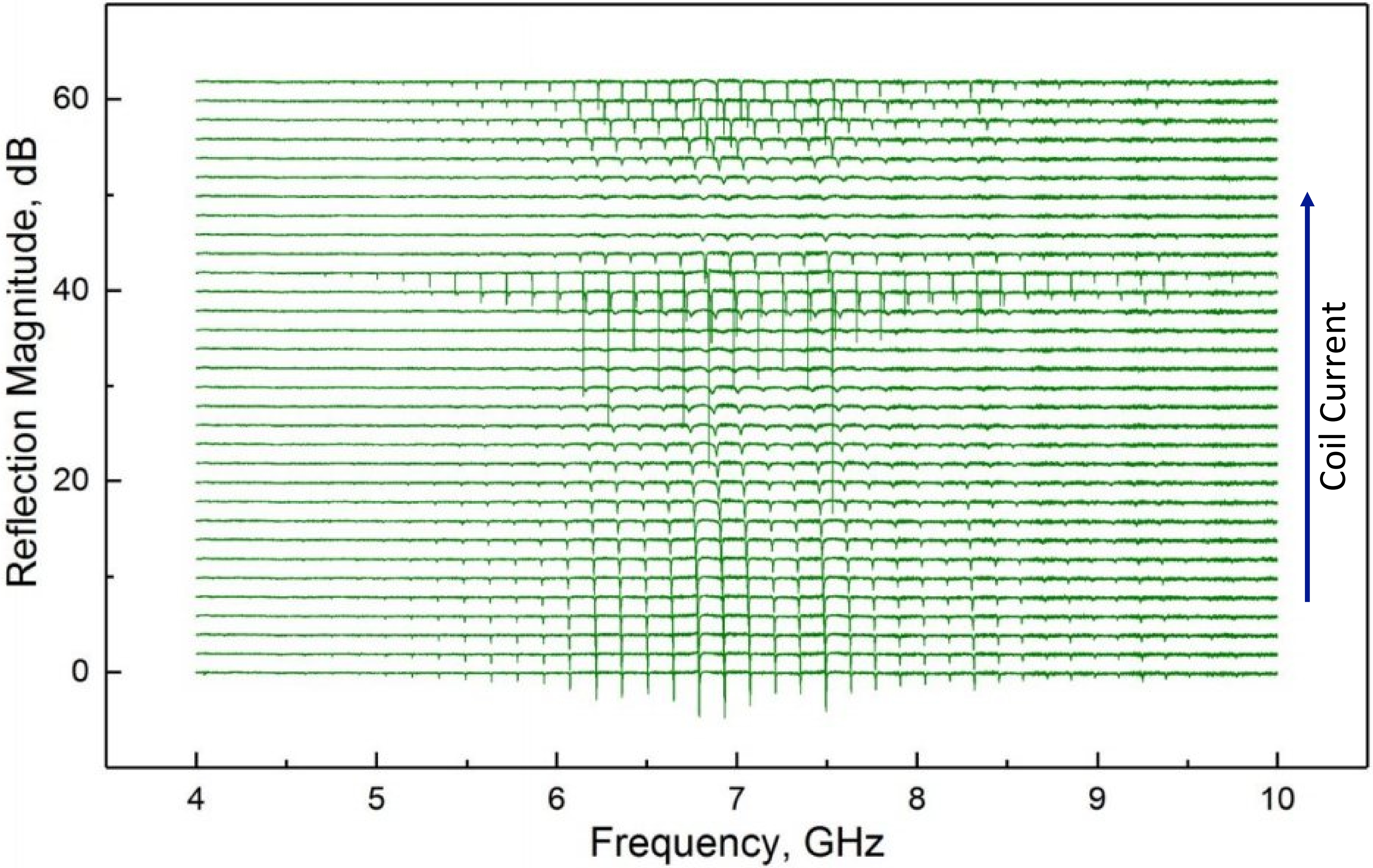
$E_J^{\phi} = E_J \left|\cos\left(\pi \frac{\Phi}{\Phi_0}\right)\right|$

$\omega_p \sim 15\ GHz$

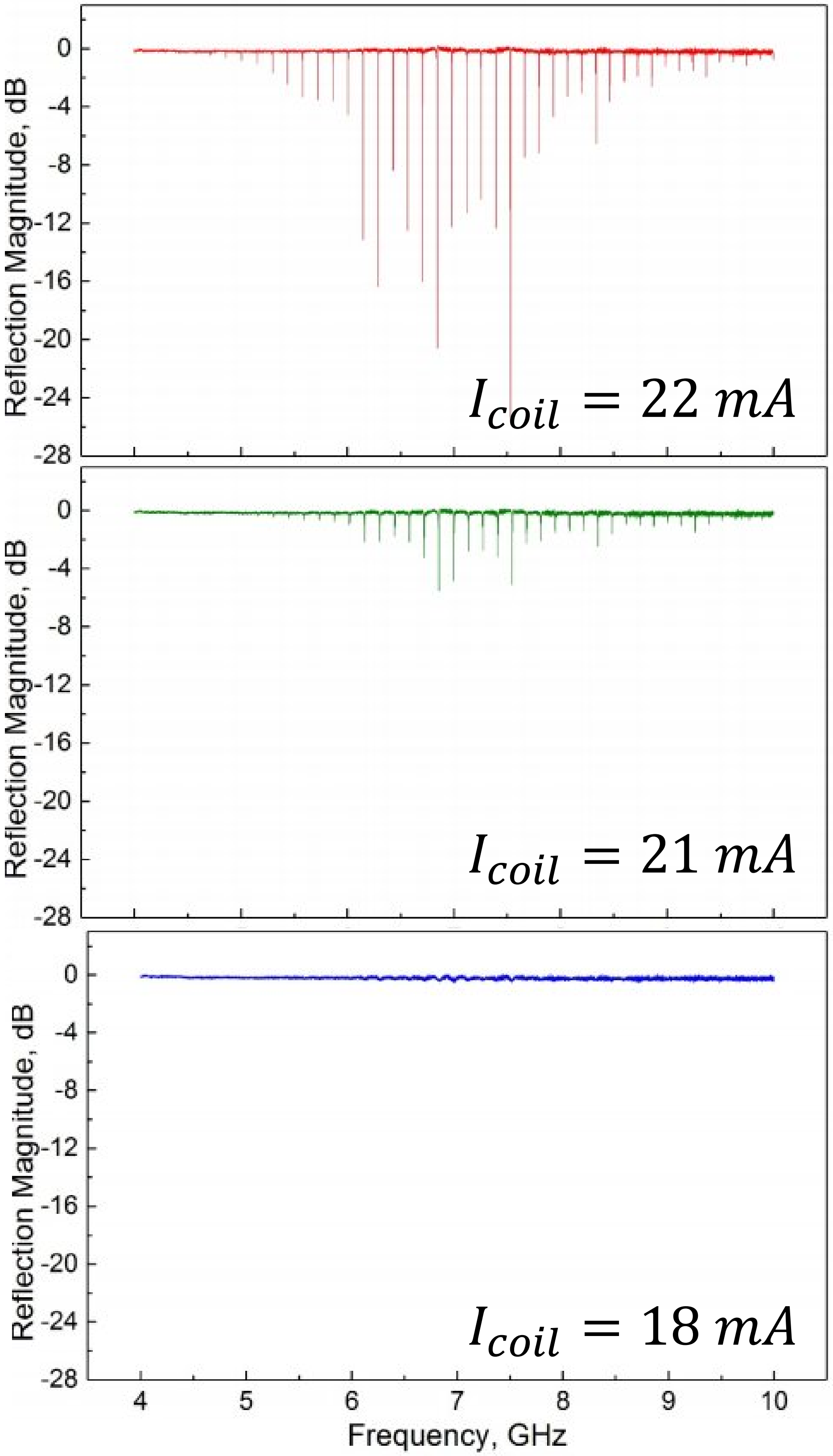
$E_J/E_c \approx 1$

$\omega_p \approx \sqrt{E_J E_C}$

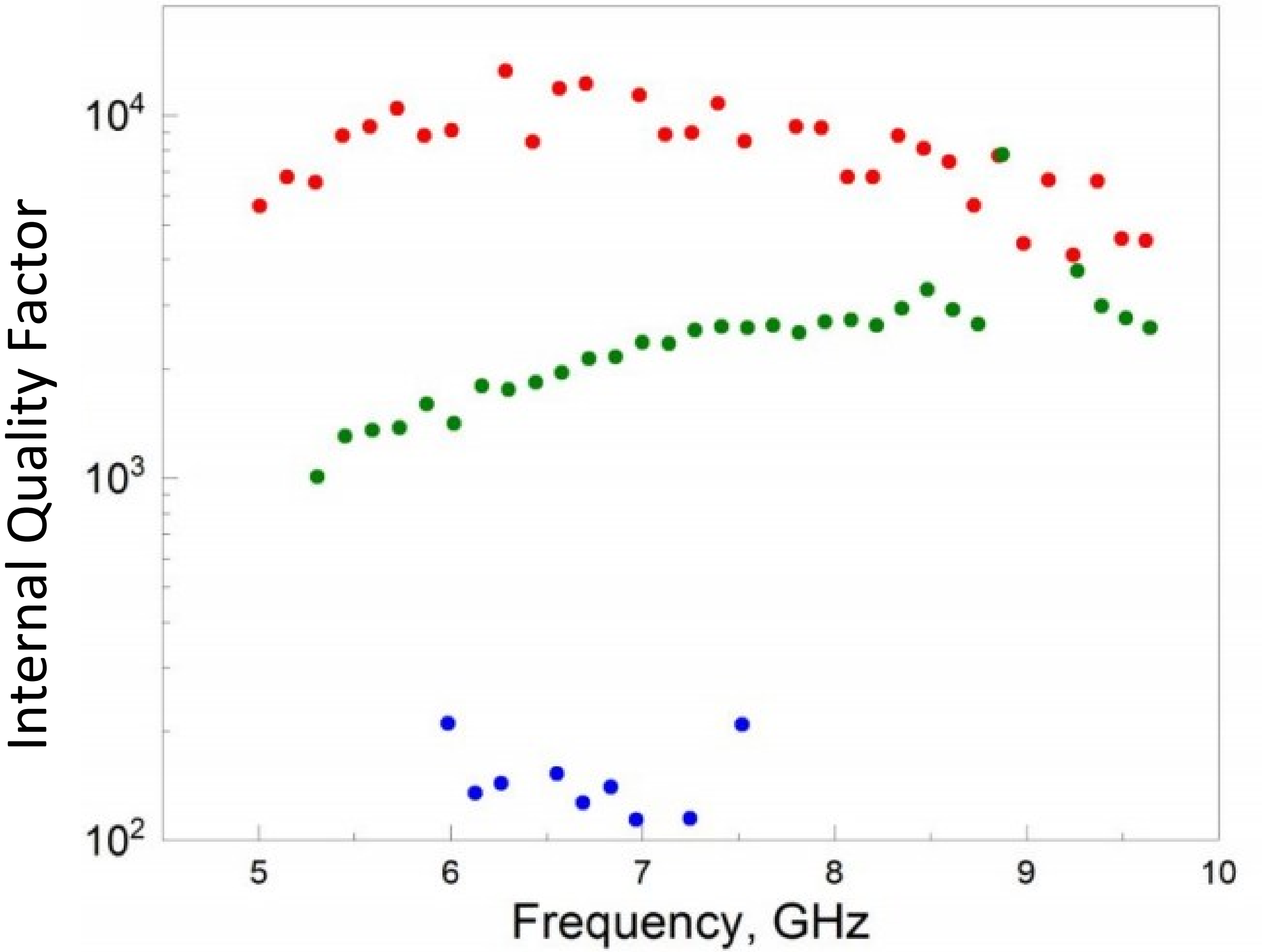
# Suppression of the waves by only one phase-slip junction



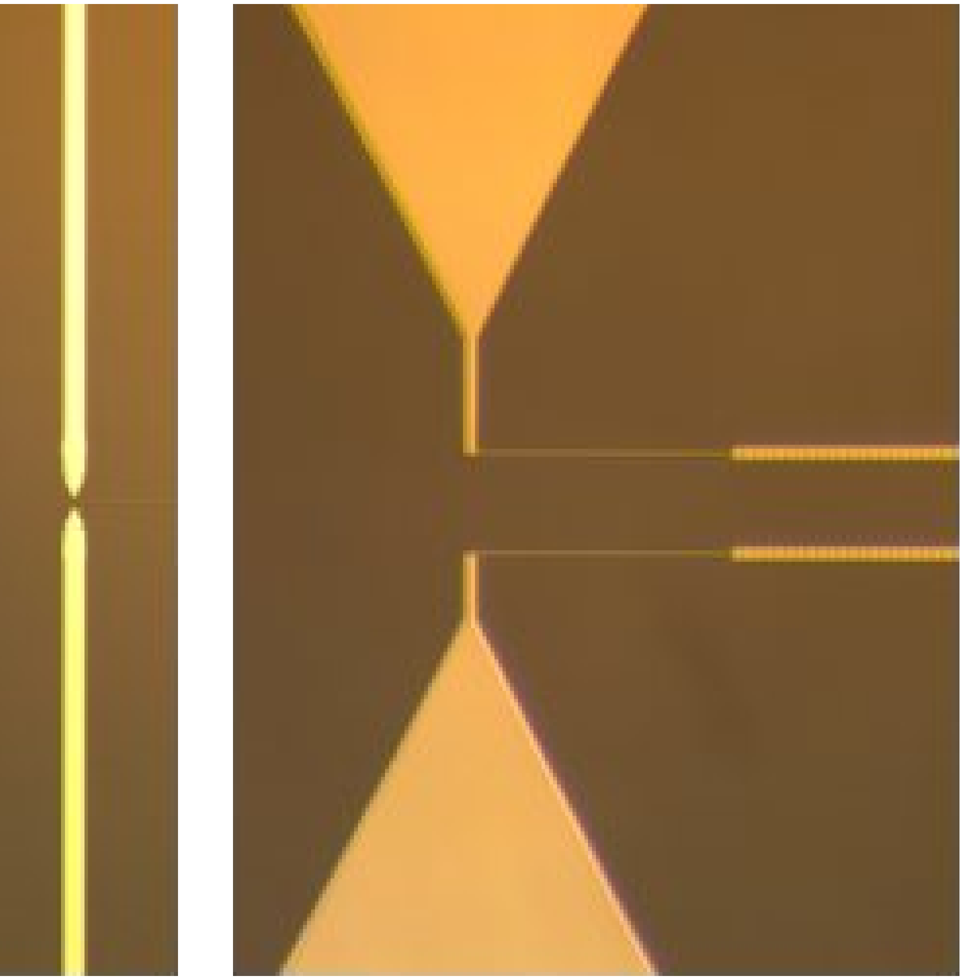
# Frequency depended dissipation



measures the rate of wave decoherence inside the chain

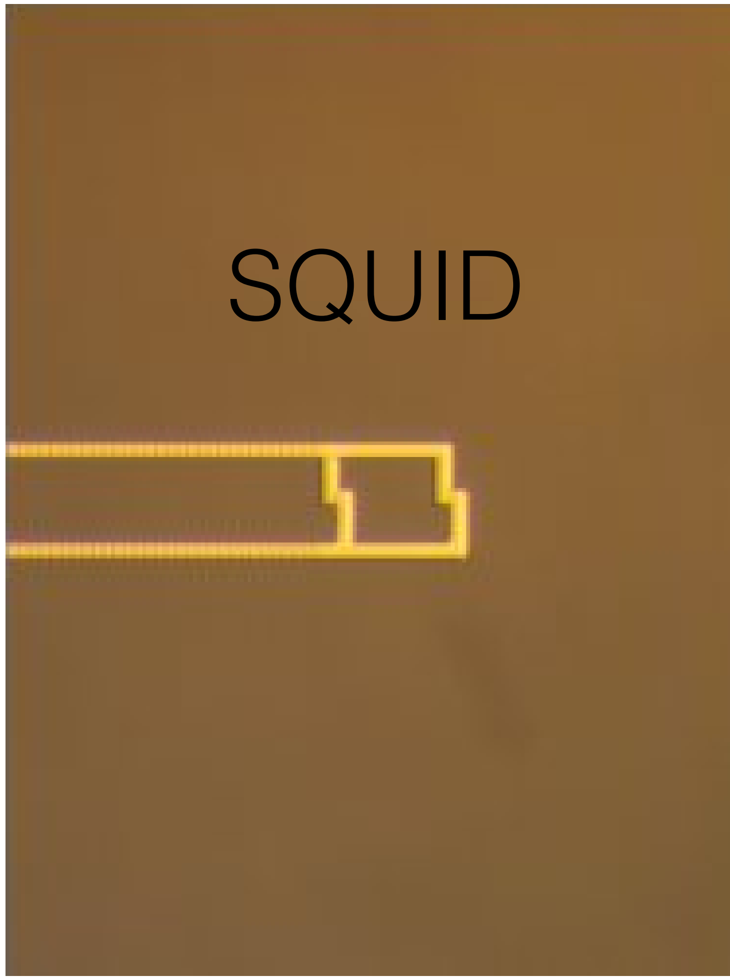


# Weakly anharmonic oscillator impurity



“Common mode”

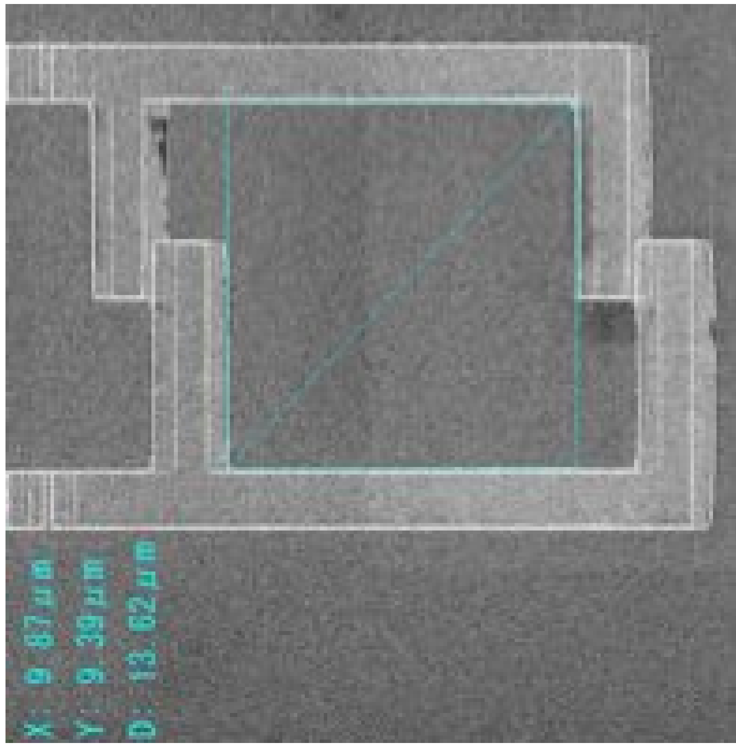
33,000 junctions
(10 mm long)

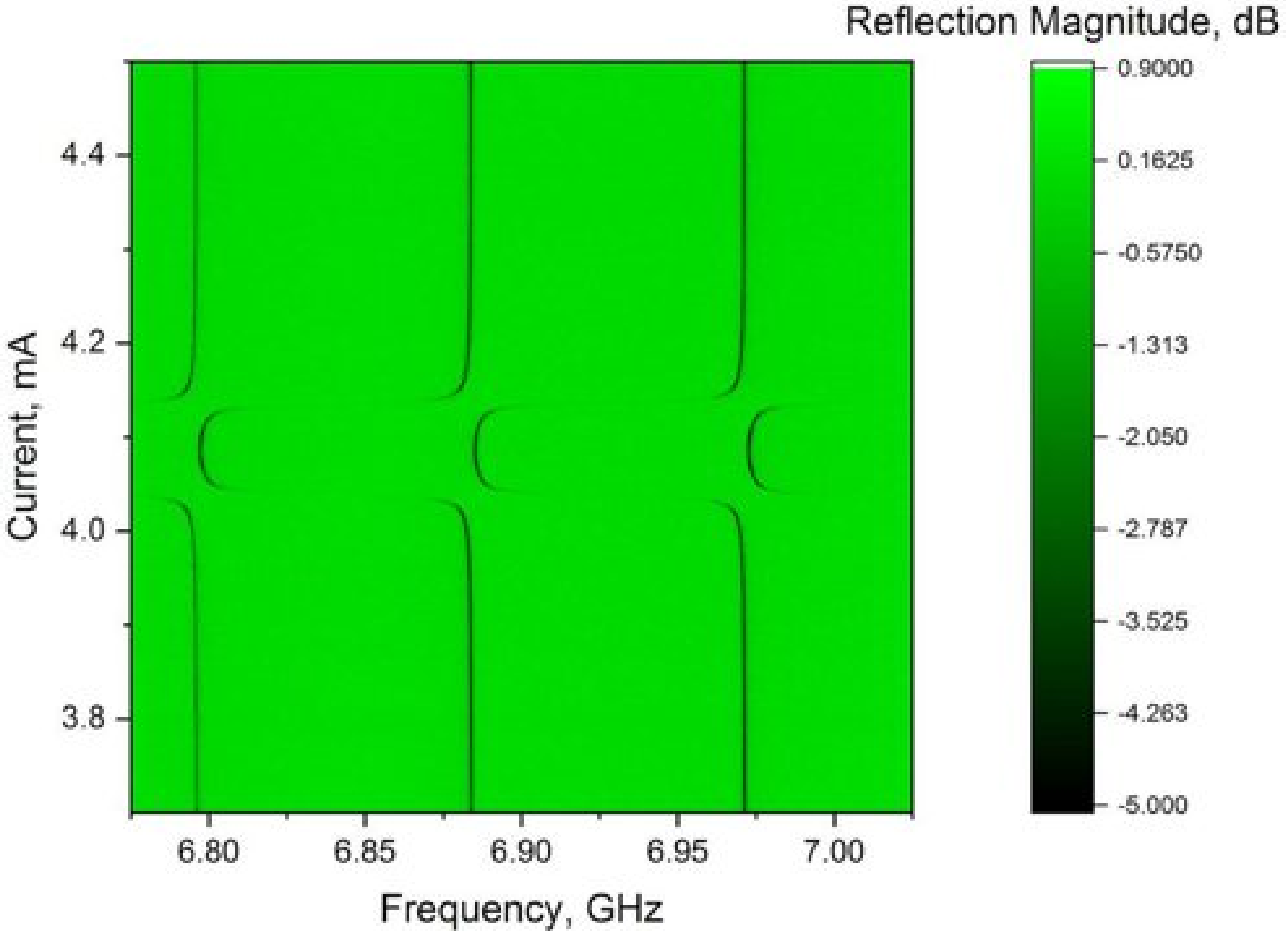


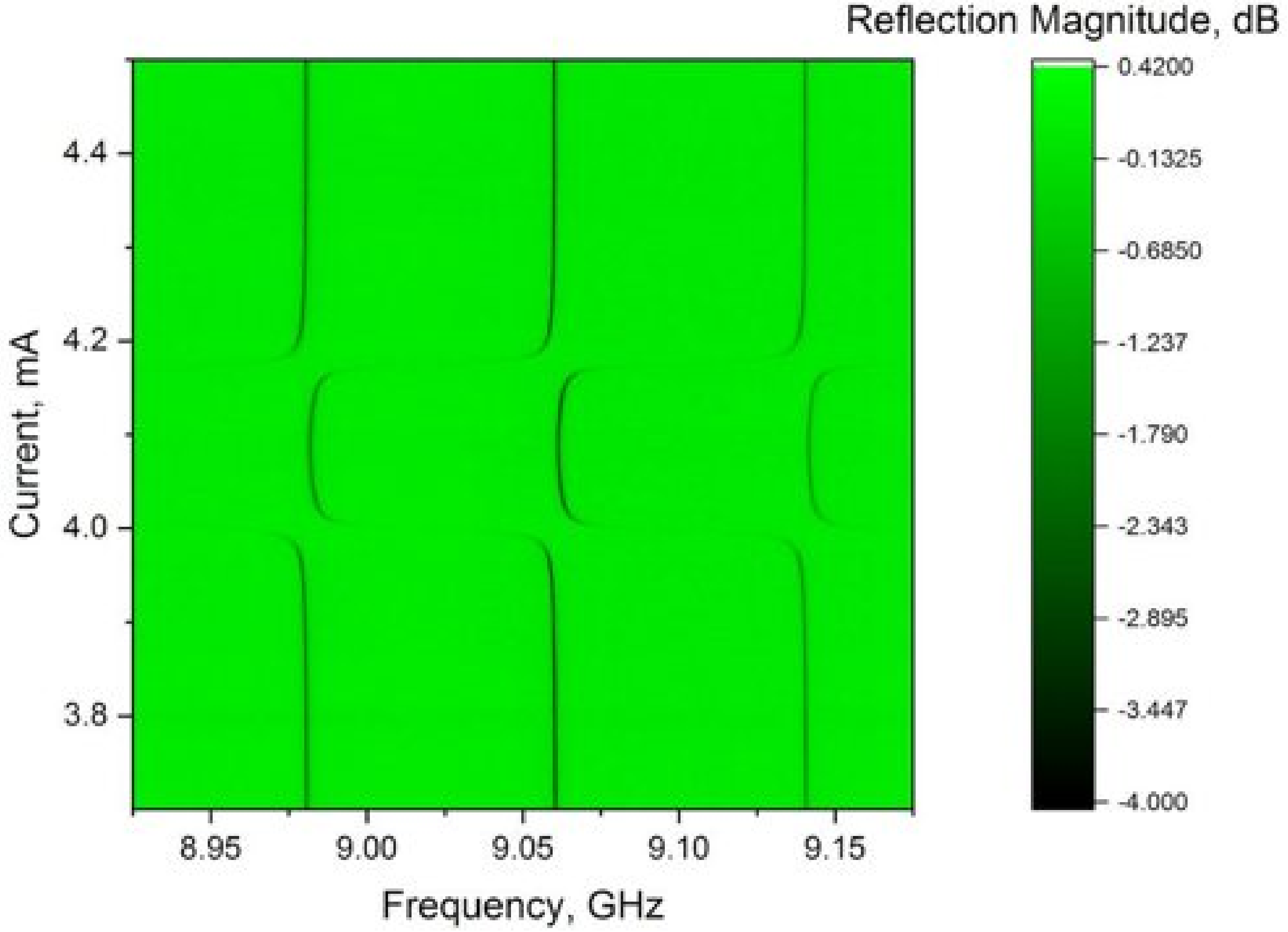
SQUID

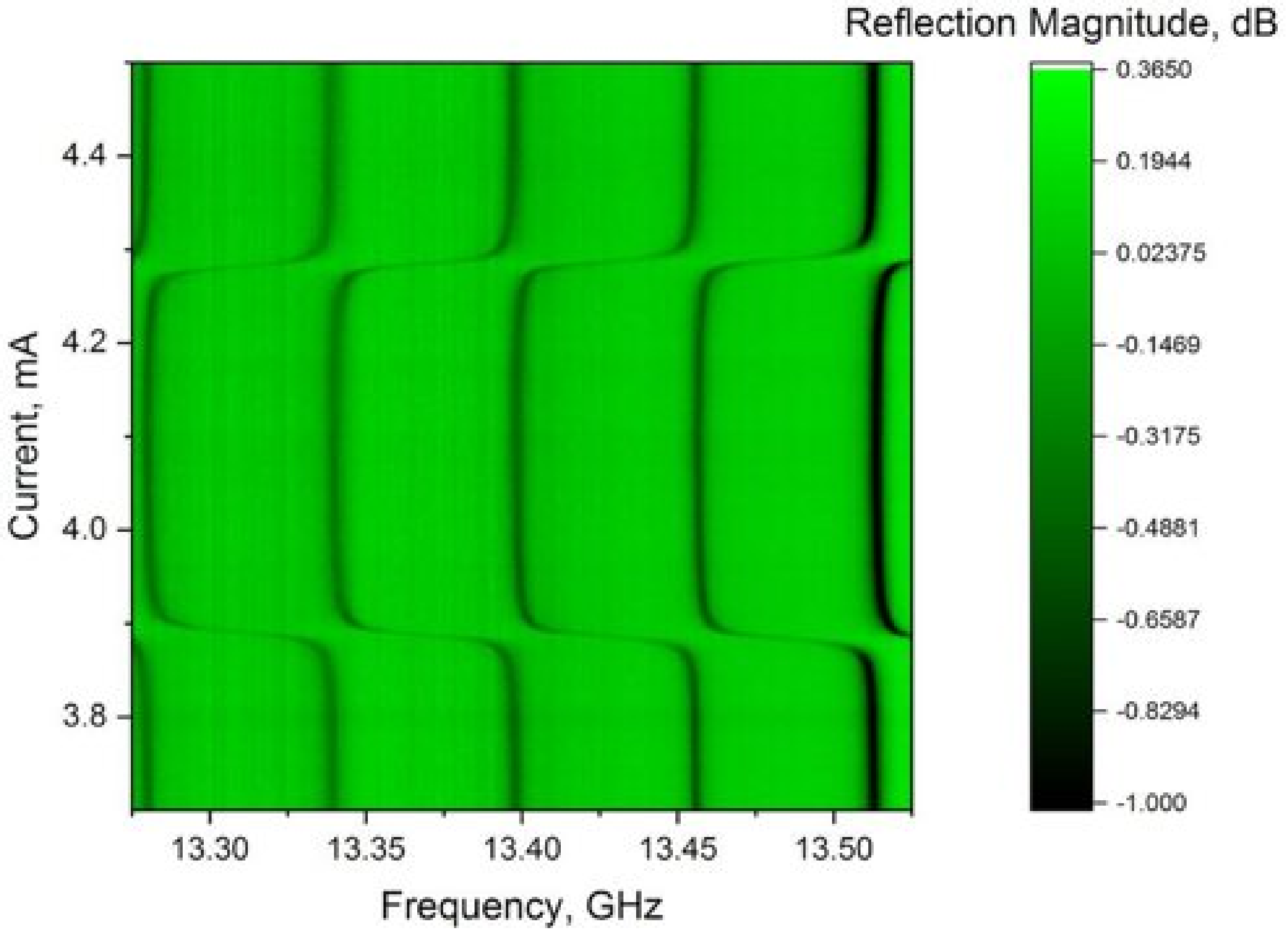
More familiar (to us) low-energy spectrum

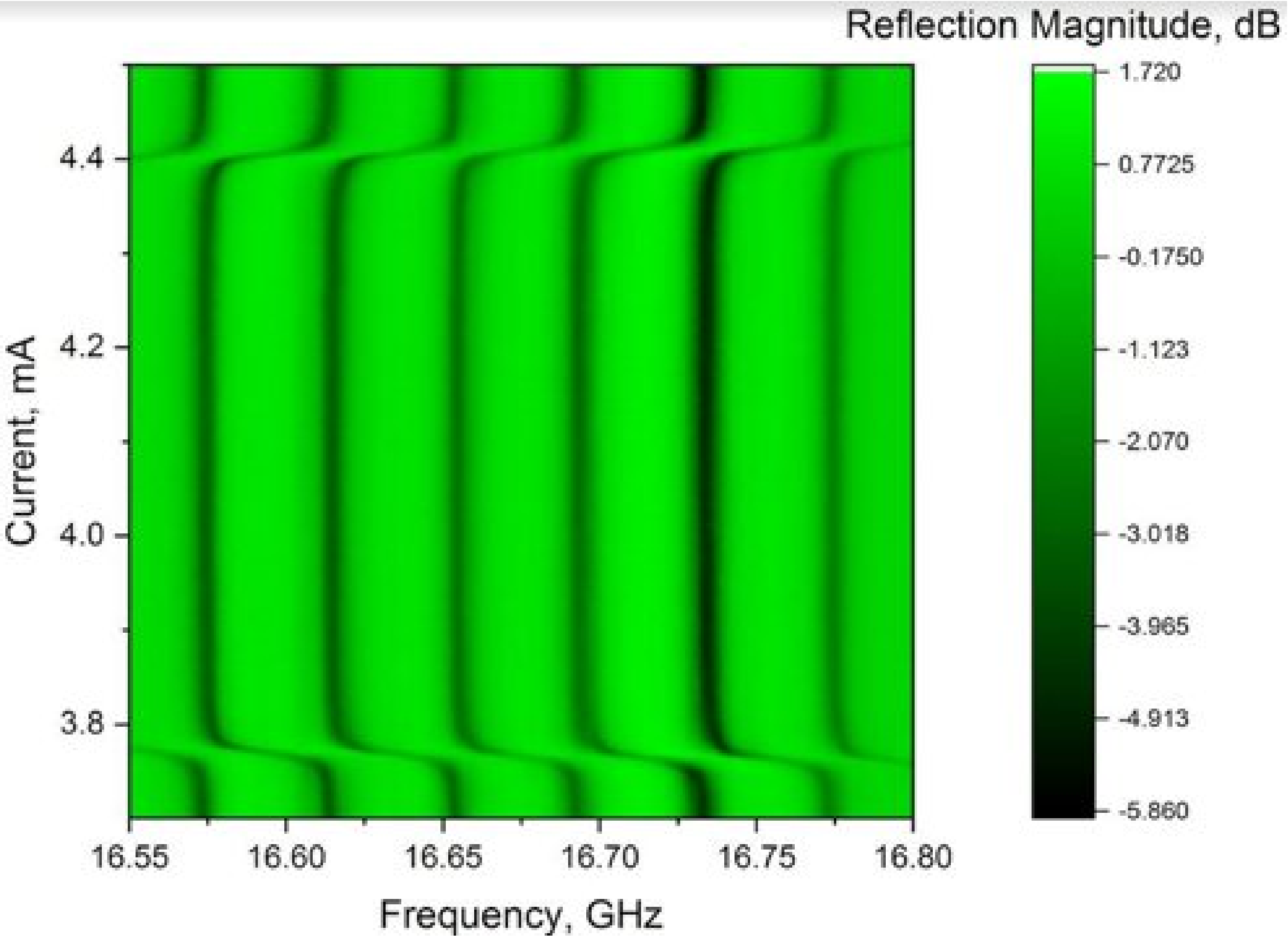
High impedance >10 kOhm —> low g
Independently measured plasma freq




Reflection Magnitude, dB
0.9000
0.1625
-0.5750
-1.313
-2.050
-2.787
-3.525
-4.263
-5.000
Current, mA
4.4
4.2
4.0
3.8
Frequency, GHz
6.80
6.85
6.90
6.95
7.00


Reflection Magnitude, dB
0.4200
-0.1325
-0.6850
-1.237
-1.790
-2.343
-2.895
-3.447
-4.000
Current, mA
4.4
4.2
4.0
3.8
Frequency, GHz
8.95
9.00
9.05
9.10
9.15

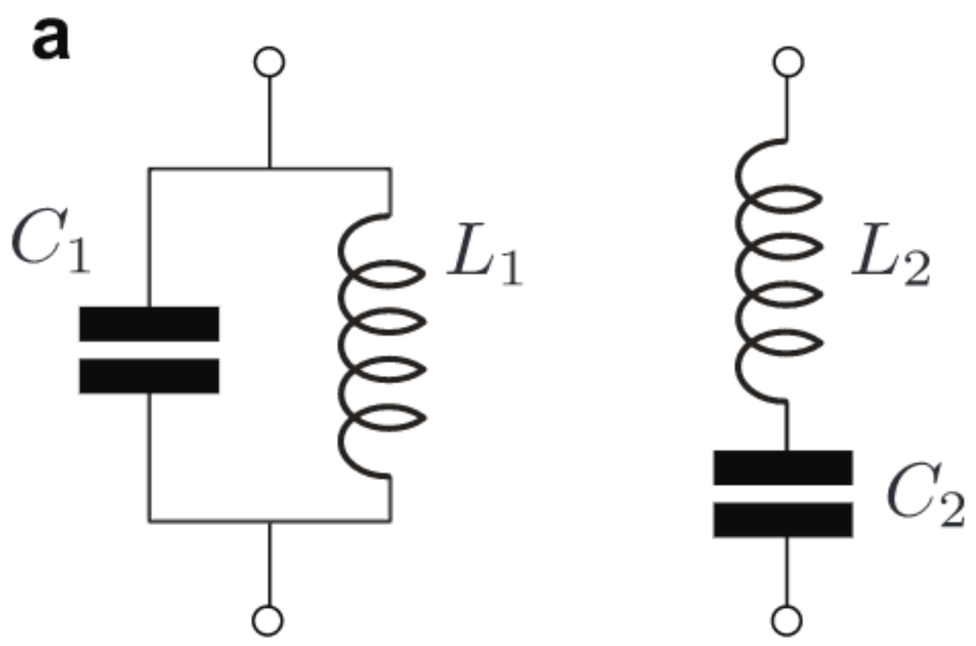

Reflection Magnitude, dB
0.3650
0.1944
0.02375
-0.1469
-0.3175
-0.4881
-0.6587
-0.8294
-1.000
Current, mA
4.4
4.2
4.0
3.8
Frequency, GHz
13.30
13.35
13.40
13.45
13.50


Reflection Magnitude, dB
1.720
0.7725
-0.1750
-1.123
-2.070
-3.018
-3.965
-4.913
-5.860
Current, mA
4.4
4.2
4.0
3.8
Frequency, GHz
16.55
16.60
16.65
16.70
16.75
16.80

# A (over)simplified circuit model

PLASMA-M

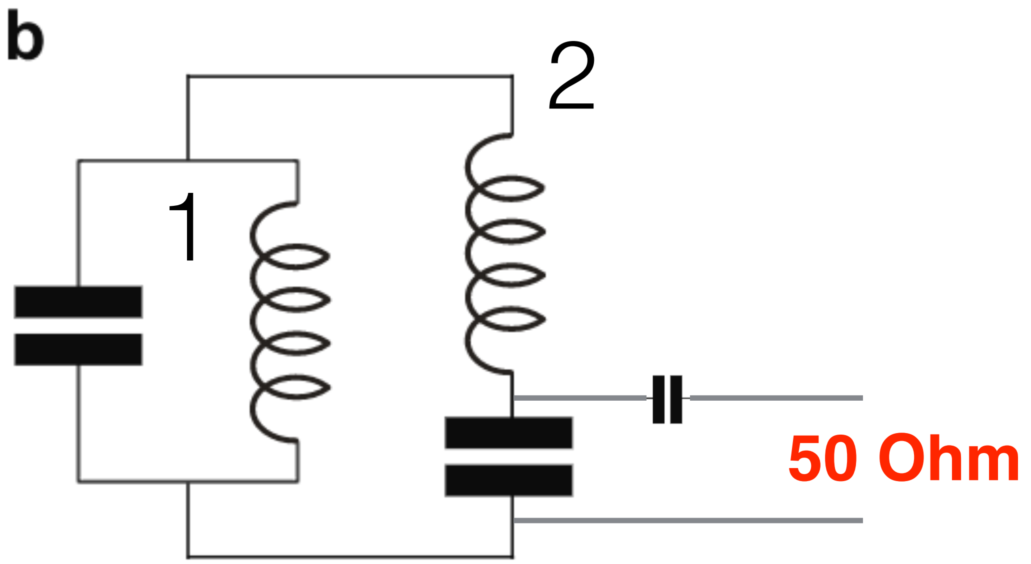
COMMON-M



$$\omega = (LC)^{-1/2}$$

$$Z = (L/C)^{1/2}$$

PLASMA-COMMON MODES COUPLING

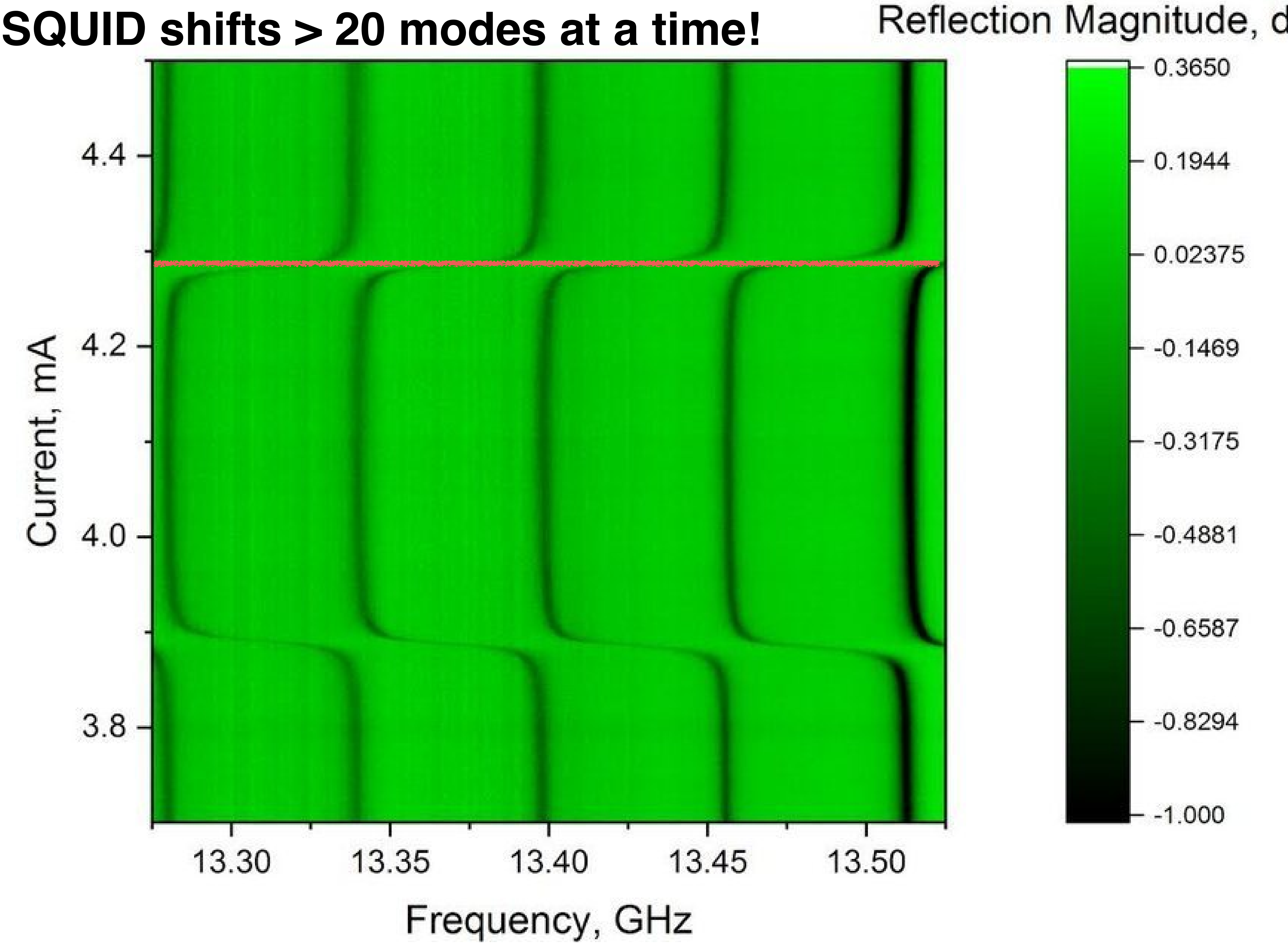


$$g_{12}/\omega = \sqrt{Z_1/Z_2}/2$$

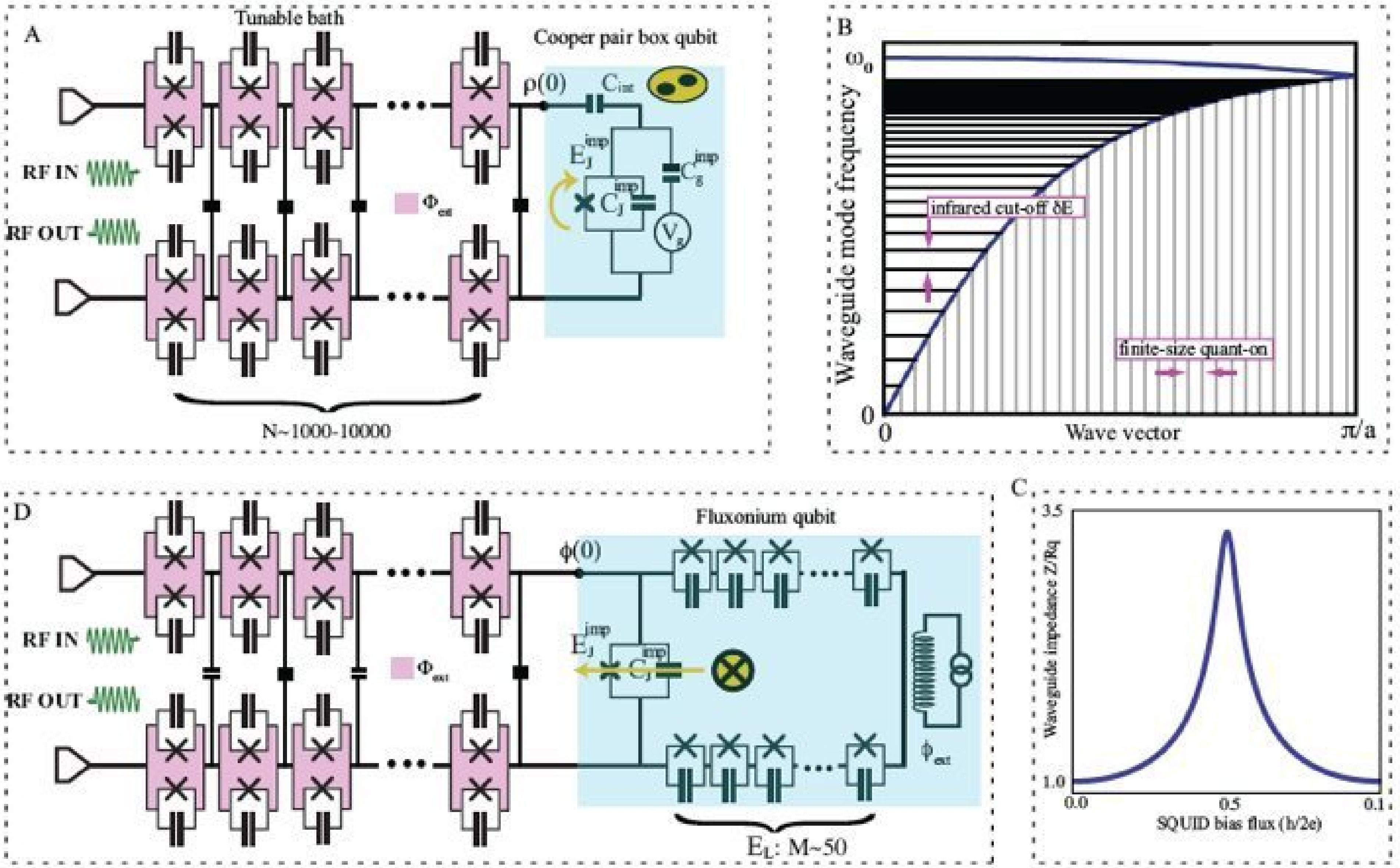
Design rule: $Z_2 >> Z_1$ to prevent mode hybridization

Vladimir E Manucharyan *et al* 2017 *J. Phys. A: Math. Theor.* **50** 294001

# SQUID shifts > 20 modes at a time!



# Simulation of Kondo impurities



Fast control knobs:
- Infrared cut-off (length)
- exchange anisotropy (impedance)
- magnetic field (charge/flux offsets)

Relevant theory:
G. Ripoll et al. (2007)
K. Le Hur et al. (2012)
M. Goldstein et al. (2012)

Relevant experiments
K. Lehnert et al. (2008)
O. Astafiev et al. (2010)
A. Weiss et al. (2015)
P. Forn Diaz et al. (2016)

# Summary

High-impedance Josephson transmission lines
are tunable Luttinger liquids with low disorder

Junctions and qubits act as impurities with arbitrary
strong coupling: Boundary sin-Gordon & Kondo

Experiment probes frequency-dependent elastic
and inelastic scattering instead of conductance

Many experiments ahead, almost no theory!