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OSCILLATIONS AND WAVES IN PLASMA

Exactly Solvable Model of Resonance Tunneling of an Electromagnetic Wave in Plasma Containing Short-Scale Inhomogeneities

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Abstract—Different variants of resonance tunneling of a transverse electromagnetic wave through a plasma layer containing short-scale (subwavelength) inhomogeneities, including evanescence regions to which approximate methods are inapplicable, are analyzed in the framework of an exactly solvable one-dimensional model. Complex plasma density profiles described by a number of free parameters determining the permittivity modulation depth, the characteristic scale lengths of plasma structures, their number, and the thickness of the inhomogeneous plasma layer are considered. It is demonstrated that reflection-free propagation of the wave incident on the layer from vacuum (the effect of wave-barrier transillumination) can be achieved for various sets of such structures, including plasma density profiles containing a stochastic component. Taking into account cubic nonlinearity, it is also possible to obtain an exact solution to the one-dimensional problem on the nonlinear transillumination of nonuniform plasma. In this case, the thicknesses of the evanescence regions decrease appreciably. The problem of resonance tunneling of electromagnetic waves through such barriers is of interest for a number of practical applications.

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1. INTRODUCTION

The interaction of electromagnetic waves with nonuniform media (including plasmas) containing large-amplitude short-scale (subwavelength) inhomogeneities is currently an active field of research (see, e.g., [1–9]). Special attention is paid to the analysis of reflection-free wave propagation on the basis of exactly solvable models. Clearly, analysis of the effects caused by short-scale inhomogeneities is of great interest for a number of practical applications, e.g., for microwave heating of dense plasma, explanation of mechanisms for the emergence of radiation generated by sources located in dense plasma [10], and correct interpretation of observational data on the astrophysical electromagnetic radiation and the location of its sources. In addition, this problem is important for increasing the efficiency of antireflective and absorbing coatings in radiophysics, including the development of thin transparent fairings for radio wave antennas [11], for which it is necessary to find an optimum permittivity profile that minimizes the coefficient of reflection or ensures efficient transmission of electromagnetic signals from antennas covered with a dense plasma layer [10]. The development of exactly solvable models will allow one to reveal fundamentally new features of the oscillation dynamics and wave propagation in highly inhomogeneous media and demonstrate the possibility of interesting practical applications in the case of media with controlled parameters. It is also important to investigate the effect of resonance tunneling of electromagnetic waves through stratified plasma in the presence of fairly broad evanescence regions in which the square of the refractive index is negative. Exactly solvable models allow one to analyze how the parameters of the electromagnetic wave should be matched to those of a nonuniform plasma layer in order to achieve reflection-free wave propagation through inhomogeneous plasma. Earlier, the problem of increasing the efficiency of absorption of electromagnetic waves in plasma resonance layers was analyzed by traditional methods, e.g., in [12–14].

In the present work, we analyze complementary (to those considered in [5, 8, 9]) exactly solvable models that allow one to analyze resonance tunneling of an electromagnetic wave through a broad nonuniform plasma layer containing large-amplitude short-scale inhomogeneities of the plasma density. Analytical models describing this effect are presented, and different versions of the initial parameters of the problem (which substantially affect the inhomogeneity profile) are considered. The appearance of intense spikes of the wave field and the possibility of reflection-free propagation in the presence of a fairly large (formally unlimited) number of evanescence layers are investigated. The essentially nonlocal coupling between the wave vector and the effective plasma permittivity is demonstrated. The used mathematical model of barrier transillumination in the interaction of an electromagnetic wave with nonuniform plasma is based on solving the Helmholtz equation. In this case, the number of the free parameters determining the number of structures and evanescence regions, the amplitudes and characteristic dimensions of inhomogeneities, their spatial profiles, and the distances between spikes of the wave field can be fairly large. This allows one to implement very different spatial profiles of plasma inhomogeneities for which complete transillumination of the wave barriers can be achieved.

In the approach used in this study, it is of fundamental importance that plasma contains large-amplitude subwavelength inhomogeneities, i.e., traditional approximate methods of solving this problem are inapplicable. The analysis performed in this work shows that the development of exactly solvable models capable of describing reflection-free interaction of an electromagnetic wave with nonuniform plasma allows one to substantially improve the existing concepts of the dynamics of electromagnetic waves and pulses in highly inhomogeneous nonstationary dielectric media and laboratory and space plasmas.

2. BASIC EQUATIONS AND ANALYSIS OF RESONANCE WAVE TUNNELING

Let us consider the one-dimensional problem on the resonance tunneling of an electromagnetic wave through a nonuniform plasma layer containing large-amplitude subwavelength plasma density inhomogeneities. The simplest situation takes place when either an *s*-polarized electromagnetic wave propagates in plasma in the absence of an external magnetic field or the wave propagates across a homogeneous external magnetic field in magnetoactive plasma. In this case, using representation of the wave field in the form $E(x,t) = \mathrm{Re}[F(x)\exp(-i\omega t)]$, where ω is the wave frequency, we obtain the Helmholtz equation for the function $F(x)$,

$$d^2F/dx^2 + k_0^2\varepsilon_f(x)F = 0, \tag{1}$$

where the x axis is directed across the plasma layer, $k_0 = \omega/c$ is the vacuum wavenumber, and $\varepsilon_f(x)$ is the effective permittivity. When the wave propagates in unmagnetized plasma, we have $\varepsilon_f(x) = 1 - [\omega_{pe}(x)/\omega]^2$, where $\omega_{pe}(x)$ is the electron plasma frequency. When the wave propagates in magnetoactive plasma across the external magnetic field, we have $\varepsilon_f(x) \equiv N^2(x) = \varepsilon_\perp - (\varepsilon_c^2/\varepsilon_\perp)$, where N is the refractive index, with $\varepsilon_{xx} = \varepsilon_{yy} \equiv \varepsilon_\perp$, and $\varepsilon_{xy} = -i\varepsilon_c$ being the components of the plasma permittivity tensor [14]. For further analysis, it is convenient to introduce the dimensionless coordinate $\xi = k_0x$ and dimensionless wavenumber $\mathbf{p}(\xi) = ck_x(x)/\omega$. As in [2, 5, 12], the solution to Eq. (1) can be written in the form

$$\begin{gathered} F(\xi) = F_0\exp[i\Psi(\xi)][1/p(\xi)]^{1/2}, \\ d\Psi/d\xi = p(\xi), \quad F_0 = \mathrm{const}. \end{gathered} \tag{2}$$

Taking into account Eqs. (1) and (2), we find that the effective permittivity $\varepsilon_f(x)$is related to the dimensionless wavenumber $\mathbf{p}(\xi)$ by the nonlinear equation

$$\begin{gathered} \varepsilon_f(\xi) = [p(\xi)]^2 \\ + (d^2p/d\xi^2)/2p - 0.75(dp/d\xi)^2/p^2. \end{gathered} \tag{3}$$

We see that, in contrast to classical relationships, there is a nonlocal coupling between the functions $\varepsilon_f(\xi)$ and и $p(\xi)$ in the exact solution. Let us also introduce the normalized wave amplitude $|F/F_0| \equiv A(\xi) = [1/p(\xi)]^{1/2}$. In this case, expression (3) can be written in the form of the nonlinear equation for the wave amplitude A,

$$d^2A/d\xi^2 + \varepsilon_f(\xi)A - [1/A(\xi)]^3 = 0. \tag{4}$$

Nonlinear equation (4) determines the spatial profile of the dimensionless amplitude of the electromagnetic wave for a given profile of the effective permittivity $\varepsilon_f(\xi)$. It should be noted that, in uniform plasma with $\varepsilon_f(\xi) = \mathrm{const} > 0$, solution to Eq. (4) for a fixed wave frequency (the equation for a dissipationless nonlinear oscillator) describes the propagation of both a constant-amplitude wave with $A_0 = 1/\varepsilon_f^{1/4}$and a spatially modulated wave packet with parameter (oscillator energy) characterizing the modulation depth of the amplitude A ($A_{\min} < A_0 < A_{\max}$), which can be fairly large.

Similar to [2, 5, 12], analysis of solutions to Eq. (4) consists in specifying the function $p(\xi)$ by analytic expressions with a set of parameters, followed by the calculation of the effective permittivity $\varepsilon_f(\xi)$ corresponding to resonance tunneling of an electromagnetic wave through a nonuniform plasma layer by formula (3).

Let us consider the reflection-free propagation of a transverse electromagnetic wave through a plasma layer occupying the region $0 \leqslant \xi \leqslant b$, which is interfaced by vacuum from the left ($\xi = 0$) and right ($\xi = b$). As the model that ensures the conditions of reflection-free matching with the electromagnetic waves incident from vacuum ($\xi < 0$) at the left boundary of the plasma layer and that propagating to the right ($\xi > b$) at the right boundary, we choose the model with $p(\xi) = 1 - 0.5f(\xi)[1 - \cos(\gamma\xi)]$, where $f(\xi)$ is a bounded (generally, arbitrary) function and $\gamma = 2\pi/b$. The factor $[1 - \cos(\gamma\xi)]$ ensures the condition of reflection-free matching of the wave fields at the plasma–vacuum

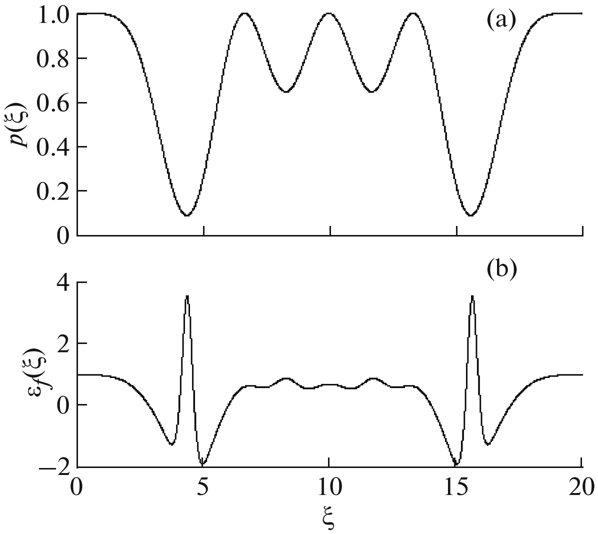

Fig. 1. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_f(\xi)$ in a nonuniform plasma layer with the function $f(\xi)$ of form (5) for $\mu = 3$ and $b = 20$.

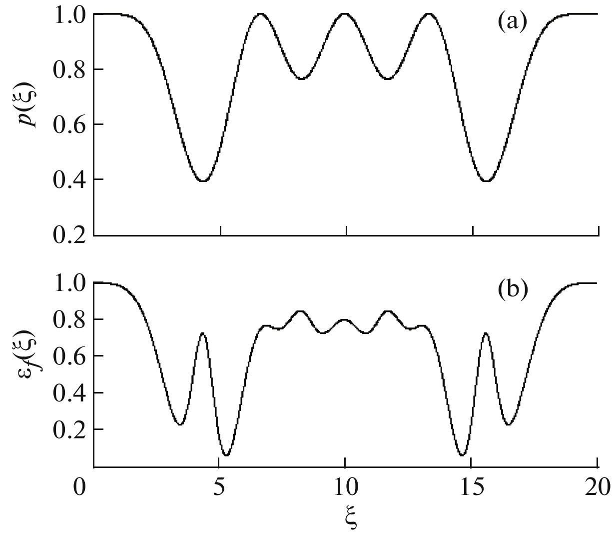

Fig. 2. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_f(\xi)$ for a nonuniform plasma layer with the function $f(\xi)$ of form (5) for $\mu = 2$ and $b = 20$.

interfaces, i.e., $p(0) = p(b) = 1$ and $dp/d\xi = 0$ at $\xi = 0$ and $\xi = b$.

As an example, let us choose the following function $f(\xi)$, satisfying the boundary conditions $f(0) = f(b) = 0$:

$$f(\xi) = 0.25\mu[1 + 0.5\cos(\gamma\xi) - \cos(2\gamma\xi) - \cos(3\gamma\xi) + 0.5\cos(5\gamma\xi)], \tag{5}$$

where μ is a free parameter. Using expression (5), the effective permittivity $\varepsilon_f(\xi)$ can be calculated by formula (3). Due to its complexity, the expression for $\varepsilon_f(\xi)$ is not presented here. The profiles of the functions $p(\xi)$ and $\varepsilon_f(\xi)$ for $b = 20$ and $\mu = 3$ are shown in Figs. 1a and 1b, respectively. One can see that the profile of the wavenumber $p(\xi)$ has two deep minima near the left and right boundaries of the plasma layer, where $p_{\min}$ is on the order of 0.1. In the central part of the plasma layer, $p(\xi)$ drops by about 30%. The profile of the permittivity $\varepsilon_f(\xi)$ has two narrow peaks with the maximum values of $\varepsilon_f(\xi)$ of about 3.8, which correspond to magnetoactive plasma, and four minima. In the central part of the plasma layer, one can see small variations of $\varepsilon_f(\xi)$.

Let us now reduce the parameter μ. The profiles of the functions $p(\xi)$ and $\varepsilon_f(\xi)$ for $b = 20$ and $\mu = 2$ are shown in Figs. 2a and 2b, respectively. According to Fig. 2a, a decrease in μ leads to a decrease in variations of the wavenumber $p(\xi)$. At the same time, the amplitude of variations of the permittivity $\varepsilon_f(\xi)$ in the central part of the plasma layer increases. However, the peak values of $\varepsilon_f(\xi)$ do not exceed unity, i.e., this version of resonance tunneling of an electromagnetic wave is possible in unmagnetized plasma. We also note that the decrease in μ resulted in a considerably increase in the widths of the extrema of $\varepsilon_f(\xi)$ near $\xi \sim 5$ and $\xi \sim 15$.

The above tendencies in the change of the $p(\xi)$ and $\varepsilon_f(\xi)$ profiles persist as the parameter μ decreases further. The profiles of the functions $p(\xi)$ and $\varepsilon_f(\xi)$ $b = 20$ and $\mu = 0.9$ are shown in Figs. 3a and 3b, respectively. This case is also realized in unmagnetized plasma. It can be seen from Fig. 3a that the amplitude of variations of the wavenumber $p(\xi)$ has decreased. The local maxima of the permittivity $\varepsilon_f(\xi)$ near $\xi \sim 5$ and $\xi \sim 15$ are weakly pronounced, and variations of $\varepsilon_f(\xi)$ in the center of the plasma layer are small.

Let us now reduce the layer thickness. Figure 4 shows the profiles of the functions $p(\xi)$ and $\varepsilon_f(\xi)$ for $b = 8$ and $\mu = 2$. The profile of the wavenumber $p(\xi)$ in Fig. 4a is similar to that shown in Fig. 3a. At the same time, the profile of the permittivity $\varepsilon_f(\xi)$ changed dramatically and corresponds to magnetoactive plasma. As compared to Fig. 3b, variations of $\varepsilon_f(\xi)$ increased considerably. In addition, there appeared evanescence regions in which $\varepsilon_f(\xi)$ is negative. We note that, in an exactly solvable problem, the square of $p(\xi)$ is positive everywhere, including the evanescence regions.

Let us analyze a version with a more complicated (as compared to formula (5)) function $f(\xi)$, namely,

$$f(\xi) = 0.125\mu[1 - 0.25\cos(\gamma\xi) - 0.5\cos(2\gamma\xi) - 1.25\cos(3\gamma\xi) - \cos(4\gamma\xi) - 0.25\cos(52\gamma\xi) - 0.5\cos(6\gamma\xi) - 0.5\cos(7\gamma\xi) - 0.25\cos(9\gamma\xi)]. \tag{6}$$

Let us assign the values $b = 20$ and $\mu = 0.2$ to the model parameters. The results of calculations corresponding to this case are presented by the profiles of the functions $p(\xi)$ and $\varepsilon_f(\xi)$ in Fig. 5. We see that, when the function $f(\xi)$ is chosen in form (6), the maximum variations of $p(\xi)$ and $\varepsilon_f(\xi)$ occur in the center of the plasma layer (at $\xi = 10$). In the central zone of the

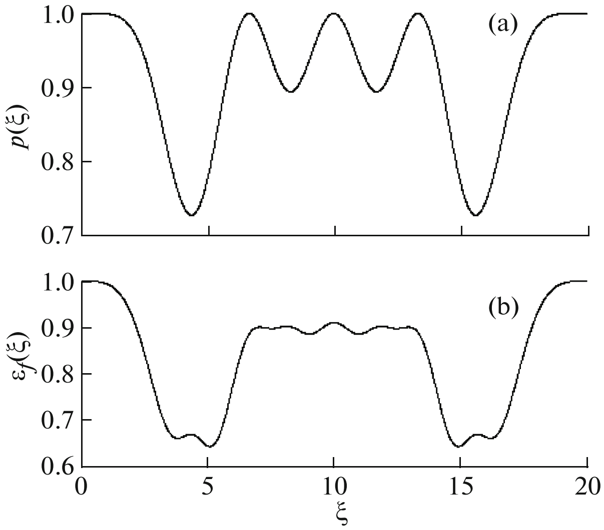

Fig. 3. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_f(\xi)$ for a nonuniform plasma layer with the function $f(\xi)$ of form (5) for $\mu = 0.9$ and $b = 20$.

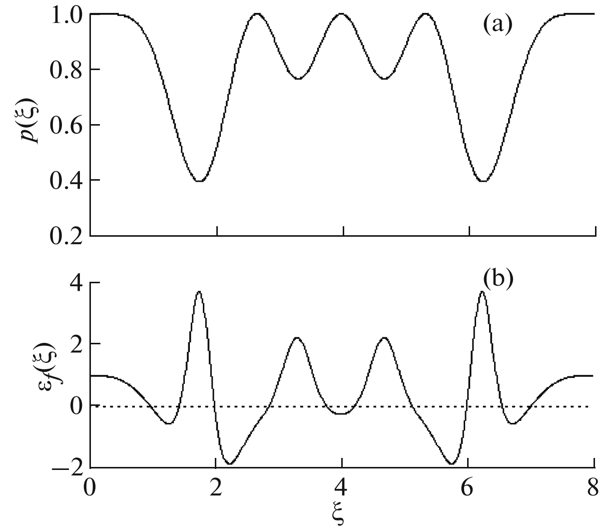

Fig. 4. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_f(\xi)$ for a nonuniform plasma layer with the function $f(\xi)$ of form (5) for $\mu = 2$ and $b = 8$.

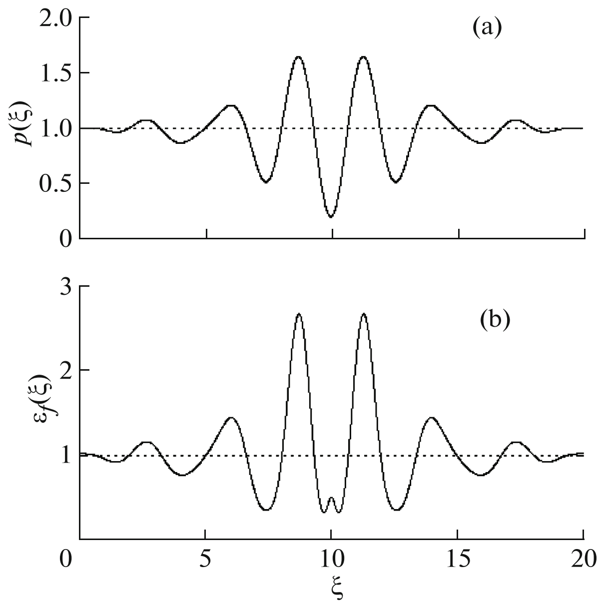

Fig. 5. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_{ef}(\xi)$ for a nonuniform plasma layer with the function $f(\xi)$ of form (6) for $\mu = 0.2$ and $b = 20$.

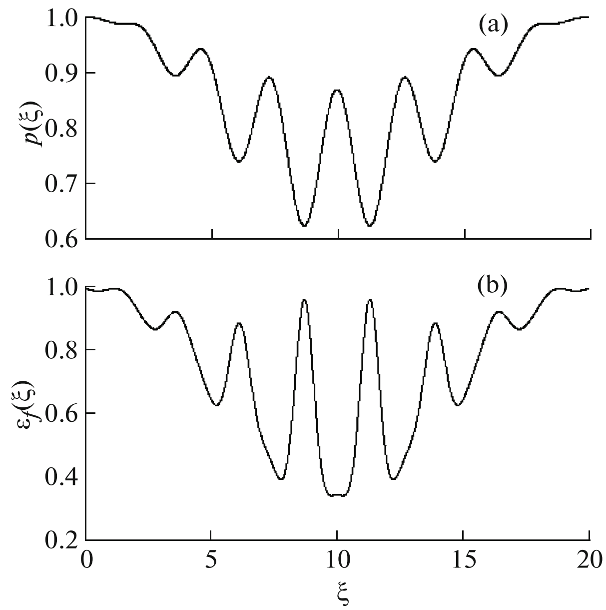

Fig. 6. Profiles of the (a) dimensionless wavenumber $p(\xi)$ and (b) permittivity $\varepsilon_f(\xi)$ for a nonuniform plasma layer with the function $f(\xi) = \mu[1 - \alpha\sin(\beta\gamma\xi)]$ for $b = 20$, $\mu = 0.26$, $\alpha = 0.5$, and $\beta = 7.5$.

plasma layer, the ratio max$p$/min$p$ reaches a value of about 4. In general, the profile of the permittivity $\varepsilon_f(\xi)$ in Fig. 5b is similar to that of $p(\xi)$, but has a small-amplitude local maximum at $\xi = 10$. Note that there are no evanescence zones for this set of model parameters.

Finally, let us choose the function $f(\xi) = \mu[1 - \alpha\sin(\beta\gamma\xi)]$ with the additional parameters α and β, which allow one to vary the functions $p(\xi)$ and $\varepsilon_f(\xi)$. The case corresponding to $b = 20$, $\mu = 0.26$, $\alpha = 0.5$, and $\beta = 7.5$ is illustrated in Fig. 6. According to Fig. 6a, the wavenumber $p(\xi)$ experiences modulation that considerably increases toward the center of the plasma layer. As can be seen from Fig. 6b, this case can be realized in unmagnetized plasma, because $\varepsilon_f(\xi)$

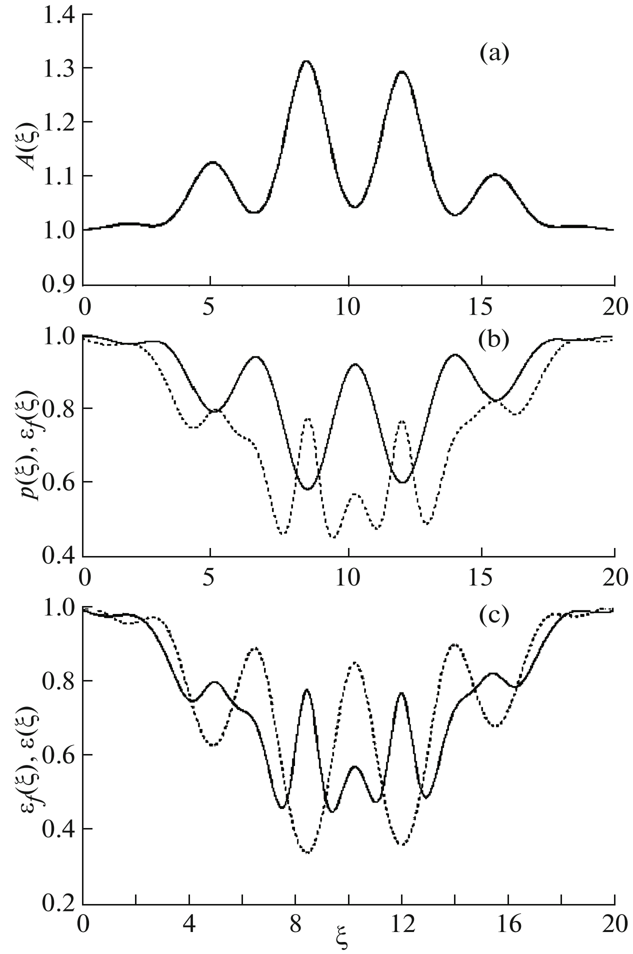

Fig. 7. Profiles of the (a) dimensionless amplitude $A(\xi) = 1/[p(\xi)]^{1/2}$ of the wave field, (b) functions $p(\xi)$ (solid line) and $\varepsilon_f(\xi)$ (dashed line), and (c) functions $\varepsilon_f(\xi)$ (solid line) and $\varepsilon(\xi)$ (dashed line) for $b = 20$, $\mu = 0.26$, $\alpha = 0.7$, and $\beta = 5.37$.

does not exceed unity. Note that variations of the permittivity $\varepsilon_f(\xi)$ are essentially nonsinusoidal.

Let us consider the results of calculations for the parameters $b = 20$, $\mu = 0.26$, $\alpha = 0.7$, and $\beta = 5.37$, which correspond to unmagnetized plasma. Figure 7a shows the profile of the dimensionless amplitude of the wave field $A(\xi) = 1/[p(\xi)]^{1/2}$. We see that variations of $A(\xi)$ in the plasma layer reach 30% and the profile of the amplitude is asymmetric with respect to the layer center $\xi = 10$. For convenience of comparison, Fig. 7b shows the profiles of the wavenumber $p(\xi)$ and permittivity $\varepsilon_f(\xi)$. According to Fig. 7b, the profiles of these functions are differ substantially due to the nonlocal nonlinear of their coupling (see formula (3)). Let us introduce the function $\varepsilon(\xi) \equiv p^2(\xi)$. The functions $\varepsilon_f(\xi)$ and $\varepsilon(\xi)$ are shown in Fig. 7c. We see that the profiles of $\varepsilon_f(\xi)$ and $\varepsilon(\xi)$ differ substantially due to the nonlocal nonlinear coupling of $p(\xi)$ and $\varepsilon_f(\xi)$ in the exactly solvable model. Thus, the presence of a set of large-amplitude short-scale structures in a nonuniform plasma layer causes the appearance of strong gradient dispersion, due to which the profiles of $p^2(\xi)$ and $\varepsilon_f(\xi)$ are substantially different, both qualitatively and quantitatively.

It should be noted that, in the exactly solvable models under study, reflection-free propagation of an electromagnetic wave through the layer is preserved if the thickness of the plasma layer is increased a whole number of times, i.e., relatively thick wave barriers are also transilluminated. For another choice of the initial parameters, large-amplitude spikes of the wave field can appear in a nonuniform plasma layer. Moreover, reflection-free propagation of an electromagnetic wave through a nonuniform plasma layer can also be realized when cubic nonlinearity of the form $\varepsilon_f(\xi) = \varepsilon_L(\xi) + \chi|A|^2$, where χ is the parameter of nonlinearity and $\varepsilon_L(\xi)$ is the linear plasma permittivity, is taken into consideration. In this case, in the presence of evanescence zones in the profile of $\varepsilon_L(\xi)$ for the nonlinear permittivity, the value of $|\varepsilon_f(\xi)|$ in the evanescence zones decreases substantially, i.e., cubic nonlinearity facilitates the transillumination of gradient wave barriers in plasma. Thus, similar to the results obtained in [8], in the exactly solvable model, due to nonlinearity and resonance tunneling, an electromagnetic wave can propagate through nonuniform plasma without reflection, generating large-amplitude spikes of the electromagnetic field in some sublayers.

3. CONCLUSIONS

In this work, based on the exactly solvable Helmholtz equation, we have studied several new variants of resonance tunneling of a transverse electromagnetic wave through a nonuniform plasma layer containing short-scale (subwavelength) inhomogeneities, including relatively broad evanescence regions [5, 8, 9]. The spatial profile of the plasma density depends on a number of free parameters determining the modulation depth of the plasma permittivity, the characteristic scale lengths of the plasma structures, their number, the thickness of the nonuniform plasma layer, and the parameters of evanescence regions. Since the relation between the amplitude $A(\xi)$ of the electromagnetic wave field and the effective permittivity $\varepsilon_f(\xi)$ is described by a nonlinear equation, there is nonlocal coupling between $p^2(\xi)$ and $\varepsilon_f(\xi)$ due to the presence of large-amplitude subwavelength structures in plasma. We have analyzed complex profiles of the plasma permittivity and showed that reflection-free propagation of electromagnetic waves through the plasma layer can be achieved for a rather diverse set of such structures. Resonance tunneling of an electromagnetic wave through a nonuniform plasma layer is possible for both unmagnetized and magnetoactive plasmas.

The one-dimensional problem on the transillumination of a nonuniform plasma layer can be solved exactly also when cubic nonlinearity is taken into account. In this case, due to nonlinearity, the thicknesses and depths of the evanescence regions, in which $\varepsilon_{ef}(\xi) < 0$, decrease substantially. Generalizing the developed approach, it would be of great interest to analyze the possibility of implementing another effect, namely, strong reflection of a high-frequency electromagnetic wave from a moderate-thickness plasma layer.

The problem on resonance tunneling of electromagnetic waves through gradient barriers is of interest for a number of practical applications, because the exactly solvable models allow one to reveal new specific features in the oscillation dynamics and propagation of electromagnetic waves through nonuniform plasma and the development of nonlinear processes under the conditions of strong inhomogeneity. Such models can also demonstrate the possibility of interesting practical applications in the case of media with controlled parameters. As an example, we can mention microwave heating of dense plasma, explanation of the mechanism for the emergence of electromagnetic radiation generated in the dense plasma of astrophysical objects, the possibility of remote sensing through a dense plasma shell, generation of electromagnetic waves in nonuniform plasma by charged particle beams, and transmission of signals through dense plasma layers.

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