Two-dimensional collapse of Langmuir waves

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Results are presented of a numerical experiment that simulates the development of the two-dimensional collapse of Langmuir waves.

The question of the existence of a Langmuir collapse, namely a strongly nonlinear mechanism of Langmuir-wave absorption, accompanied by the formation of regions of decreased plasma density ("caverns"), is one of the important problems in plasma-turbulence physics. The initial paper[1] dealt with a spherically-symmetrical collapse, but in the later papers[2,3] it was shown that a spherical collapse should have certain "pathological" singularities and can hardly be realized in nature. This is connected with the fact that the electric field of spherically-symmetrical potential oscillations vanishes at the origin, where there is consequently not a maximum but a minimum of the high-frequency pressure. The formation of a spherically-symmetrical cavern having a maximum depth at the origin is therefore impossible, and a spherically-symmetrical collapse can be realized only in the form of a quasiplane layer converging towards the center (spherical soliton[3]), which is unstable[4] relative to azimuthal perturbations.

This difficulty is overcome if the requirement of spherical symmetry is dispensed with and it is proposed that a field configuration with a maximum of the high-frequency pressure is realized at the origin. The result of a numerical simulation of a two-dimensional configuration of this type is described in this article.

We consider the subsonic case (relative oscillation energy density $W/nT \ll m/M$) and describe the plasma with the aid of a complex vector of the time-dependent envelope of the electric field

$$\mathbf{E} = (1/2)\{e^{-i\omega_p t}\vec{\mathcal{E}} + e^{i\omega_p t}\vec{\mathcal{E}}^*\}(8m\omega_p\sigma/e^2)^{1/2}(T_i + T_e)^{1/2}.$$

The vector $\vec{\mathcal{E}}$ satisfies the equation[5]

$$i(\partial\vec{\mathcal{E}}/\partial\tau) + \Delta\vec{\mathcal{E}} - \epsilon\,\mathrm{rot\,rot}\,\vec{\mathcal{E}} = \sigma|\vec{\mathcal{E}}|^2\vec{\mathcal{E}}$$

where

$$\tau = \omega_p t, \quad \epsilon = c^2/3\omega_p^2 r_D^2 - 1.$$

In the numerical calculation it was assumed that $\sigma = -10$ and $\epsilon = 5$, which corresponds to a hot plasma with $T = 50$ keV.

The problem was solved in the rectangle $0 < x < l_\parallel$, $0 < y < l_\perp$ with boundary conditions

$$\left.\frac{\partial\mathcal{E}_y}{\partial x}\right|_{x=0,\,l_\parallel} = 0, \quad \left.\frac{\partial\mathcal{E}_y}{\partial y}\right|_{y=0,\,l_\perp} = 0, \quad \left.\frac{\partial\mathcal{E}_x}{\partial y}\right|_{y=l_\perp} = 0,$$

$$\mathcal{E}_x\Big|_{x=0,\,l_\parallel} = 0, \qquad \mathcal{E}_x\Big|_{y=0} = 0. \tag{1}$$

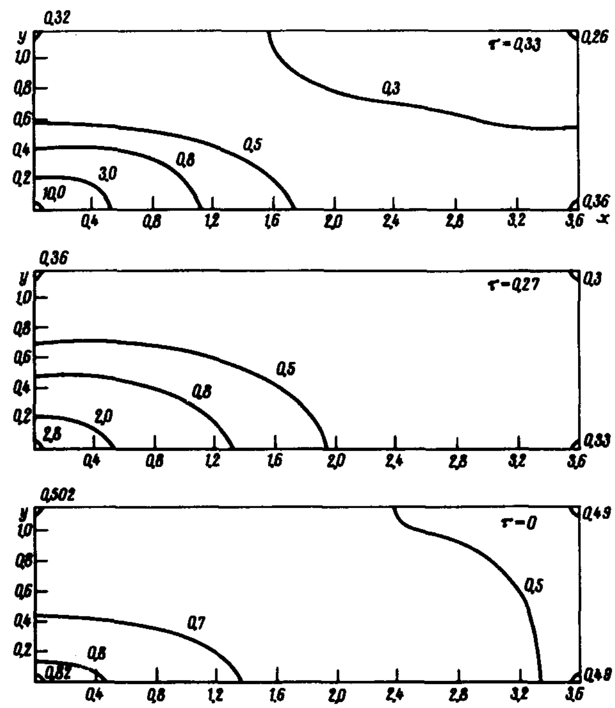
These conditions guarantee the possibility of continuing $\mathcal{E}_y$ symmetrically and $\mathcal{E}_s$ antisymmetrically through the x and y axes, so that the rectangle considered by us constitutes one-quarter of the cavern. Potential initial conditions were chosen:

$$\mathcal{E}_x = -\frac{\pi}{4}\frac{E_0}{l_\parallel}\left(y + \frac{l_\perp}{\pi}\sin\frac{\pi y}{l_\perp}\right)\sin\frac{\pi x}{l_\parallel},$$
$$\mathcal{E}_y = E_0\cos^2\left(\frac{\pi y}{2l_\perp}\right)\cos^2\left(\frac{\pi x}{2l_\parallel}\right) + \beta. \tag{2}$$

We have assumed $E_0 = 0.2$, $\beta = 0.7$, $l_\parallel = 3.6$, and $l_\perp = 1.16$. The spacing of the coordinate grid was 0.04, and the time spacing was varied during the course of the calculation between 3×10^{-3} and 5×10^{-4}.

The initial condition (2) ensures a maximum of the high-frequency pressure $u = \mathcal{E}_x^2 + \mathcal{E}_y^2$ at the origin.

The numerical experiment revealed the existence of a Langmuir collapse, viz., after a time on the order of $\tau = 0.35$ the high-frequency pressure at the origin increased by one order of magnitude and a collapsing



cavern was formed. The successive phases of development are seen from the figure, which shows the level lines of the high-frequency potential u. We see that this collapse has an approximate self-similar character, the ratio of the scales along x and y remaining approximately constant.

The observed collapse picture is preserved up to $W/nT \sim m/M$, after which the collapse acquires a supersonic character. In spite of the change of the character of the collapse, its main cause, the crowding out of the plasma from the cavern by the high-frequency pressure, is preserved in the supersonic regime and one should expect the collapse to continue until Laudau damping of the Langmuir waves sets in or until the electron trajectories intersect.

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