PHYSICAL REVIEW LETTERS

VOLUME 69 | 24 AUGUST 1992 | NUMBER 8

Rough Sea Foam

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(Received 16 April 1992)

A simple theory is developed for rough seas in which the energy cascade towards small scales is too much for a surface tension dominated wrinkling of the surface to handle. We suggest that a new phase, consisting of an air-water foam, is created and becomes the principal medium for energy dissipation. The foam depth and droplet size are calculated and we suggest how the predicted dependence on energy flux and surface tension could be verified experimentally.

PACS numbers: 92.10.Kp, 03.40.Kf

Through mechanisms still only partially understood, wind transfers energy and momentum to surface water waves. For moderately sloped longer waves which are dominated by gravity rather than surface tension forces, energy and momentum are redistributed throughout the spectrum by four-wave resonances first described by Phillips [1] and Hasselmann [2]. The flux of both quantities is towards high wave numbers where energy is dissipated and the net nonisotropic component of momentum is transferred to ocean currents. At high wave numbers, the kinetic equation of weak turbulence theory gives rise to two constant energy flux Kolmogorov spectra. For $k < k_0 = (g/\sigma)^{1/2}$, where gravity dominates [the linear dispersion relation is $\omega = (gk + \sigma k^3)^{1/2}$], one finds the dimensionless measure of spectral energy to relax to [3-6]

$$E(k) = k_0^4 \langle \eta_k^2 \rangle \sim \left(\frac{P}{P_0} \right)^{1/3} \left(\frac{k_0}{k} \right)^{7/2} = E_1(k) . \qquad (1)$$

For $k > k_0$, where surface tension dominates and energy is redistributed through three-wave resonant processes [7], the constant flux Kolmogorov spectrum is

$$E(k) \sim \left(\frac{P}{P_0} \right)^{1/2} \left(\frac{k_0}{k} \right)^{19/4} = E_2(k) . \qquad (2)$$

In Eqs. (1) and (2), g is gravity, $\sigma = S/\rho$ is surface tension divided by water density, and

$$P_0 = (\sigma g)^{3/4} \qquad (3)$$

is a critical value of the energy flux per unit area P. P can be written as $\epsilon^{3/2} V^3$, where $\epsilon = \rho_a/\rho \sim 10^{-3}$ is the ratio of air to water density and V is the wind speed. η_k is the Fourier transform of the water surface elevation and $\langle\,\rangle$ is ensemble (here spatial) average. The Kolmogorov spectra (1) and (2) are solutions of the kinetic equations relevant when the sea surface slope is small. For larger slopes for which the surface, in the absence of surface tension, can form discontinuities in slope, the Phillips spectrum [6] (obtained by arguing that the Fourier transform of the wave slope $k\eta_k$ is dominated by the k^{-1} spectrum associated with discontinuities),

$$E(k) \sim \left(\frac{k_0}{k} \right)^4 = E_3(k), \quad k < k_0 , \qquad (4)$$

is relevant and provides an upper bound for the Kolmogorov spectrum $E_1(k)$ for $k \le k_0$ provided $P < P_0 = \epsilon^{3/2} V_0^{3/2}$, which corresponds to a critical wind speed $V = V_0$ of approximately 6 m sec^{-1}, the value at which whitecapping is observed to set in [8]. For values $P < P_0$, the energy is carried to high, surface tension dominated wave numbers by $E_1(k)$ where the energy flux can be absorbed by surface tension wrinkling. Only when $P = P_0$ will the equilibrium spectrum $E_2(k)$ be exactly realized because only then is $E_1(k_0) = E_2(k_0)$. For $P < P_0$, and $k > k_0$, $E(k) > E_2(k)$. The important thing to stress, however, is that for this range of fluxes the sea surface stays smooth, energy is transferred to scales where viscosity is important by wave-wave interactions, and the topological boundary condition that a water particle on the surface stays there remains intact.

The purpose of this Letter is to describe what happens when P is significantly greater than P_0. The results are of interest for several reasons. They provide, first of all, a qualitative picture of what happens in rough seas and make predictions that could be tested directly in the laboratory by a modification of the usual Faraday experiment. Second, they are a first step towards a quantitative theory of a multiphase sea surface and towards an understanding of the energy dissipation processes for seas in which the smooth surface condition is broken. This is very important because once energy is dissipated, the momentum is transferred to ocean currents. Indeed, it is generally believed that this is the principal mechanism for transferring momentum from wind to water. Third, the theory provides an example of a situation in which a phase transition causes a change of topology and the necessity to change the governing equations of motion because the sea surface is no longer connected.

For $P>P_0$, the Kolmogorov spectrum is no longer bounded from above by the Phillips spectrum and the two intersect at

$$k=k_1=k_0\left(\frac{P_0}{P}\right)^{2/3}<k_0. \qquad (5)$$

The energy flux per unit area is too much for a smooth surface to handle. In order to absorb the energy flux, the surface must increase its area. But it cannot use a smooth surface on which surface tension waves redistribute the energy to smaller scales by three-wave interaction processes because $k_1<k_0$. The only remaining way for the surface to achieve a greater area is for it to break and to spray droplets of water into the air immediately above the interface causing the formation of an air-water foam consisting of water droplets of a size at which surface tension effects are important.

In what follows, we assume the foam to be distributed uniformly over the surface. This is an idealization. In reality, it is concentrated on short-wave crests near regions of the slope discontinuity. The influence of long waves on the process is discussed in the penultimate paragraph of this Letter.

The foam thickness and water droplet size are determined by two simple considerations. First, we assume that the various forms of energy, surface, potential, and kinetic, have the same orders of magnitude. Equating the first two, that is, assuming that the potential energy of a water droplet is balanced by its surface energy, we obtain

$$\sigma\lambda^2\sim g\lambda^3 h. \qquad (6)$$

The second assumption is that all available energy goes into various forms of energy in the foam, roughly on an equal basis. For small times, we equate the surface energy of the foam in a column of height h and unit cross section to the energy input, namely,

$$\sigma\lambda^2(h/\lambda^3)\sim Pt, \qquad (7)$$

which leads to the laws

$$h\sim(Pt/g)^{1/2},\ \ \lambda\sim\sigma/(gPt)^{1/2}. \qquad (8)$$

Observe from (6) that $\lambda h=k_0^{-2}$ so that the geometric mean of foam thickness and water droplet size is k_0^{-1}, the scale at which gravity and surface tension effects balance. For a given P, the droplets will continue to break up by surface deformation and form even smaller droplets until they reach a size λ where all the energy flux can be absorbed. At this stage, the foam will cease to grow and the water droplets will simply oscillate. In the absence of viscosity, the only relevant time scale is $(\lambda^3/\sigma)^{1/2}$ and replacing the t in (8) by this value, we obtain

$$hk_0\sim\left(\frac{P}{P_0}\right)^{2/7}>1,\ \ \lambda k_0\sim\left(\frac{P}{P_0}\right)^{-2/7}<1. \qquad (9)$$

Note that h is larger than the critical wavelength k_0^{-1} but smaller than the scale k_1^{-1} of waves which produce the spray,

$$hk_1\sim\left(\frac{P}{P_0}\right)^{-8/21}<1. \qquad (10)$$

If viscosity is present, we can form the Kolmogorov length scale

$$l=\frac{\nu}{P^{1/3}}=\frac{\nu}{P_0^{1/3}}\left(\frac{P_0}{P}\right)^{1/3}. \qquad (11)$$

However, the ratio

$$\frac{l}{\lambda}=\frac{\nu g^{1/4}}{\sigma^{3/4}}\left(\frac{P_0}{P}\right)^{1/21}$$

becomes smaller as P increases, so that even though the bubble size decreases, the Kolmogorov length scale, at which viscous effects would be expected to become important, decreases at a slightly faster rate. Thus the mechanism for dissipation of the foam must involve more complicated dynamical processes such as a weak turbulence energy transfer by small amplitude waves on the bubble surfaces. Such a study is underway.

These predictions could be directly verified in a Faraday experiment by exciting the water surface in a Petri dish. The flux P can be easily controlled and the energy can be injected at whatever wave number $k<k_0$ found to be suitable. Moreover, the surface tension can also be sensitively controlled so that a fully developed, spatially homogeneous foam is formed. In the sea, such a state is only likely to be reached under hurricanelike conditions. For the rough seas usually observed, we suggest that the spottiness of whitecaps is due to the local intensification of the energy flux P due to the passage of long, large waves. It is known [8] that the flux of energy and momentum to small scales is accompanied by an inverse

cascade of "number" of waves, with density $E(k)/\omega(k)$, to long waves. These long waves can intensify the cascade of energy to short waves beyond the amount of flux carried by four-wave resonant interactions because the particle velocities of the long waves are comparable to the phase and group velocities of short waves and cause, through local Doppler shifts, the focusing of energy into small scales in the region ahead of the crests of the long wave. The flux locally exceeds P_0, the surface breaks into a spray, and a cloud of bubbles is locally formed. However, once the long wave has passed, the energy drains out of the foam, the water droplets coalesce to form larger droplets in order to lower the total surface energy of the foam, and the new phase eventually disappears. We suggest that the behavior near $P=P_0$ is equivalent to a phase transition of second order and that the presence of large long waves acts as a "geometric imperfection" which makes the phase transition continuous.

The presence of large long waves is likely to lead a somewhat nonuniform distribution of foam on the sea surface and this would have to be taken into account in calculating the energy dissipation rate and momentum transfer. It is also likely that the composition of the foam is more complicated than the single phase of water droplets in air. During the wave-breaking process, in addition to the explosion of water into droplets, one can also have entrainment of air into the water and the formation of a cloud of air bubbles surrounded by a connected water region. For this situation, and for a combination of air bubbles in water and water bubbles in air, similar calculations would obtain. However, there is also the possibility of an emulsion stage in which neither the air nor water is in bubble form but each component of the mixture is thoroughly mixed (although also connected) throughout the foam. Indeed, it is possible that a layer of foam could contain all three phases, stratified according to their relative densities, water droplets in air near the top, a thoroughly mixed emulsion in the middle, and air bubbles in water at the bottom. In addition to the multiphase possibilities, a more sophisticated model should take account of strong shear layers in the top few centimeters and the turbulence in the water itself. While the inclusion of all these effects will be necessary in order to gain a complete picture of energy dissipation and momentum transfer, the simple point of view and results of this Letter are a step in the right direction.

Research support from AFOSR Contract No. FQ8671-900589, NSF Grants No. DMS 8922179 and No. DMS 9021253, and ONR Contract No. N00014-92-J-1343 is gratefully acknowledged.

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