Wave collapse

1988 Sov. Phys. Usp. 31 672

(http://iopscience.iop.org/0038-5670/31/7/A06)

Scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the Academy of Sciences of the USSR (20–21 January 1988)

Usp. Fiz. Nauk **155**, 529–533 (July 1988)

A joint scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the USSR Academy of Sciences was held on January 20 and 21, 1988 at the S. I. Vavilov Institute of Physics Problems. The reports enumerated below were presented at the session.

Scientific session dedicated to the 80th anniversary of the birth of Academician L. D. Landau (1908–1968)

January 20

1. *A. M. Polyakov*. Problems in quantum geometry.
2. *P. B. Vigman*. Superconductivity in strongly correlated electronic systems.

January 21

3. *O. A. Kirzhnits*. Electrodynamics of the magnetic monopole.
4. *V. E. Zakharov*. Wave collapse.

A summary of one report is presented below.

V. E. Zakharov. *Wave collapse.* Solitons—stable localized excitations—are actively employed in different areas of modern theoretical physics. Solitons comprise the best-known class of strongly nonlinear wave phenomena.

The purpose of this report is to call attention to another class of such phenomena: wave collapse—explosive concentration of wave energy in a shrinking volume. The theory of collapse is closely related with the theory of solitons: in those cases when the soliton becomes unstable, the nonlinear stage of the instability is usually a wave collapse. The collapse could result in the formation of a smaller soliton, but the most typical situations are those when the energy flowing into the collapse is absorbed owing to appearance of dissipative effects which were previously unimportant. In this case collapse is strongly nonlinear and could be a very important mechanism for dissipation of wave energy.

Steady-state excitations of the soliton type can contribute to the thermodynamic properties of matter and can be studied by means of statistical physics. Wave collapse is a fundamentally non-steady-state phenomenon, required for understanding many strongly nonequilibrium processes of the turbulent type. They include wave turbulence, arising when strong light pulses propagate in a nonlinear dielectric (here the collapse is a steady-state or non-steady-state self-focusing); flows of strong currents in a plasma (the collapse consists of catastrophic z-pinch compression); propagation of collisionless shock waves at an angle to the magnetic field. Collapse of Langmuir waves plays a central role in the theory of Langmuir plasma turbulence, arising when a plasma is heated by intensive methods, e.g., electromagnetic radiation or beams.[1]

The number of physical situations for the understanding of which collapse has to be invoked is constantly growing and it could turn out that the significance of collapse in physics is comparable to that of solitons.

The simplest variant of the theory of collapse is the theory of the appearance of singularities in the solutions of nonlinear wave equations. Thus the nonlinear Schrödinger equation (a general model for quasimonochromatic wave packets in a nonlinear medium)

$$i\Psi_t + \Delta\Psi + |\Psi|^2\Psi = 0 \qquad (1)$$

with dimension $d = 1$ describes stable solitons, but for $d \gtrsim 2$ the initial condition for which the integral of the motion (Hamiltonian) is negative

$$H = \int \left(|\nabla\Psi|^2 - \frac{1}{4}|\Psi|^4 \right) \mathrm{d}\mathbf{r} < 0,$$

terminates in a singularity at a finite time $t = t_0$.[2] The question of the nature of this singularity has been solved only very recently. Two variants of the singularity were studied[3]: "weak collapse," when the energy flowing into the singularity formally equals zero and an integrable singularity of the wave-energy distribution forms at the point of collapse and "strong collapse," when the energy concentrated at the point of collapse is finite. The choice between these two possibilities strongly affects the formula for the effective coefficient of nonlinear damping, contributed by the collapse. It has now been established[4,5] that for $d = 3$ the collapse is weak (although unstable regimes of strong collapse are possible[6]) and self-similar

$$\Psi \to \frac{1}{(t_0 - t)^{1/2 + i\varepsilon}} F\left(\frac{r}{(t_0 - t)^{1/2}} \right), \quad \varepsilon = 0.545, \qquad (2)$$

and in addition as $t \to t_0$

$$|\Psi|^2 \to \frac{2c^2}{r^2}, \quad c=1.01.$$

The theory in the two-dimensional case $d = 2$ is more difficult. Back in 1975 it was hypothesized[7] that the collapse is quasi-self-similar:

$$|\Psi| \to \frac{1}{f(t)} R_0\left(\frac{r}{f(t)}\right), \qquad (3)$$

and in addition the function $R_0(\xi)$ is identical to the profile of a steady-state two-dimensional soliton, exhibiting in this case neutral stability. This hypothesis was proved in Ref. 8, where the form of the function $f(t)$ was also established:

$$f(t) = \left(\frac{t}{la(t)}\right)^{1/2}, \quad a \ln a = \alpha \,|\ln(t_0 - t)| \quad (\alpha = \text{const}).$$

The asymptotic behavior (3) means that the collapse is strong, and a constant and fixed amount of energy (critical energy) flows into the singularity. The numerical solution of Eq. (1) played an important role in the construction of the theory of collapse on the basis of that equation. Special numerical schemes with an adaptive grid, enabling very close approach to the singularity ($|\Psi_{max}|^2/|\Psi_0|^2 \sim 10^{18}$), were developed. In general, numerical simulation is fundamental for the theory of wave collapse.

The collapse of Langmuir waves consists of the formation of shrinking regions of low density in the plasma—cavities, which act as resonators for electromagnetic oscillations of the dipole type. Cavities have a lenticular shape and their initial size exceeds by several orders of magnitude the Debye radius. The basic collapse process is described by a system of equations for the complex envelope of the high-frequency potential Ψ and variation of the density n, related to Eq. (1):

$$\Delta(i\Psi_t + \Delta\Psi) = \operatorname{div} n \nabla\Psi,$$
$$n_{tt} - \Delta n = \Delta |\nabla\Psi|^2. \qquad (4)$$

The system (4) has been modeled numerically many times.[1] Combined analytical and numerical studies have established that in the most interesting region of scales

$$1 \ll \frac{L}{r_D} \ll \left(\frac{m_e}{m_i}\right)^{1/2}$$

the collapse is strong, and in addition energy of the order of

$$E \sim A\left(\frac{m_i}{m_e}\right)^{1/2} r_D^3 nT;$$

where A is a structure factor, flows into the singularity.

The vibrational energy concentrated in the collapse is transferred at the final stage of the process to the electrons in the plasma; the collapsing cavity also generates acoustic oscillations. To study this stage of collapse the plasma must be modeled numerically by the method of particles in cells, and a substantially three-dimensional problem must be solved. This modeling, which falls at the limit of the capabilities of modern computers, has recently been performed at the Computational Complex of the Institute of Space Studies of the USSR Academy of Sciences.[10] The modeling showed that the final size of the cavity is quite large ($L/r_D \sim 15$–20, which agrees with previous, apparently unexplainable data from a laboratory experiment[11]), and it confirmed the hypothe sis that collapse is an efficient mechanism for accelerating hot electrons, usually observed when a plasma is heated with a laser.

The study of the elementary act of collapse is a prerequisite for the development of the theory of "light-induced turbulence," described by Eq. (1), and of Langmuir turbulence of a plasma, described by the system (4). In both cases the turbulence is accompanied by instantaneous collapse, in which dissipation of wave energy and generation of small-scale disturbances occur. The numerical modeling of turbulence of these types is a very important problem, and only the first steps toward its solution have been taken.[12] Other types of plasma turbulence that are accompanied by collapse and have been studied to some degree are the lower-hybrid turbulence[13] and magnetosonic turbulence in the case of positive dispersion of sound,[14] occurring when acoustic waves are propagating at an angle to the magnetic field. It should be noted that many different types of turbulence can develop in a plasma in a magnetic field, and many of them undoubtedly involve collapse.

There exists an important general physical situation that leads to collapse. In strongly nonequilibrium media (in the presence of charged-particle beams, flows, etc.) waves with negative energy, whose propagation reduces the total energy of the system, can exist. In such media explosive instability is possible. It can occur when triplets of waves exist with the wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ and frequencies ω_1, ω_2, and ω_3 satisfying the resonance conditions

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0, \quad \omega_1 + \omega_2 + \omega_3 = 0 \quad (i = 1, 2, 3). \qquad (5)$$

This process is described by the system of equations for envelope waves

$$\frac{\partial\Psi_i}{\partial t} + (\mathbf{v}_i\nabla)\Psi_i = i\Psi_j^*\Psi_k^* \quad (i \neq j \neq k); \qquad (6)$$

here $\mathbf{v}_i$ are the group velocities of the wave packets. The system (6) can be solved exactly with the help of the method of the inverse scattering problem (see Ref. 15); application of this method shows that a sufficiently strong initial condition of a general type leads to the formation of a point singularity—a collapse. The integrability of the equations with the help of the method of the inverse scattering problem not only does not eliminate the possibility of collapse, but, on the contrary, most integrable equations in the "general position" exhibit collapse. Thus the Korteweg–de Vries equation

$$u_t + uu_x + u_{xxx} = 0 \qquad (7)$$

for the complex function u leads to collapse, which is easily verified by analyzing the simplest exact solution of the one-soliton type. In the real case, for equations of the type

$$u_t + u^p u_x + u_{xxx} = 0 \qquad (8)$$

collapse occurs for $p \gtrsim 4$.

Thus turbulence with the participation of waves with negative energy is accompanied by collapse. Concrete physical examples of such turbulence have not yet been studied; they should occur in plasma physics and hydrodynamics. In general, the question of the role of collapse in hydrodynamic turbulence is one of the most important questions. A number of experimental data—intermittency, strong difference between a turbulent random process and a Gaussian process (large "excesses" in high order correlation functions), etc.—suggest that the dissipation of turbulent energy is of a

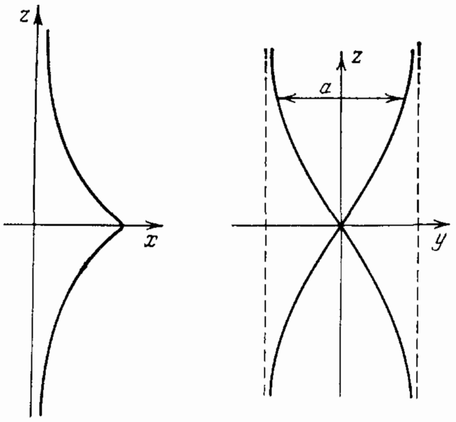

FIG. 1.

nonuniform, "spotty" character, which could be explained by collapse. However the possibility of collapse in hydrodynamics has not yet been proved. But a numerical experiment,[16] describing collapse in the problem of the interaction of two vortex filaments of finite thickness (on the basis of a quite rough model of such interaction), has been performed. We shall give the analytic explanation of this experiment. Let two vortex filaments, symmetric about the x axis and having equal and opposite vorticity Γ and radius d be given; let the distance between the filaments be a (see Fig. 1); and, let x be the coordinate of the center of the system. Then in the approximation $R \gg a \gg d$

$$\frac{a_{zz}}{a} \sim \frac{1}{R^2},$$

the following system of equations is obtained:

$$\Delta = \Gamma \ln \frac{a}{d},$$
$$\dot{X} = \frac{\Gamma}{a} - \Delta \frac{\partial^2 a}{\partial z^2},$$
$$\dot{a} = \Delta \frac{\partial^2 x}{\partial z^2}. \qquad (9)$$

This system has a collapsing self-similar solution of the form

$$a = (t_0 - t)^{1/2} f\left(\frac{z}{(t_0 - t)^{1/2}}\right),$$

which agrees qualitatively and quantitatively with the data of the experiment of Ref. 16. The state of the filaments at the moment of collapse is shown in Fig. 1. The system (9), however, is no longer applicable when $a \sim d$, and the question of the further behavior of the system of filaments remains open.

There is no doubt that collapse plays a role in the turbulence of potential oscillations of a compressible liquid (acoustic turbulence). Here shock waves play the role of collapses; the distinction from the preceding situation lies in the fact that the shock waves are long-lived flat regions of energy dissipation. The question of whether or not point or filamentary regions of this type can exist is very interesting (they could be called "density funnels"; see Ref. 17). Investigation shows that point density funnels exist in the nonlinear Schrödinger equation (1), if the dimension of the space is $d \gtrsim 4$.

[1]V. E. Zakharov, Fundamentals of Plasma Physics [in Russian], Energoatomizdat, M., 1984, Vol. 2, p. 79.
[2]S. N. Vlasov, L. V. Piskunova, and V. I. Talanov, Zh. Eksp. Teor. Fiz. **75**, 1602 (1978) [Sov. Phys. JETP **48**, 808 (1978)].
[3]V. E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].
[4]D. W. Mclaughlin, G. G. Papanicolaou, S. Sulem, and P. L. Sulem, Phys. Rev. A **34**, 1200 (1986).
[5]V. E. Zakharov, A. G. Litvak, E. A. Rakova, A. M. Sergeev, and V. F. Shvets, Zh. Eksp. Teor. Fiz. **94**, No. 5 (1988) [Sov. Phys. JETP **67**, No. 5 (1988)].
[6]V. E. Zakharov and E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **91**, 1310 (1986) [Sov. Phys. JETP **64**, 773 (1986)].
[7]V. E. Zakharov and V. S. Sypakh, Zh. Eksp. Teor. Fiz. **68**, 940 (1975) [Sov. Phys. JETP **41**, 465 (1975)].
[8]V. E. Zakharov, "On the character of the singularity associated with critical collapse," Zh. Eksp. Teor. Fiz. (1988) [Sov. Phys. JETP (1988)].
[9]V. E. Zakharov and V. F. Shvets, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 227 (1988) [JETP Lett. **47**, 275 (1988)].
[10]V. E. Zakharov, A. N. Pushkarev, A. M. Rubenchik, R. Z. Sagdeev, and V. F. Shvets, "Final stage of three-dimensional Langmuir collapse," *ibid.* (1988).
[11]P. Y. Cheung and A. Y. Wong, Phys. Fluids **28**, 1538 (1985).
[12]L. M. Degtyarev, I. M. Ibragimov, R. Z. Sagdeev, T. I. Solov'ev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 455 (1984) [JETP Lett. **40**, 1282 (1984)].
[13]S. L. Musher and V. I. Sturman, *ibid.* **22**, 537 (1975) [JETP Lett. **22**, 265 (1975)].
[14]E. A. Kuznetsov, S. L. Musher, and A. V. Shafarenko, *ibid.* **37**, 204 (1983) [JETP Lett. **37**, 241 (1983)].
[15]V. E. Zakharov and S. V. Manakov, *ibid.* **18**, 413 (1973) [JETP Lett. **18**, 243 (1973)].
[16]A. Pumir and E. D. Siggia, Phys. Fluids **30**, 1606 (1987).
[17]V. E. Zakharov and L. N. Shur, Zh. Eksp. Teor. Fiz. **81**, 2019 (1981) [Sov. Phys. JETP **54**, 1064 (1981)].

Translated by M. E. Alferieff