Noisy Quantum Measurements: a nuisance or fundamental physics?

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Content

- Quantum measurement: projection and weak measurements
- Quantum dynamics: Keldysh contour
- Facets of weak quantum measurements
    1. Counting statistics and quantum transport
    2. Keldysh-ordered expectations are quasiprobabilities
    3. Time-reversal symmetry breaking
    4. General non-markovian weak measurement

Classical vs. quantum measurement

Classical:
- Pointer indicates the value of a system variable
- Correlations of different variables can be measured straightforwardly

Quantum mechanical:
- Probabilistic and invasive, wavefunction collapses to the state of the measured eigenvalue (projection)
- Correlations are non-trivial: Measurement of non-commuting observables is unclear

A way out: weak measurement preserves the system state
→ Measurement of non-commuting observables becomes possible

Quantum mechanical projection postulate:
A measurement of a quantum variable $\hat{A}$ yields one of the eigenvalues (with some probability) and the state collapses to the corresponding eigenstate!

Why needed
- Quantum dynamics only describes probability amplitudes
- Prediction of experimental results need extra rule
- Correctly predicts „one-click“ results

Why questionable
- Many experiments are not projective
- Collapse of the wave function seemingly contradicts relativity
- Time duration of projection?
- What about correlations?

Correlations? **Order of operators** matters!

$$\langle a(t)b(s)\rangle \overset{?}{\to} \begin{cases} i\langle[\hat{B}(s),\hat{A}(t)]\rangle/2 \\ \langle\hat{B}(s)\hat{A}(t)\rangle \\ \langle\hat{A}(t)\hat{B}(s)\rangle \\ \langle\{\hat{B}(s),\hat{A}(t)\}\rangle/2 \end{cases}$$

Textbook (LL Vol. V):

The operators $\hat{x}(t)$ and $\hat{x}(t')$ relating to different instants do not in general commute, and the correlation function must now be defined as

$$\phi(t'-t) = \tfrac{1}{2}\overline{[\hat{x}(t)\hat{x}(t')+\hat{x}(t')\hat{x}(t)]}, \qquad (121.9)$$

Quantum optics:

photodetector measures ‚normal ordered' expectations (one click)
homodyning and heterodyning are highly specific

Von Neumann measurement: from strong to weak

Idea: couple system ($\hat{A}$) to a pointer wavefunction $\sqrt{P(x)}$

$$|\psi_i\rangle \otimes \sqrt{P(x)}$$

$$|\psi_i\rangle = \alpha_1 |A_1\rangle + \alpha_2 |A_2\rangle$$

$$\hat{U}_{int} = e^{ig\hat{p}\hat{A}}$$

$$\rightarrow \alpha_1 |A_1\rangle \sqrt{P(x+gA_1)} + \alpha_2 |A_2\rangle \sqrt{P(x+gA_2)}$$

<u>Strong measurement (large g)</u>: projective measurement on well separated pointer positions implies projection of system state

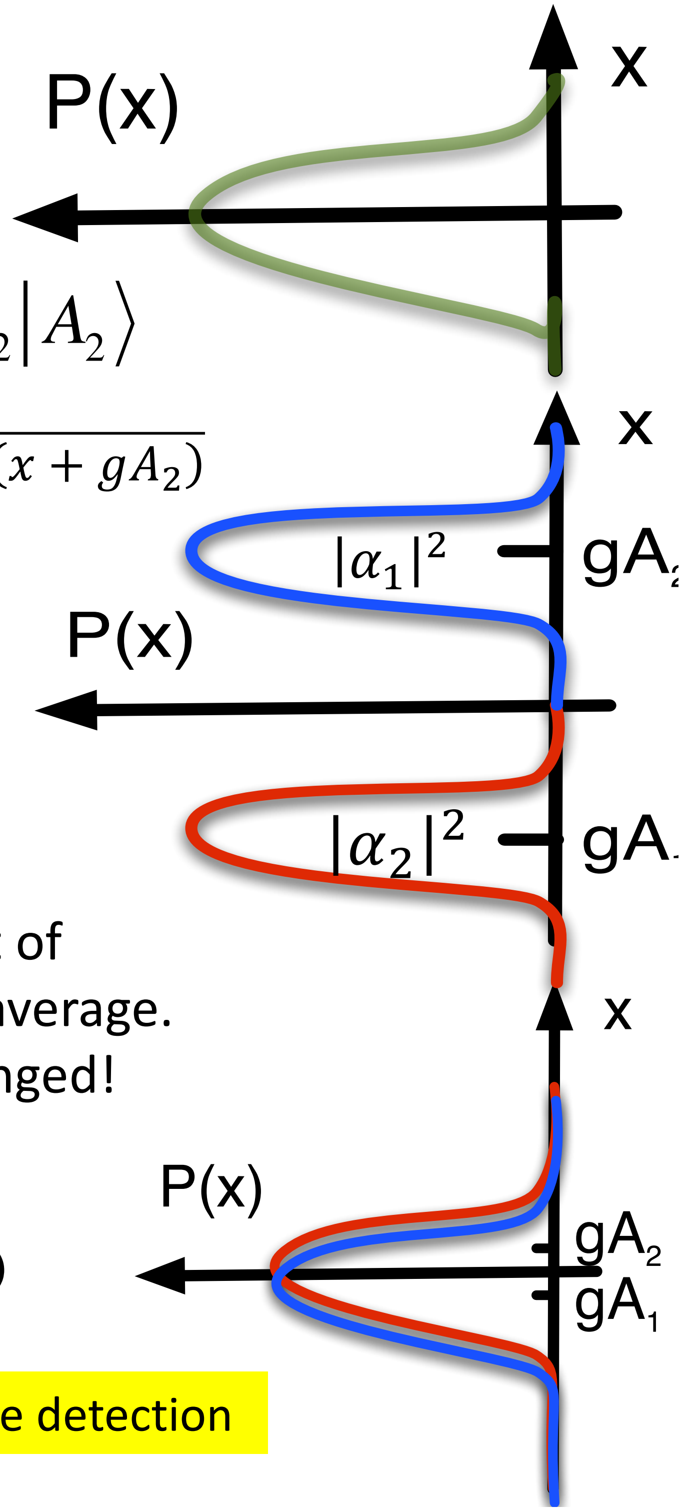
$$|\psi_f\rangle = |A_1\rangle \quad \text{or} \quad |\psi_f\rangle = |A_2\rangle$$

<u>Weak measurement (small g):</u> projective measurement of pointer state gives almost no information, but correct average. The system state in one measurement is almost unchanged!

After reading the pointer

$$|\psi_f\rangle \approx \alpha_1 |A_1\rangle + \alpha_2 |A_2\rangle + O(g^2)$$

Price to pay for non-invasiveness: large uncertainty of the detection



Quantum dynamics: time evolution of a quantum system

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle \rightarrow |\Psi(t)\rangle = U_{\rightarrow}(t)|\Psi(0)\rangle$$

Forward time-evolution

$$-i\hbar\frac{\partial}{\partial t}\langle\Psi(t)| = \langle\Psi(t)|\hat{H} \rightarrow \langle\Psi(t)| = \langle\Psi(0)|U_{\leftarrow}(t)$$

Backward time-evolution

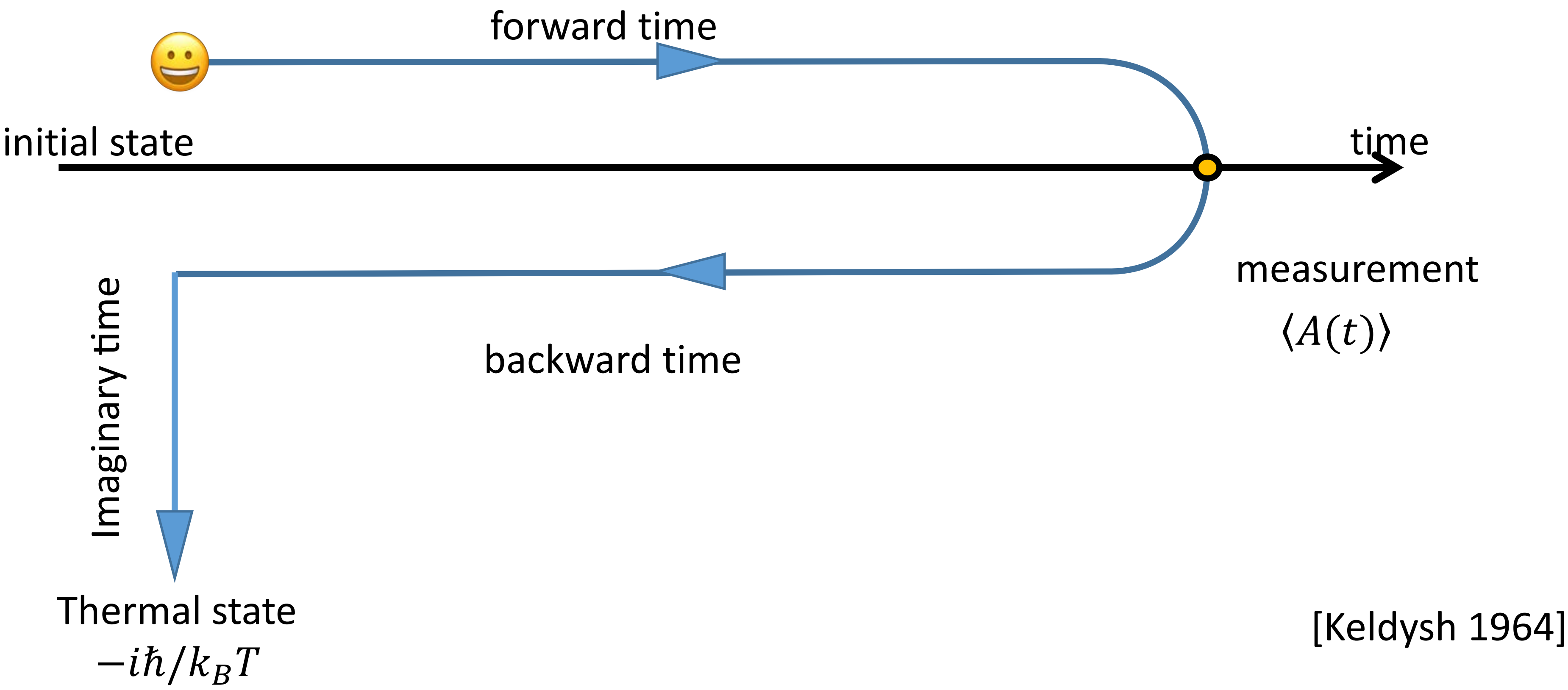
Physical expectations

$$\langle A(t)\rangle = \langle\Psi(t)|\hat{A}|\Psi(t)\rangle = \langle\Psi(0)|U_{\leftarrow}(t)\hat{A}U_{\rightarrow}(t)|\Psi(0)\rangle$$

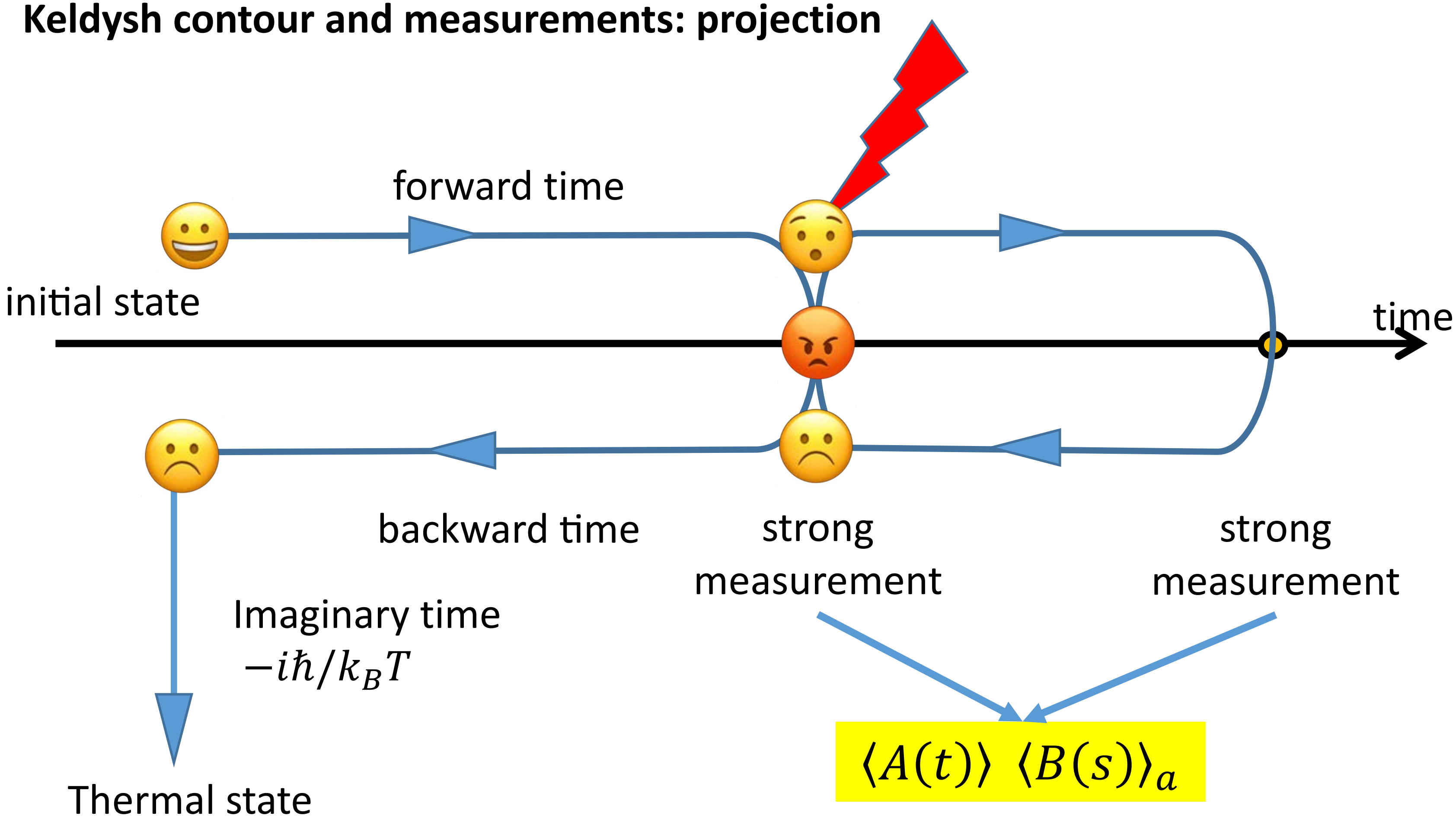
Backward and **Forward**
time-evolution

Quantum dynamics requires **Forward** and **Backward** time-evolution → **Keldysh contour**

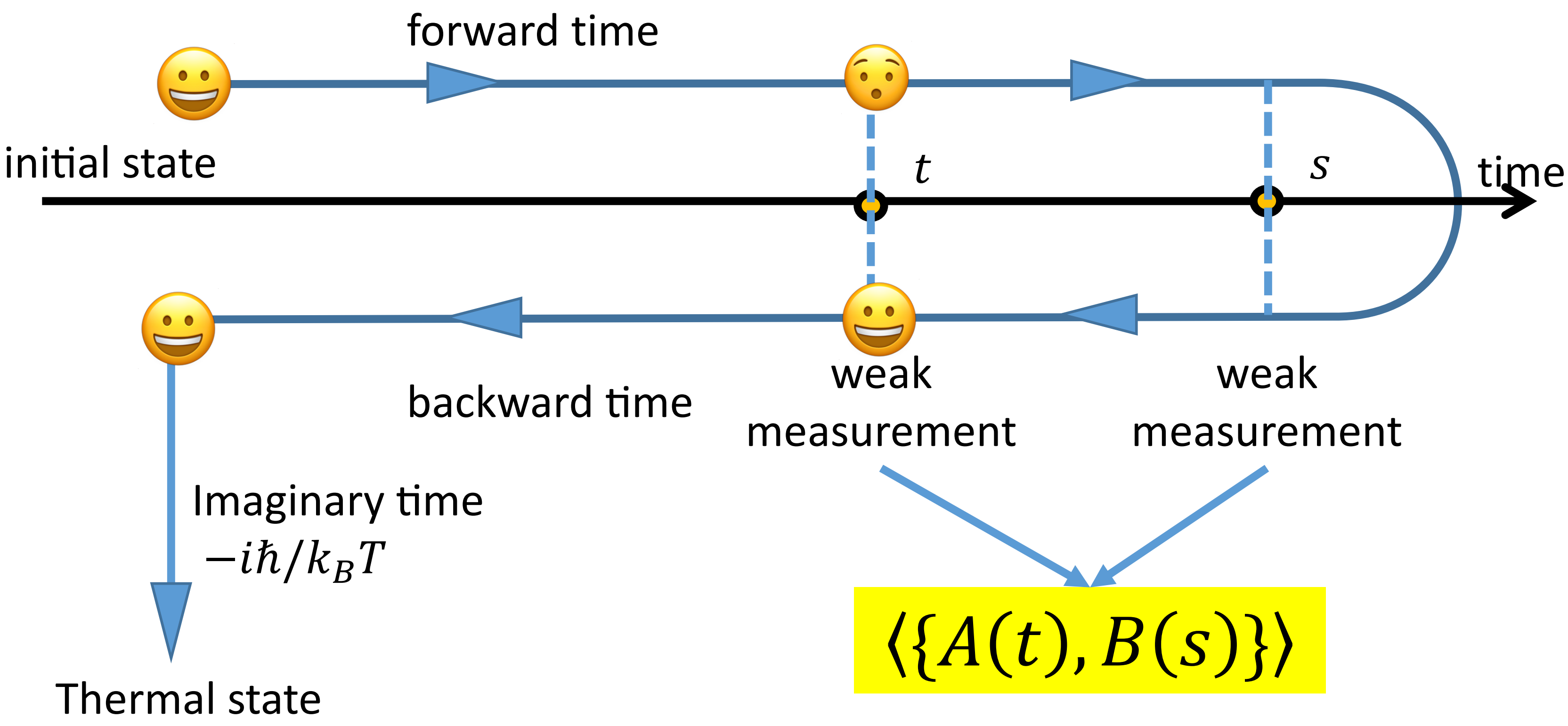
The Keldysh contour: expanding the time dimension



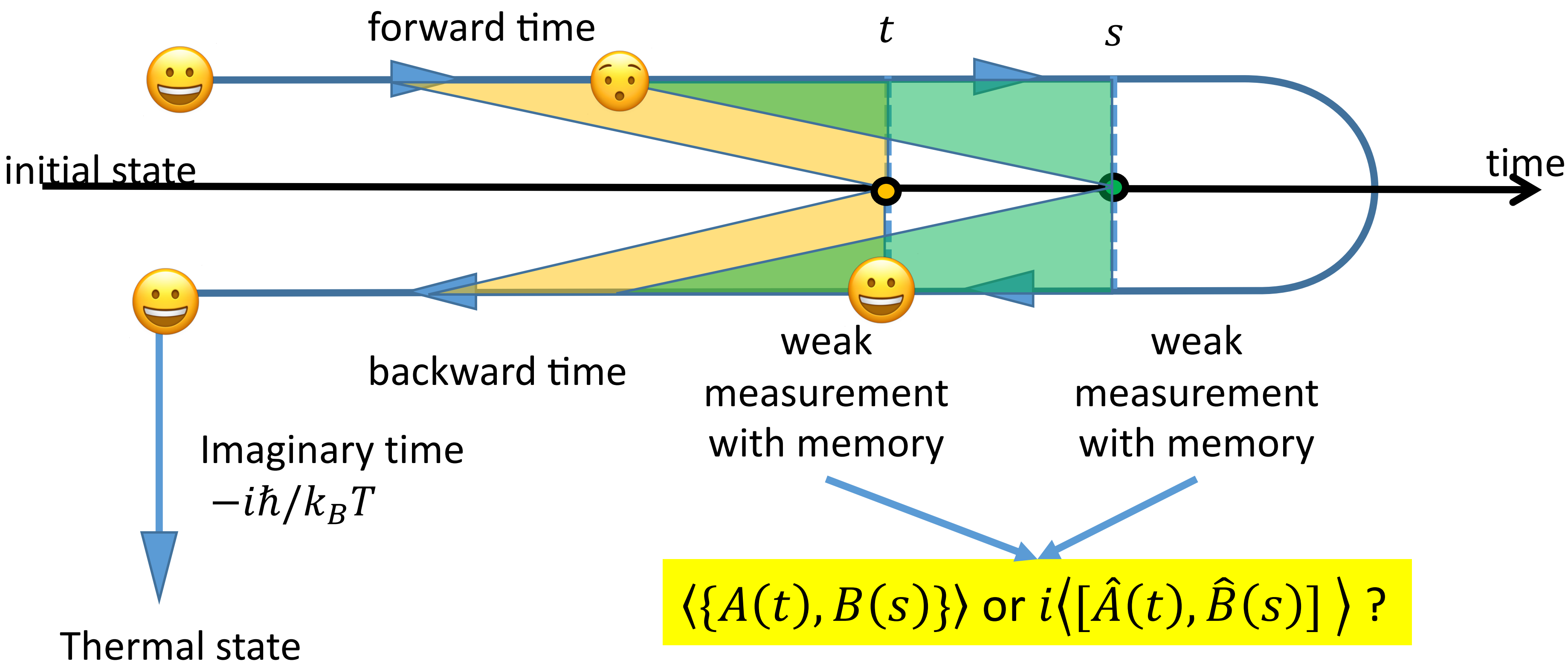
Keldysh contour and measurements: projection



Keldysh contour and measurments: weak and markovian (instantaneous)



Keldysh contour and measurements: weak and continuous



Overview of 1)

- Full Counting Statistics (FCS) and Cumulant Generating Function (CGF)
- Keldysh-ordered generating function, interpretation problem
- Examples: Levitov-Lesovik formula, Andreev scattering
- FCS of a quantum point contact under arbitrary time-dependent voltage
- Generalization of Keldysh generating functional to finite frequency and Positive Operator Valued Measure (POVM) formulation of finite-frequency FCS

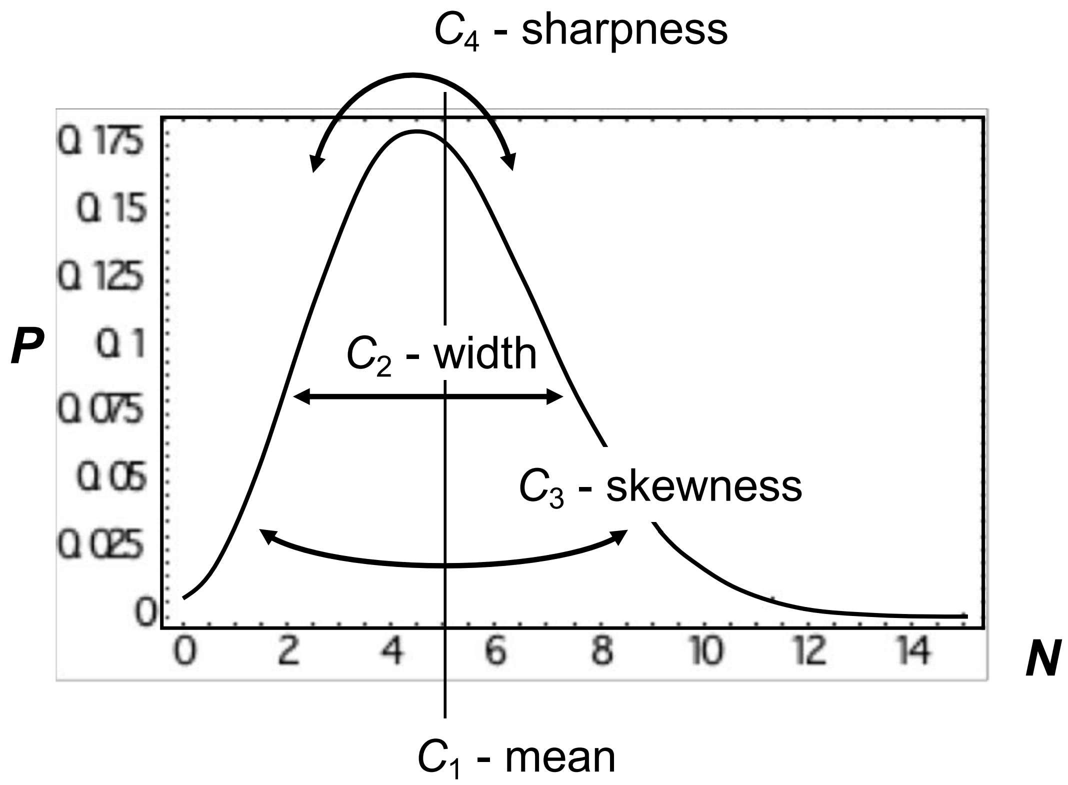
Literature:

M. Vanevic, Yu. V. Nazarov, and W. Belzig, Phys. Rev. Lett. 99, 076601 (2007)

A. Bednorz and W. Belzig, Phys. Rev. Lett. 101, 206803 (2008)

Full Counting Statistics:

$$P_{t_0}(N)$$ probability that a total charge Ne is transferred in given time t_0



Classical distribution is characterized by **cumulants:**

$$
\begin{aligned}
C_1 &= \langle N \rangle = \sum_N N P_{t_0}(N) \\
C_2 &= \langle (N-\bar{N})^2 \rangle \\
C_3 &= \langle (N-\bar{N})^3 \rangle \\
C_4 &= \langle (N-\bar{N})^4 \rangle - C_2^2
\end{aligned}
$$

Cumulant generating function (CGF):

(equivalent to distribution)

$$S_{(t_0)}(\chi)$$

$$e^{S_{t_0}(\chi)} = \langle e^{i\chi N} \rangle_{t_0} = \sum_N P_{t_0}(N) e^{i\chi N}$$

Cumulants follow from derivatives (dropping dependence on t_0)

$$S(\chi) = \sum_k C_k \frac{\chi^k}{k!}$$

$$C_k = \frac{1}{i^k} \frac{\partial^k S(\chi)}{\partial \chi^k}\bigg|_{\chi=0}$$

Obtaining the distribution

$$P(N) = \int_0^{2\pi} \frac{d\chi}{2\pi} e^{S(\chi) - iN\chi}$$

How to calculate the CGF quantum mechanically?
Quantum mechanical current detection has to account for non-commuting current operators!

$$\left[\hat{I}(t), \hat{I}(t')\right] \neq 0$$

$$e^{S(\chi)} = \mathrm{Tr}\left[\hat{\rho}\tilde{T} e^{i\frac{\chi}{2}\int_0^{t_0}\hat{I}(t)dt} \mathcal{T} e^{i\frac{\chi}{2}\int_0^{t_0}\hat{I}(t)dt}\right]$$

Microscopic justification: time evolution of **ideal current detector** and **projective measurement** (Projection can be problematic for superconductors, due to charge-phase uncertainty)

General result for the CGF of a **quantum point contact** can be obtained using Keldysh technique

$$S(\chi) = Tr\ln\left[1 + \frac{T}{4}\left(\{\check{G}_1, \check{G}_2\} - 2\right)\right]$$

Belzig, Nazarov, PRL 01

Important difference
to classical definition
(see also Levitov, Lesovik 93)

$$e^{S_{cl}(\chi)} = \left\langle e^{i\chi\int_0^{t_0}\hat{I}(t)dt}\right\rangle$$

Relation to current correlators:

Average current: $C_1 = t_0 \left\langle \hat{I} \right\rangle$

Zero frequency noise:

$$C_2 = \int_0^{t_0} dt_1 \int_0^{t_0} dt_2 \left\langle \delta\hat{I}(t_1)\delta\hat{I}(t_2) \right\rangle = \frac{t_0}{2} \int_{-\infty}^{\infty} dt \left\langle \left\{ \delta\hat{I}(t), \delta\hat{I}(0) \right\} \right\rangle$$

Zero frequency third cumulant:

$$C_3 \neq C_3^{cl} = \int_0^{t_0} dt_1 \int_0^{t_0} dt_2 \int_0^{t_0} dt_3 \left\langle \delta\hat{I}(t_1)\delta\hat{I}(t_2)\delta\hat{I}(t_3) \right\rangle$$

E.g. tunnel junction at zero temperature:

$$C_3 = C_2 = C_1 \qquad\qquad C_2^{cl} = C_1^{cl} \qquad \boxed{C_3^{cl} = 0}$$

Poisson process | Gaussian process

Difference for finite times: Chtchelkatchev&Lesovik (2006)

The interpretation problem:

Quantum CGF predicts outcomes of measurements of charge detector
But: what does the CGF tell us about the transport process?

Possible tools:

CGFs of independent processes are independent

$$S(\chi) = S_1(\chi) + S_2(\chi)$$

CGF of a known distribution, e.g. multinomial or Poisson

$$S(\chi) = \ln\left[1 + \sum_n p_n \left(e^{i\chi n} - 1\right)\right] \quad \text{or} \quad S(\chi) = \sum_n \bar{N}_n \left(e^{i\chi n} - 1\right)$$

Obtaining P(N) by Fourier transformation

$$P(N) = \int_0^{2\pi} \frac{d\chi}{2\pi} e^{S(\chi) - iN\chi} \longrightarrow \text{usually intransparent}$$

Yu. V. Nazarov:
“The calculation of the FCS is often ‘straightforward’, but the interpretation is a nightmare!”

Example 1: Interpretation of Levitov-Lesovik formula

Voltage biased quantum point contact with transmission T between Fermi leads

$$S(\chi) = \frac{t_0}{e}\int \frac{dE}{2\pi} \ln\left[1 + \underbrace{T f_L(1-f_R)}_{p_1(E)}\left(e^{i\chi}-1\right) + \underbrace{T f_R(1-f_L)}_{p_{-1}(E)}\left(e^{-i\chi}-1\right)\right]$$

Trinomial distribution at each energy

$$f_{L/R} = \frac{1}{e^{(E-eV_{L/R})/k_B T_e}+1}$$

Zero temperature limit $M = \frac{t_0 eV}{2\pi}$

$$S(\chi) = M \ln\left[1 + T\left(e^{i\chi}-1\right)\right]$$

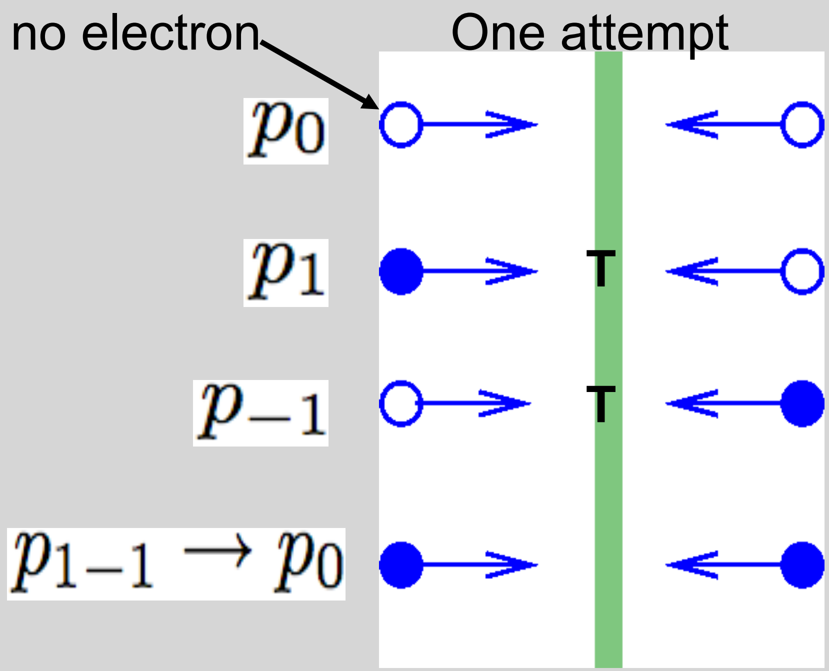
Binomial distribution

Landauer formula

$$C_1 = MT = t_0 \frac{e^2}{2\pi} TV$$

Quantum shot noise suppression

$$C_2 = MT(1-T) = C_1(1-T)$$



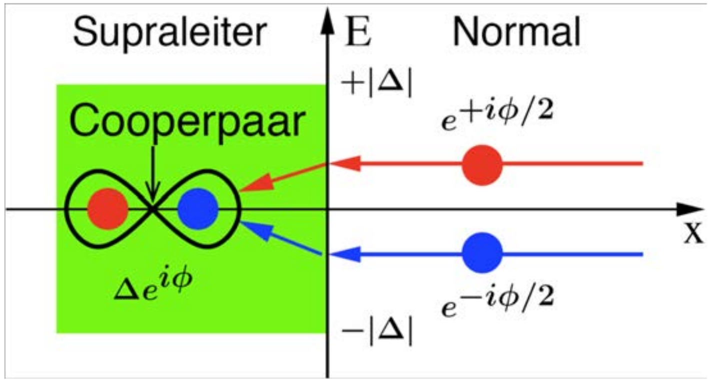
Levitov and Lesovik, JETPL 93; Lee, Levitov and Lesovik, JMP 96

Example 2: Interpretation of Andreev scattering (normal/super contact)

$$S(\chi) = t_0 \int \frac{dE}{2\pi} \ln\left[1 + p_2\left(e^{i2\chi}-1\right) + p_1\left(e^{i\chi}-1\right) + p_{-1}\left(e^{-i\chi}-1\right) + p_{-2}\left(e^{-i2\chi}-1\right)\right]$$

Muzykantskii and Khmelnitskii, PRB 95

Multinomial distribution, p_n corresponds to elementary event of charge ne, signaled by counting factors $\left(e^{in\chi}-1\right)$



Andreev reflection $k_B T_e \ll eV \ll \Delta$

$$S(\chi) = M \ln\left[1 + R_A\left(e^{i2\chi}-1\right)\right]$$

Binomial distribution of 2e-transfers

Andreev reflection probability $R_A = \frac{T^2}{(2-T)^2}$

Binomial form, despite Cooper pairs are not fermions

Example 3: Multiple Andreev Reflections (MAR)

Mechanism of charge transfer between two superconductors:
at subgap voltages charges are transferred by subsequent Andreev reflections with threshold voltages (k integer)

$$eV = \frac{2\Delta}{k}$$

Leads to subharmonic gap structure in the current-voltage characteristics

Elementary events:
Multiple charges or Cooper pairs+electron?

Cuevas, Belzig PRL 03

FCS of MAR

at zero temperature

$$S(\chi) \quad = \quad t_0 \int_0^{eV} \frac{dE}{\pi} \ln \left[1 + \sum_k P_k(E, eV) \left(e^{ik\chi} - 1 \right) \right]$$

Cumulant generating function reveals multiple charge transfers
(gives expressions for energy resolved probabilities)

Low transparency:

Cuevas, Belzig PRL 03
Cuevas, Belzig PRB 04

Time-Dependent Counting Statistics of a Quantum Point Contact
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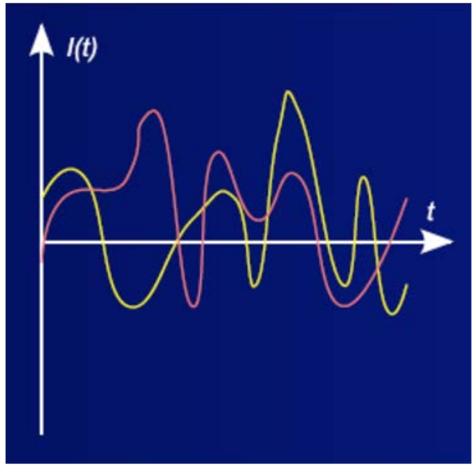
High-Frequency Noise and the Probabilistic Interpretation of FCS

Generalization of FCS to Time-Dependent Counting:



“Probability” density functional for given current profile I(t):

$$\varrho[I] = \int \mathcal{D}\chi \; e^{S[\chi] - \frac{i}{e}\int dt\, \chi(t) I(t)}$$

Inverse transformation

$$e^{S[\chi]} = \int \mathcal{D}I \; \varrho[I] e^{\frac{i}{e}\int dt\, \chi(t) I(t)} \equiv \left\langle e^{\frac{i}{e}\int dt\, \chi(t) I(t)} \right\rangle_{\varrho}$$

classical average

Keldysh ordered!

Quantum definition of CGF for time-independent FCS

$$e^{S(\chi)} = \mathrm{Tr}\left[\hat{\rho}\tilde{\mathcal{T}} e^{i\frac{\chi}{2}\int_0^{t_0} \hat{I}(t)dt} \mathcal{T} e^{i\frac{\chi}{2}\int_0^{t_0} \hat{I}(t)dt}\right]$$

Generalization of standard Keldysh functional **to time dependent counting**

$$e^{S[\chi]} = \mathrm{Tr}\left[\hat{\rho}\tilde{\mathcal{T}} e^{\frac{i}{2e}\int dt \chi(t)\hat{I}(t)} \mathcal{T} e^{\frac{i}{2e}\int dt \chi(t)\hat{I}(t)}\right]$$

[see also e.g. Nazarov and Kindermann, EPJB 03; Golubev, Zaikin, Galaktionov, PRB 06]

Can we interpret this as probability density generating functional?

No! Analogous to Wigner function we can have **negative probabilities**

Problem: current operators at different times do **not commute**
The current cannot be measured at all times, but only up to some uncertainty

Handling non-projective (weak) measurements:

Orthogonal measurements

$$\{\hat{P}_A = |A\rangle\langle A|\} \quad \sum_A \hat{P}_A = \hat{1}$$

$$\hat{P}_A \hat{P}_B = \hat{P}_A \delta_{A,B}$$

Probability to find A

$$p_A = \mathrm{Tr}\hat{\rho}\hat{P}_A$$

State after measurement

$$\hat{\rho}_A = \hat{P}_A \hat{\rho} \hat{P}_A / p_A$$

Non-projective measurements:

Kraus operators $\left\{\hat{K}_A\right\}$ $\hat{F}_A = \hat{K}_A^\dagger \hat{K}_A$

$$\sum_A \hat{K}_A \hat{K}_A^\dagger = \hat{1}$$

$$\hat{F}_A \hat{F}_B \neq \hat{F}_A \delta_{A,B}$$

Positive
Operator
Valued
Measure

$$p_A = \mathrm{Tr}\hat{\rho}\hat{K}_A^\dagger \hat{K}_A$$

$$\hat{\rho}_A = \hat{K}_A \hat{\rho} \hat{K}_A^\dagger / p_A$$

Neumarks Theorem: Every POVM corresponds to a projective measurement in some extended Hilbert space

See e.g. Milburn & Wiseman, Quantum Measurement and Control (Cambridge, 2009)

Proposed solution: weak measurement a la **POVM**

Kraus operator (instead of projection operator)

$$\hat{K}[I] = \int \mathcal{D}\varphi \, T e^{\int dt \frac{i}{e}\varphi(t)(\hat{I}(t) - I(t)) - \frac{\varphi^2(t)}{\tau}}$$

Causality

Noise of the detector
+ uncertainty

Positive operator valued probability measure (=projection in extended space)

Neumarks theorem

Positive definite probability distribution:

$$\rho[I] = \mathrm{Tr}\left[\hat{\rho}\hat{K}^\dagger[I]\hat{K}[I]\right]$$

A. Bednorz and W. Belzig, Phys. Rev. Lett. **101**, 206803 (2008)

Final result for current generating functional

Generalized Keldysh functional

$\varphi(\varphi^\dagger) = \phi \pm \chi/2$

$$e^{S[\chi,\phi]} = \mathrm{Tr}\left[\hat{\rho}\tilde{\mathcal{T}} e^{\frac{i}{e}\int dt(\chi(t)/2+\phi(t))\hat{I}(t)} \mathcal{T} e^{\frac{i}{e}\int dt(\chi(t)/2-\phi(t))\hat{I}(t)}\right]$$

Current generating functional with additional backaction and noise due to detector

$$e^{S[\chi]} = \int \mathcal{D}\phi e^{S[\chi,\phi]} e^{-\int dt(2\phi^2(t)+\chi^2(t)/2)/\tau}$$

Gaussian noise of the detector

Backaction of the detector (partial projection)

Limiting cases:

$\tau \longrightarrow \infty$ full projection → Strong backaction

$\tau \longrightarrow 0$ large detector noise → Weak measurement

$\varrho[I] = \int \mathcal{D}\chi\, e^{-i\int dt \chi(t) I(t)} \Phi[\chi]$ with $\Phi[\chi] = e^{S[\chi,0]+\int dt \chi^2(t)/2\tau}$

$\tau \to 0$: Large Gaussian noise substracted!

Generalized Wigner functional

WB and Y. V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001)
Phys. Rev. Lett. **87**, 067006 (2001)
A. Bednorz and WB, PRL (2008,2010)

The generating function of a **markovian** quantum measurement is Keldysh-ordered:

$$\Phi[\chi] = e^{S[\chi,0]-S_{det}} = \langle \tilde{\mathcal{T}}\left[e^{i\int dt\chi(t)\hat{A}(t)}\right]\mathcal{T}\left[e^{i\int dt\chi(t)\hat{A}(t)}\right]\rangle$$

Quasiprobability density generating functional!
No! Analogous to Wigner function we can have **negative probabilities**

The generating function of a **non-markovian** quantum measurement is ...
... (even) more complicated

The **answer** to the question of operator order: $C \sim \delta^2\Phi[\chi]/\delta\chi(t)\delta\chi(s)$

Markovian: $\langle a(t)a(s)\rangle \rightarrow \langle\{\hat{A}(s),\hat{A}(t)\}\rangle/2$

Higher order Markovian: $\langle abc\rangle \rightarrow \frac{1}{4}\left\langle\left\{\hat{A},\{\hat{B},\hat{C}\}\right\}\right\rangle$

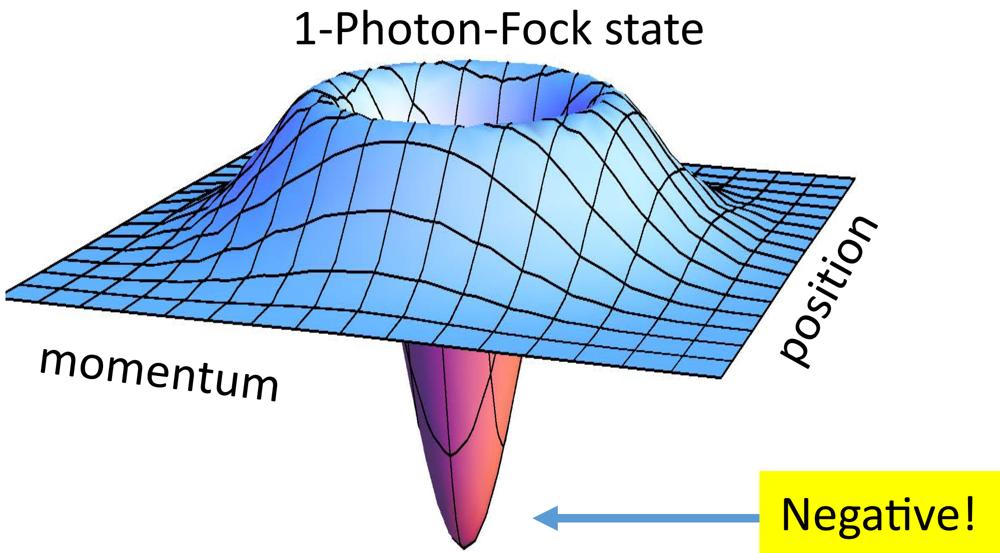
Non-Markovian: $\langle a(t)a(s)\rangle \rightarrow g\otimes\langle\{\hat{A}(t),\hat{A}(s)\}\rangle + f\otimes\langle[\hat{A}(t),\hat{A}(s)]\rangle$

g, f depend on the detector, but arbitrary ordering possible (→ engineering)

2) Keldysh-ordered expectations are quasiprobabilities

Quasiprobability?

Example:
Wigner-function $W(x, p)$
= Probability for x **and** p



Cannot be measured directly, but through a **noisy and weak measurement**

Signatures of negativity (=non-classicality)?
Violation of classical inequalities, e.g. Bell, CHSH, Leggett-Garg, weak values....

Bednorz and WB, Phys. Rev. Lett. 2008

Wigner quasiprobability:

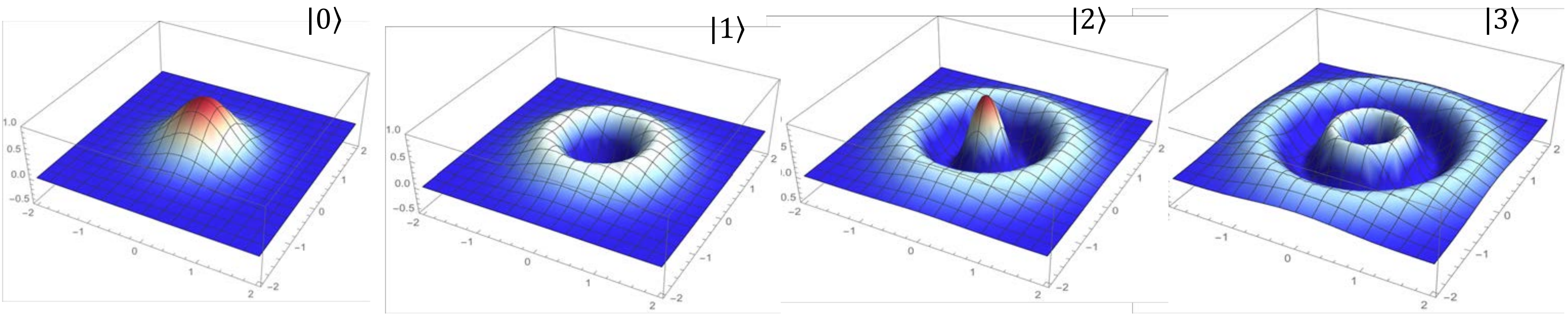
Attempt to define a **phase space** distribution of **incompatible** observables.

Wigner function $$W(x,p) = \frac{1}{2\pi}\int dy e^{\frac{ipy}{\hbar}} \left\langle x - \frac{y}{2} \middle| x + \frac{y}{2} \right\rangle = W(\alpha = \frac{x+ip}{\sqrt{2}})$$

Alternative definition:
(via generating function)

$$W(\alpha) = \frac{1}{2\pi}\int d^2\eta e^{\eta^*\alpha - \eta\alpha^*} \underbrace{\langle e^{\eta a + \eta^* a^+} \rangle}_{\Phi_W(\eta)}$$

Examples: Energy eigenstate of a harmonic oscillator („**Fock states**“ of photons)



Weak positivity in the markovian scheme

Weak markovian measurement scheme: [Bednorz & Belzig, PRB 2011

$$C_{ij} = \langle A_i A_j \rangle = \frac{1}{2}\langle \{\hat{A}_i, \hat{A}_j\} \rangle$$ = positive definite correlation matrix

C can be simulated by classical probability distribution, e.g.

$$p(A_1, A_2, \ldots) \sim e^{-\sum_{ij} A_i C_{ij}^{-1} A_j / 2} \geq 0$$

With symmetrized **second order** correlation functions a violation of classical inequalities is impossible → the corresponding quasiprobability is **weakly positive**

Note: does not assume dichotomy, corresponding e.g. to $\left\langle \left(A^2 - 1\right)^2 \right\rangle = 0$

Possible inequality → Cauchy-Bunyakowski-Schwarz (CBS) inequality

$$\langle X^2\rangle\langle Y^2\rangle \geq \langle XY\rangle^2$$

→ Fullfilled for all **positive** probabilities $P(X,Y)$

Weak positivity of the Wignerfunction:
A violation of an inequality by the Wigner function **has** to invoke **4th-order correlations**, since all 2nd-order correlators of the Wigner function can be explained classically
[A. Bednorz and WB, Phys Rev A **83**, 052113 (2011)]

Test of CBS with Wigner functional for current fluctuations

Current operator in frequency space: $\hat{I}_\omega = \int dt e^{i\omega t}\, \hat{I}(t)$

We choose: $\hat{X} = \int_{\Delta_X} d\omega \delta\hat{I}_\omega \delta\hat{I}_{-\omega}$ and $\hat{Y} = \int_{\Delta_Y} d\omega \delta\hat{I}_\omega \delta\hat{I}_{-\omega}$

measurement bandwidth $\Delta_{X/Y}$ centered at $\omega_{X/Y}$

$$\langle XY\rangle \sim \langle \delta\hat{I}_\omega, \delta\hat{I}_{-\omega}, \delta\hat{I}_\omega \delta\hat{I}_{-\omega}\rangle$$

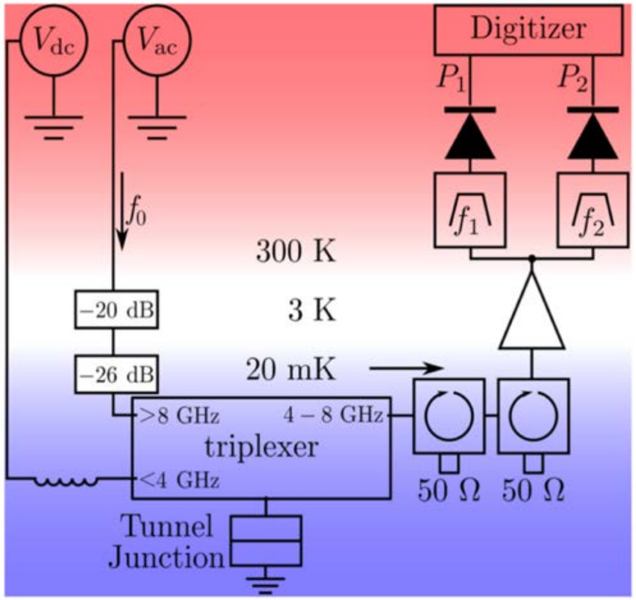
$$\langle X^2\rangle = \left\langle \left(\delta\hat{I}_\omega \delta\hat{I}_{-\omega}\right)^2\right\rangle + \langle \delta\hat{I}_\omega \delta\hat{I}_{-\omega}\rangle^2$$

2nd and 4th-order correlators from tunnel Hamiltonian

$$\langle X^2\rangle\langle Y^2\rangle \geq \langle XY\rangle^2$$

Violation would be a proof of negativity of Wigner functional!

Typical experimental setup

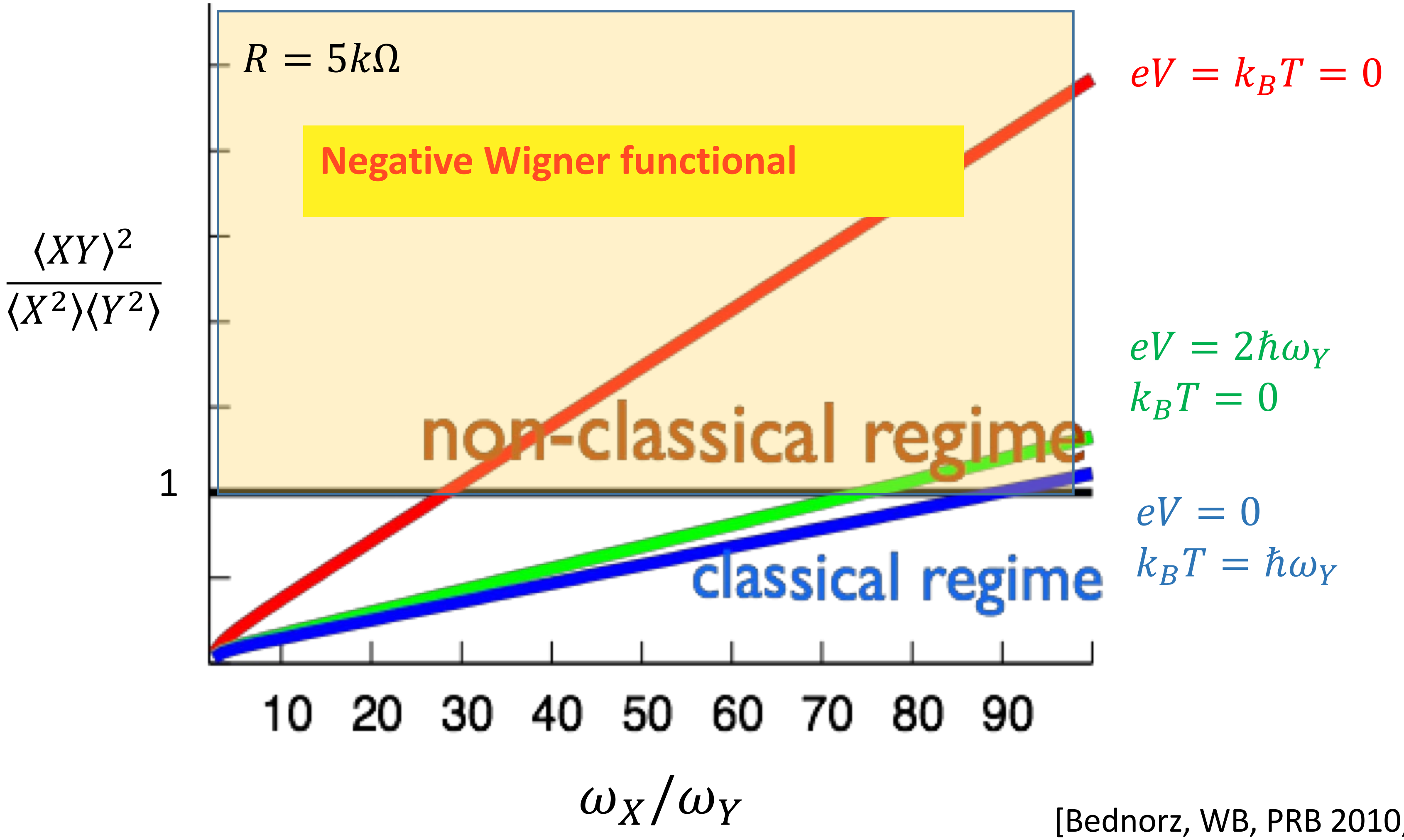


Forgues, Lupien, Reulet, PRL (2014)
See also
Zakka-Bajjani et al. PRL (2010)

Bednorz and WB,
Phys. Rev. Lett. **105**, (2010)
Phys. Rev. B **81**, 125112 (2010)

Violation of CBS for a tunnel junction

Maximally extended non-overlapping frequency intervals $\omega_X \approx \Delta_X, \omega_Y \approx \Delta_Y$



[Bednorz, WB, PRB 2010, PRL 2010]

Violation: Quantum many-body entanglement of electrons in different dynamical modes

E.g. nonequlibrium many-body wave function, Vanevic, Gabelli, Belzig, Reulet, PRB 2016

3) Time-reversal symmetry breaking

Does the observation of a system in thermal equilibrium show time-reversal symmetry (T)?

Measurement	Classical	Quantum
strong (invasive)	T is broken (order of disturbances influences the dynamics)	T is broken (order of projections influences the state)
weak (non invasive)	T is observed (measurement is completely independent of the dynamics)	?

Bednorz, Franke, WB, New J. Phys. (2013)

Time-resolved weak measurements

Quantum prediction for three measurements?

$A \to B \to C$ → $\langle\{A,\{B,C\}\}\rangle$

$\neq$

Opposite order:

$C \to B \to A$ → $\langle\{C,\{B,A\}\}\rangle$

Three point correlator for $t', t > 0$ (e.g. thermal equilibrium)

$$\langle\{A,\{A(t),A(t+t')\}\}\rangle \neq \langle\{A,\{A(t'),A(t+t')\}\}\rangle$$

time-reversal (and shift by $t+t'$)

Classical expectation is not matched:
A quantum system observed weakly in equilibrium
seemingly breaks time-reversal symmetry

Bednorz, Franke, WB, New J. Phys. (2013)

Experimental confirmation that time-ordering matters in third order weak measurements

Curic, Richardson, Thekkadath, Flórez,
Giner, Lundeen, Phys. Rev. A (2018)

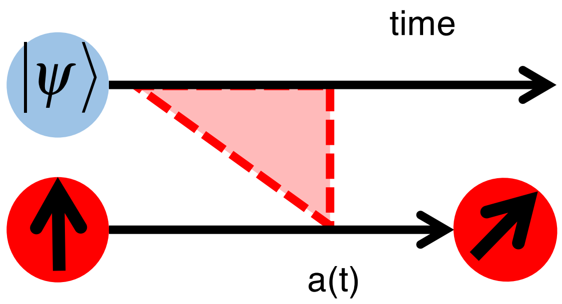
$\langle\{A, B\}\rangle = \langle\{B, A\}\rangle$ → (+ third measurement) → $\langle\{A, \{B, C\}\}\rangle \neq \langle\{B, \{A, C\}\}\rangle$

4) General non-markovian weak measurement

The measured observable depends on the history!

A single measurement (of A):

$$\langle a(t)\rangle = \int_{-\infty}^{t} dt' g(t-t')\langle \hat{A}(t')\rangle$$



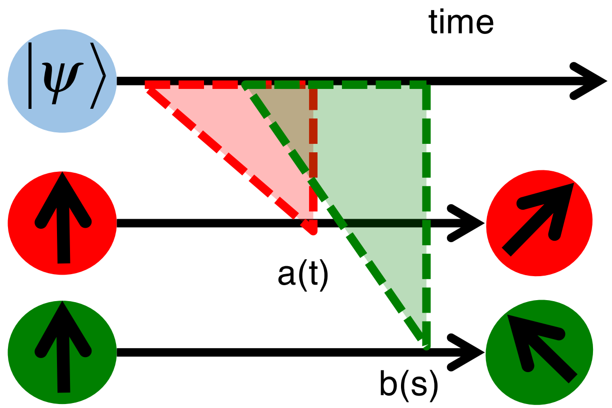
Two measurements (first A, then B)

Derived using time-non-local Kraus operators

Standard Markovian

$$\langle a(t)b(s)\rangle = g \otimes \langle\{\hat{B},\hat{A}\}\rangle(t,s) + f \otimes \langle[\hat{B},\hat{A}]\rangle(t,s)$$

memory functions



Result: Introducing memory function allows measurement of the commutator

→ non-Markovian scheme

Bednorz, Bruder, Reulet, WB, PRL 2013

Microscopic picture of non-Markovian weak measurments

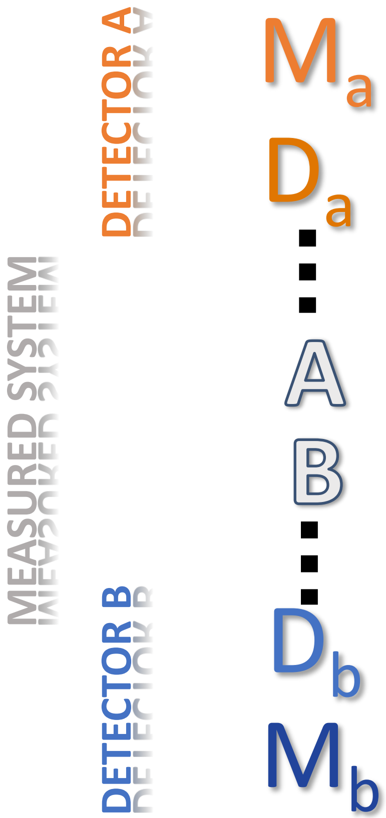
- One system, two detectors weakly coupled: $\widehat{H} = \widehat{H}_{sys} + \widehat{H}_a + \widehat{H}_b + \widehat{H}_{int}$
- Initial product state of the density matrices
- Unitary time evolution, interrupted by readout of the detectors (Kraus operators → talke as weak measurments)
- Expansion of the time evolution to 2^{nd} order in the coupling constant
- Final density matrix provides probability for the correlation function

Non-Markovian: $\langle a(t)b(s)\rangle \rightarrow g \otimes \left\langle\left\{\hat{A}, \hat{B}\right\}\right\rangle(t,s) + f \otimes \left\langle[\hat{A}, \hat{B}]\right\rangle(t,s)$

Result: Separation into three processes $C = \langle a(t)b(s)\rangle = C^{sym} + C_a^{det} + C_b^{det}$

J. Bülte, A. Bednorz, C. Bruder, and WB, Phys. Rev. Lett. **120**, 140407 (2018).

Interaction Hamiltonian



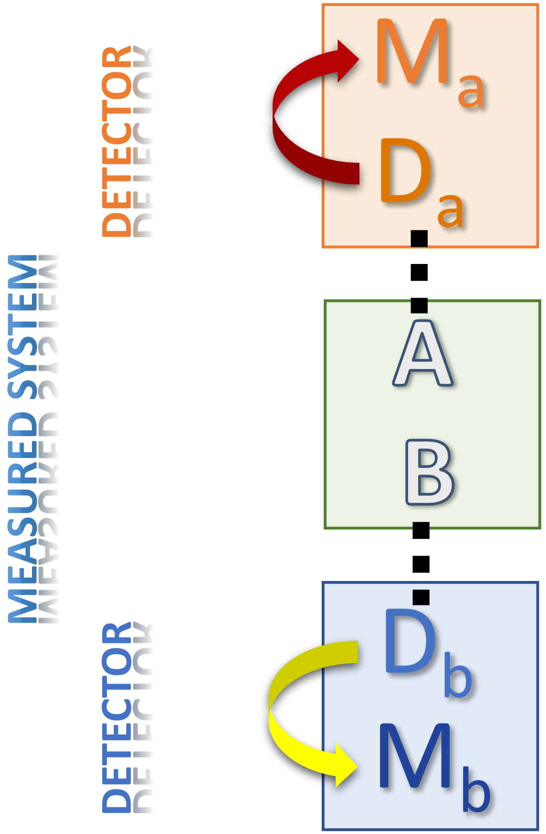
Interaction:

$$\hat{H}_{int} = \lambda_a \hat{D}_a \hat{A} + \lambda_b \hat{D}_b \hat{B}$$

The meter variables are $\hat{M}_a (\hat{M}_b)$:

$$C = \langle a(t) b(s) \rangle = \frac{1}{\lambda_a \lambda_b} \left\langle \{ \hat{M}_a(t), \hat{M}_b(s) \} \right\rangle$$

Decomposition into elementary processes

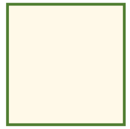


$$C = C^{\mathrm{sym}} + C_a^{\mathrm{det}} + C_b^{\mathrm{det}}$$

All contributions are expressed by ($\alpha = a, b, sys$)

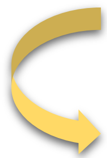
- Symmetrized noise

$$S^{\alpha}_{XY}(t,t') = \frac{1}{2}\langle\{\hat{X}_\alpha(t), \hat{Y}_\alpha(t')\}\rangle$$

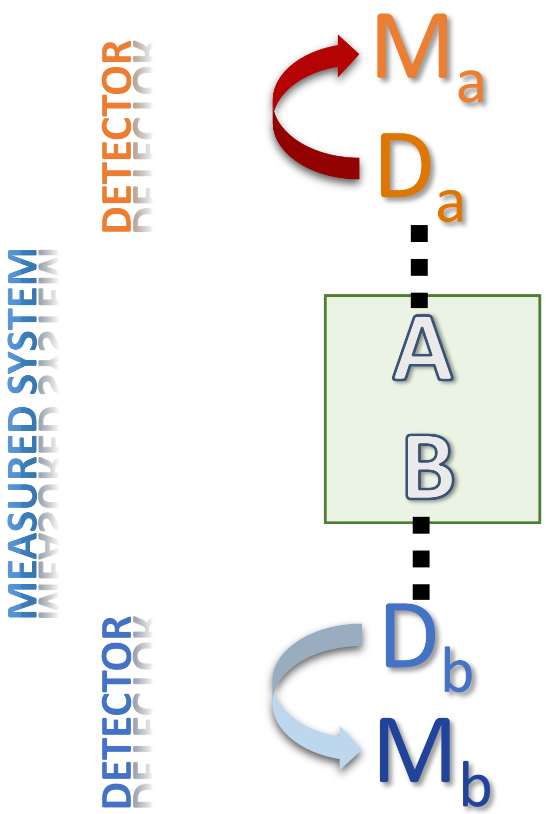


- Response function

$$\chi^{\alpha}_{XY}(t,t') = -\frac{i}{\hbar}\theta(t-t')\langle[\hat{X}_\alpha(t), \hat{Y}_\alpha(t')]\rangle$$



The markovian (symmetrized) contribution

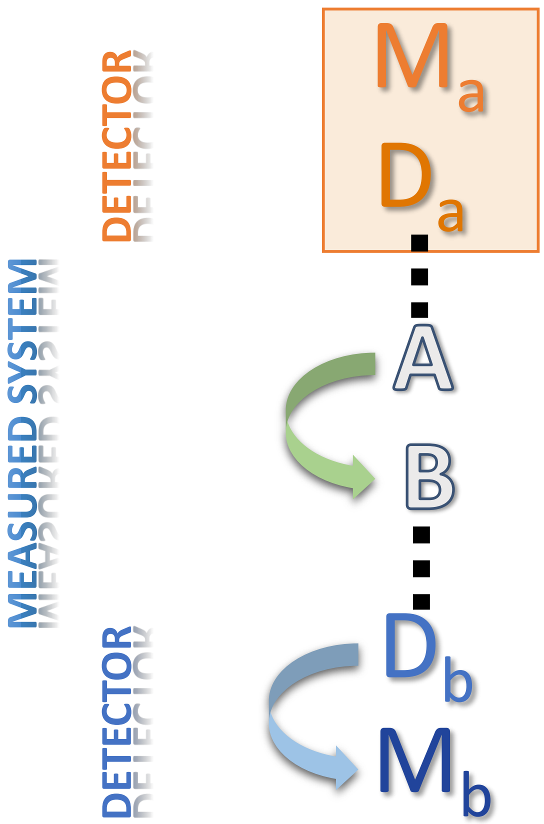


$$C^{\mathrm{sym}} = \int \mathrm{d}t\mathrm{d}t' \; \chi^{a}_{MD}(t_a, t) \; \chi^{b}_{MD}(t_b, t') \; S^{0}_{AB}(t, t')$$

“$g \otimes \langle\{\hat{A}, \hat{B}\}\rangle(t, s)$“

→ Corresponds to classical frequency filter!

The non-markovian (non-symmetrized) contribution

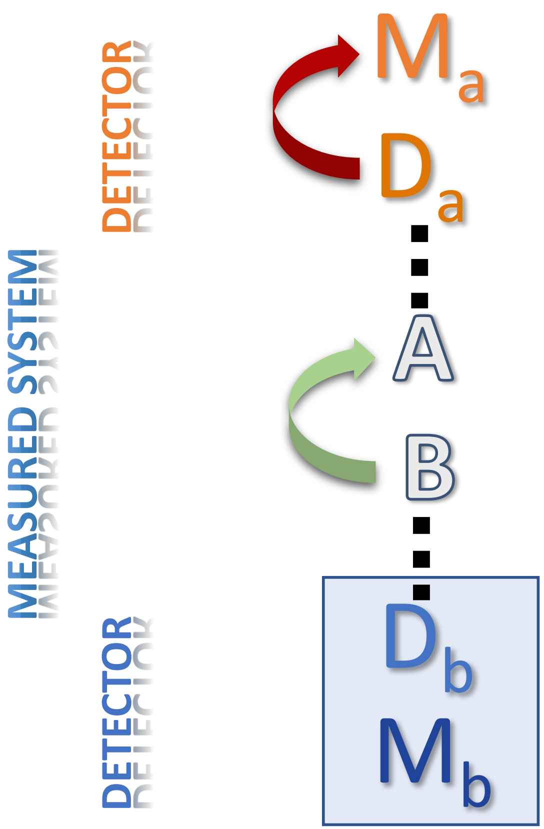


$$C_a^{\mathrm{det}} = \int \mathrm{d}t\mathrm{d}t' \; S^a_{MD}(t_a, t) \; \chi^b_{MD}(t_b, t') \; \chi^0_{BA}(t', t)$$

$$\langle a(t) b(s) \rangle \quad \sim \quad f \otimes \langle [\hat{A}, \hat{B}] \rangle (t, s)$$

System-mediated detector-detector interaction:
The noise of detector a measured by the response of the system seen by detector b.

The non-markovian (non-symmetrized) contribution (part II)



$$C_b^{\mathrm{det}} = \int \mathrm{d}t\mathrm{d}t' \; \chi^a_{MD}(t_a, t) \; S^b_{MD}(t_b, t') \; \chi^0_{AB}(t, t')$$

$$\langle a(t) b(s) \rangle \quad \sim \quad f \otimes \langle [\hat{A}, \hat{B}] \rangle (t, s)$$

The other way round......

System-mediated detector-detector interaction:
The noise of detector b measured by the response of
of the system seen by detector a

Result of microscopic treatment

Expressed by **noises** and **responses** of the **system** and the **detectors:**

Symmetrized noise $i\langle[\hat{B}(s),\hat{A}(t)]\rangle/2$

Response function $\langle\{\hat{B}(s),\hat{A}(t)\}\rangle/2$

$$C = \langle a(t)b(s)\rangle = C^{sym} + C_a^{det} + C_b^{det}$$
$$= \chi_a\chi_b \otimes S_{sys} + \chi_a\chi_{sys} \otimes S_b + \chi_b\chi_{sys} \otimes S_a$$

Frequency-filtered markovian response

System-mediated detector-detector interaction

$\langle a(t)b(s)\rangle$

Detector engineering

$\langle\{\hat{B}(s),\hat{A}(t)\}\rangle/2$

$\xi_s\langle\{\hat{B}(s),\hat{A}(t)\}\rangle + i\xi_a\langle[\hat{B}(s),\hat{A}(t)]\rangle$

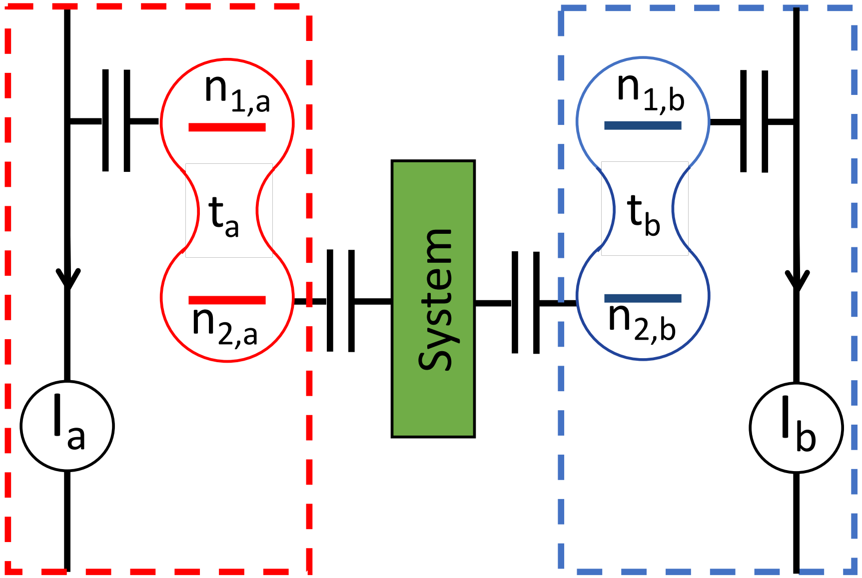
$i\langle[\hat{B}(s),\hat{A}(t)]\rangle/2$

Corresponds to a family of quasiprobabilities (Wigner, Q, P,....)

J. Bülte, A. Bednorz, C. Bruder, and WB, Phys. Rev. Lett. **120**, 140407 (2018).

Proposed implementation: two double-dot detectors

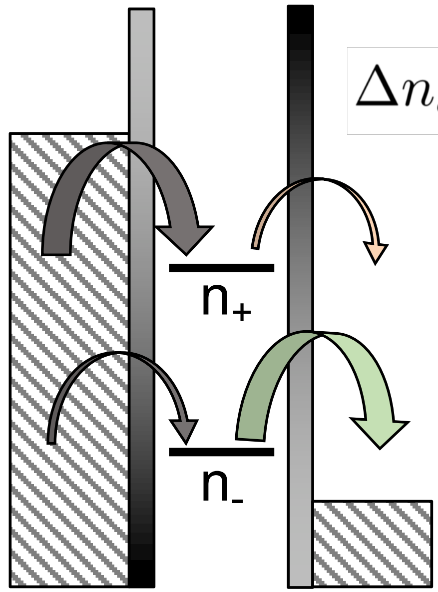
Occupation recorded by a bypassing current



$$\hat{H}_\alpha = \epsilon_\alpha \hat{\sigma}_z^\alpha + t_\alpha \hat{\sigma}_x^\alpha$$

$$\sigma_z^\alpha = n_{1,\alpha} - n_{2,\alpha}$$

$$\hat{H}_{\text{int}} = \eta_a \hat{\sigma}_z^a \hat{A} + \eta_b \hat{\sigma}_z^b \hat{B}$$

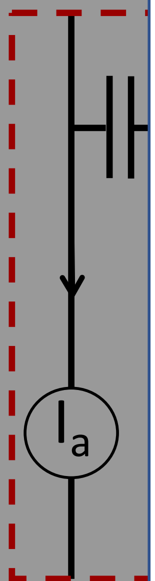


$$\Delta n_\alpha = n_\alpha^- - n_\alpha^+$$

- Double dot characterized by occupation difference of the energy eigenlevels
- Tuning Δn_α from positive to negative switches the detector from absorption to emission mode

Example: double quantum dot detectors

Occupati



Resulting noise and response functions:

$$\omega_\alpha = 2\sqrt{t_\alpha^2 + \epsilon_\alpha^2}$$

$$\chi^\alpha_{\sigma_z\sigma_z}(\tau) = -\,\theta(\tau)\frac{8t_\alpha^2}{\omega_\alpha^2}\sin(\omega_\alpha\tau)\Delta n_\alpha$$

$$S^\alpha_{\sigma_z\sigma_z}(\tau) = \frac{8t_\alpha^2}{\omega_\alpha^2}\cos(\omega_\alpha\tau) + \frac{8\epsilon_\alpha^2}{\omega_\alpha^2}\left(1-(\Delta n_\alpha)^2\right)$$

Important: $C_\alpha^{\text{det}} \propto \Delta n_\alpha$ $\quad C^{\text{sym}} \propto \Delta n_a \Delta n_b$

in $C = C^{\text{sym}} + C_a^{\text{det}} + C_b^{\text{det}}$

$- n_\alpha^+$

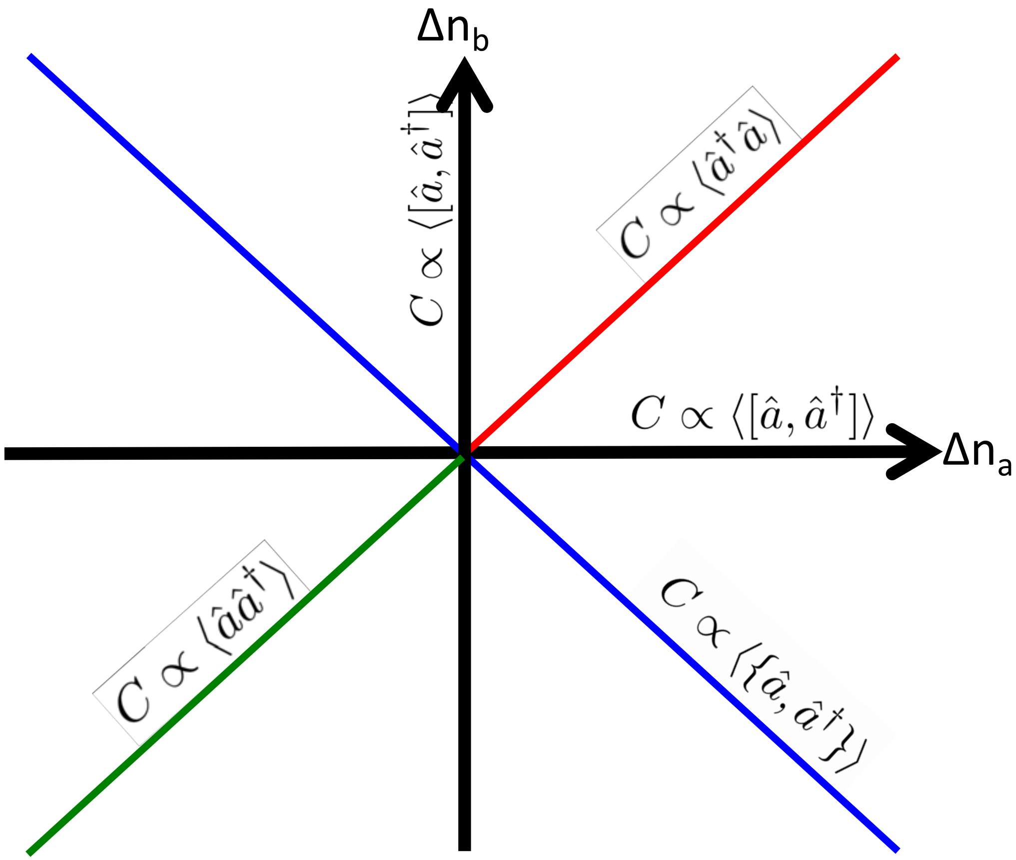
erence

es the
detector from absorption to emission mode

$\hat{H}_\alpha =$

$$\hat{H}_{\text{int}} = \eta_a \hat{\sigma}_z^a \hat{A} + \eta_b \hat{\sigma}_z^b \hat{B}$$

Measurement of a bosonic system: $a = \frac{1}{\sqrt{2}}(x + ip)$



$$C = C^{\mathrm{sym}} + C_a^{\mathrm{det}} + C_b^{\mathrm{det}}$$

By tuning Δn_a and Δn_b different system operator orders are obtained

- **Wigner**
- **normal**
- **antinormal**
- **Kubo**

J. Bülte, A. Bednorz, C. Bruder, and W. Belzig, Phys. Rev. Lett. **120**, 140407 (2018).

Conclusion

- Quantum measurement: projection and weak measurments
  - (Noisy) non-invasive measurements offer another (new) perspective on the quantum measurement problem
- Quantum dynamics: Keldysh contour
- Quantum Transport and Full Counting Statistics
  Generalized Keldysh-ordered functional
- Keldysh-ordered expectations are quasiprobabilities
  - Weakly measured non-commuting variables violate classicality
- Time-reversal symmetry breaking
  - Weak third order correlations reflect measured order (even if they shouldn't)
- General non-markovian weak measurement
  - System mediated detector-detector interaction
  - Detector engineering allows tailored operator order

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