Dipole character of the collapse of Langmuir waves

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1. Langmuir collapse[1] is an important mechanism of energy transfer from Langmuir waves to plasma particles; a study of the structure of the plasma cavern produced in the collapse is of great interest. It was reported earlier[2,3] that in a number of cases it is possible to have a collapse that has no spherical or (in the planar case) axial symmetry. The result of the present paper is the establishment of the fact that the principal role in the collapse is played by caverns that compress in self-similar fashion, inside of which the distribution of the oscillating electric charge has the character of a dipole elongated in a plane perpendicular to the dipole moment.

Just as in[1], we start from the system of equations for the complex envelope of the high-frequency potential Ψ:

$$\nabla\Psi = \frac{3}{8}\frac{\mathbf{E}}{\left(2\pi n T_e \frac{m}{M}\right)^{1/2}}, \quad \nabla\phi = \frac{1}{2}\left(\mathbf{E}e^{-i\omega_p t} + \mathbf{E}^* e^{i\omega_p t}\right)$$

and the dimensionless density variation

$$n = \frac{3}{8}\frac{\delta n}{n_0}\frac{M}{m}.$$

Here ϕ is the electrostatic potential and n_0 is the unperturbed plasma density.

These equations, expressed in dimensionless variables

$$t = \frac{4}{3}\frac{m}{M}\omega_p t, \quad r = \frac{4}{3}\sqrt{\frac{m}{M}}\frac{r}{r_D},$$

where $r_D = \sqrt{T_e/4\pi e^2 n_0}$ is the Debye radius, take the form

$$\Delta(i\Psi_t + \Delta\Psi) = \operatorname{div}(n\nabla\Psi),$$
$$n_t + \Delta\Phi = 0, \quad \Phi_t + n + |\nabla\Psi|^2 = 0, \tag{1}$$

where Φ is the hydrodynamic potential.

In the case of a small amplitude wave $|\nabla\Psi|^2 \ll 1$, $|E|^2/8\pi \ll m/M$, the system (1) reduces to a single equation[1] (static approximation)

$$\Delta(i\Psi_t + \Delta\Psi) + \operatorname{div}(|\nabla\Psi|^2\nabla\Psi) = 0. \tag{2}$$

The system (1) conserves the integrals

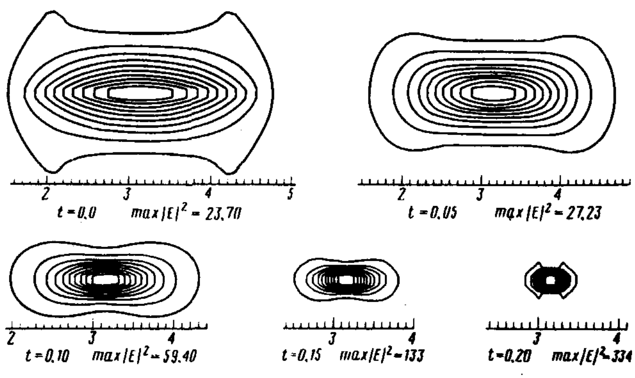

FIG. 1.

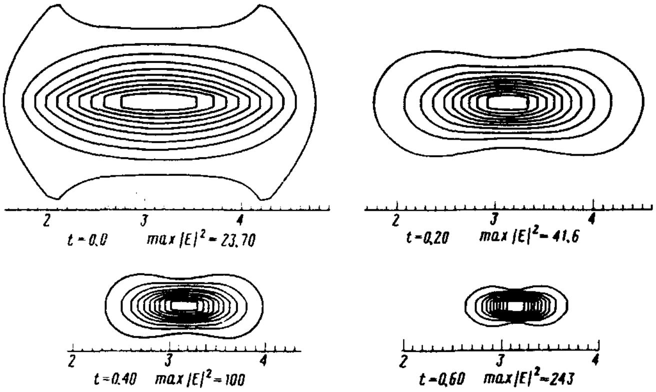

FIG. 2.

$$I_1 = \int |\nabla\Psi|^2 d\mathbf{r},$$

$$I_2 = \int \left(|\Delta\Psi|^2 + n|\nabla\Psi|^2 + \frac{1}{2}n^2 + \frac{1}{2}(\nabla\Phi)^2 \right) d\mathbf{r},$$

and the system (2) conserves the integrals I_1 and

$$I_2 = \int \left(|\Delta\Psi|^2 - \frac{1}{2}|\nabla\Psi|^4 \right) d\mathbf{r}.$$

Equations (1) and (2) have common stationary solutions $\Psi = \phi\exp(i\lambda^2 t)$, and $\phi(\mathbf{r})$ satisfies the equation

$$\Delta(-\lambda^2\phi + \Delta\phi) + \operatorname{div}(|\nabla\phi|^2\nabla\phi) = 0. \tag{3}$$

Multiplying (3) by $(\mathbf{r}\nabla\phi^*)$, adding it to the complex conjugate, and integrating, we can establish that in the two-dimensional case $I_2 = 0$ for stationary solutions; in the three-dimensional case $I_2 = \lambda^2 I_1$.

2. We present the results of some numerical calculations for the system (1) and for Eq. (2). We consider the problem in a rectangular region on the (x, y) plane, with boundary conditions of the second kind. The initial distribution of the oscillating electrostatic charge $\rho = \nabla^2\Psi$ takes the form

$$\rho(x, y, 0) = \rho_0 F(x, y) e^{im\theta}$$

$$F(x, y) = \begin{cases} z^{1/2}; & y < y_0 \\ -z^{1/2}; & y > y_0, \\ 0; & y = y_0,\ z < 0 \end{cases} \qquad z = 1 - \left(\frac{x - x_0}{a}\right)^2 - \left(\frac{y - y_0}{b}\right)^2 \tag{4}$$

ρ_0, m, a, b are parameters, $\theta = \tan^{-1}\dfrac{x - x_0}{y - y_0}$.

In the case of the system (1), we assume in addition that

$$n(x, y, 0) = -|\nabla\Psi|^2,$$

$$n_t(x, y, 0) = 0,$$

Such an initial distribution of the charge makes it possible to specify various initial configurations of the cavern. In particular, a dipole elongated parallel and transverse to the dipole moment, an axially-symmetrical density well with a rotating field in the center ($m = 1$), etc. For arbitrary initial values in (4), the qualitative character of the behavior of the solutions turns out to depend only on the value of the integral I_2. When $I_2 > 0$, a "spreading" of the initial distribution takes place. At $I_2 = 0$, the solution retains approximately the form of the initial condition. At $I_2 < 0$, both for Eq. (2) and for the system (1), a collapse is observed in the form of an explosionlike growth of the Langmuir-wave amplitude and of the frequency variation. In the collapse, just as in the static case (2) and in the "acoustic" case (1), the plasma cavern begins to contract rapidly in self-similar fashion, and its shape is independent of the initial distribution (4). In the calculation process, the intensity of the Langmuir waves in the center of the cavern increases by several dozen times, without loss of accuracy. Figures 1 and 2 show collapse in the static case of an initially axially-symmetrical distribution with rotating field ($m = 1$).

3. In the planar case, Eqs. (1) and (2) have no self-similar solutions capable of accounting for the results

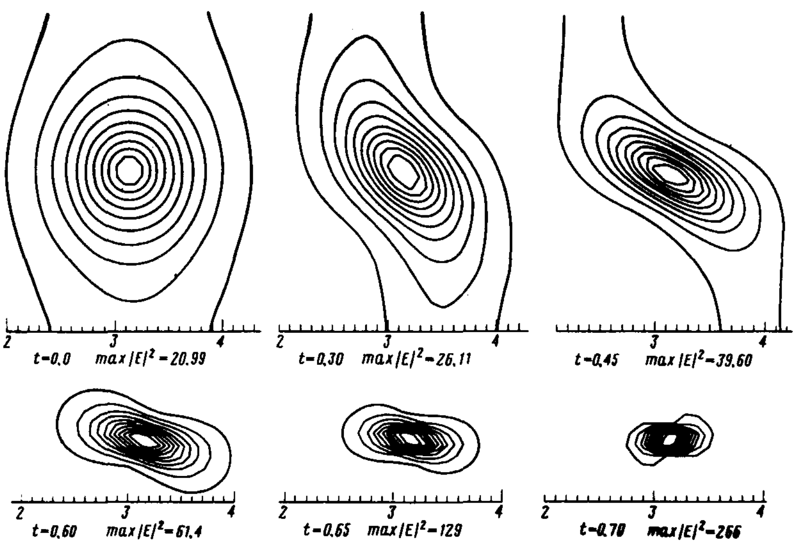

FIG. 3.

of the numerical experiments. They do, however, have asymptotic self-similar solutions whose accuracy improves as the collapse point is approached.

In the static approximation, we seek such a solution in the form

$$\Psi = Ae^{i\Phi}, \qquad A = \phi\left(\frac{r}{f(t)}\right) + E_0 r + \ldots,$$
$$\Phi = \int \frac{dt}{f^2(t)} + ff'\Phi\left(\frac{r}{f(t)}\right) + \ldots . \tag{5}$$

Here $\phi(\vec{\xi})$ is a real solution of Eq. (3), $f(t_0)=0$, and t_0 is the collapse point. If t is close to t_0 and $\Phi(\vec{\xi})$ is suitably chosen, the solution (5) in the principal orders in $1/f$ satisfies Eq. (2). Substituting (5) in I_2 and recognizing that $\int(\nabla^2\phi|^2 - \frac{1}{2}|\nabla\phi|^4 d\vec{\xi} = 0$, we obtain in the principal order in $1/f$

$$af'^2 - \frac{\beta}{f} = \text{const} ,$$

$$a = \int[(2\nabla\Phi\nabla\phi + \phi\Delta\Phi)^2 - \phi^2|\nabla\phi|^2(\nabla\Phi)^2]d\vec{\xi},$$

$$\beta = 2\int(E_0\nabla\phi)(\nabla\phi)^2 d\vec{\xi}$$

from which it follows that

$$f(t) \approx (t_0 - t)^{2/3} .$$

A similar solution[4] was constructed earlier in the theory of self-focusing.

From among all the solutions of (3), there should be realized in (5) a solution with a minimum of I_1. It can be assumed that such a solution is of the dipole type—the axially symmetrical solution has a large value of I_1 owing to the condition $|\phi_r(0)=0$ and owing to the nonmonotonic behavior of $|\phi_r(r)|^2$.

Within the framework of the system (1), an approximate self-similar solution can be obtained in the case $n \gg 1$ by putting

$$\Psi_t \approx i\lambda^2(t)\Psi.$$

Assuming

$$\lambda^2(t) = \frac{1}{(t_0-t)^2}, \quad \Psi = \phi\left(\frac{r}{t_0-t}\right), \quad n = \frac{1}{(t_0-t)^2}\, n\left(\frac{r}{t_0-t}\right) \tag{6}$$

we obtain for ϕ and n the system of equations

$$\Delta(-\phi + \Delta\phi) = \operatorname{div}(n\nabla\phi),$$
$$6n + 6\vec{\xi}\frac{\partial n}{\partial\vec{\xi}} + \xi_\alpha\xi_\beta\frac{\partial^2 n}{\partial\xi_\alpha\partial\xi_\beta} - \Delta n = \Delta|\nabla\phi|^2. \tag{7}$$

It was shown earlier[2] that the system (7) has no axially-symmetrical physically reasonable solutions, but this prohibition does not extend to distributions of the dipole type.

Both the solution (5) and the solution (6) correspond to "strong collapse," i.e., to a finite Langmuir-wave energy that falls in the singularity, and are in qualitatively good agreement with the results of the numerical experiment.

[1] V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys.-JETP 35, 908 (1972)].
[2] L. M. Legtyarev, V. E. Zakharov, and L. M. Rudakov, Preprint IPM (Appl. Math. Inst.) No. 34, 1974. Dep. No. 1449-74; Zh. Eksp. Teor. Fiz. 68, No. 1 (1975) [Sov. Phys.-JETP 41, No. 1 (1975)].
[3] V. E. Zakharov, A. F. Mastryukov, and V. S. Synakh, ZhETF Pis. Red. 20, 7 (1974) [JETP Lett. 20, 3 (1974)].
[4] V. E. Zakharov, V. E. Zakharov, V. V. Sobolev, and V. S. Synakh, ZhETF Pis. Red. 14, 564 (1971) [JETP Lett. 14, 390 (1971)].