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Nonlocal vorticity cascade in two dimensions

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The whole set of simultaneous correlation functions describing steady vorticity cascade is obtained from the Euler equation by a straightforward procedure. Nonlocality of the cascade provides for a large logarithmic parameter that enables one to obtain a universal set of the correlation functions of the vorticity ω in the inertial interval: $\langle\omega^n(\mathbf{r}_1)\omega^n(\mathbf{r}_2)\rangle \propto \ln^{2n/3}(L/\mid \mathbf{r}_1 - \mathbf{r}_2 \mid)$, with L being the scale of the external pump.

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A remarkable feature of incompressible fluid turbulence is the presence of an essential interaction between eddies of strongly different scales due to a sweeping of small eddies by large ones. Such a nonlocality in k space manifests itself differently in two and in three dimensions. In the three dimensional (3D) case, it was shown that any divergences (powerlike as well as logarithmic ones) that appear in Eulerian variables for the Kolmogorov spectrum disappear after passing into Lagrangian variables [1,2]. This means that sweeping has a strong effect upon the time dependencies of the correlation functions while the simultaneous correlators are determined by a dynamical interaction which is local in k space. Due to simple geometrical reasons, the sweeping has stronger consequences in two rather than in three dimensions. Even if one considers simultaneous correlators, logarithmic infrared divergences are present for the energy spectrum $E(k) \propto k^{-3}$ obtained for a vorticity cascade from a dimensional analysis [3]. The presence of the divergences means that nonlocal interaction should play a substantial role in shaping the energy spectrum. One can show that the powers of the logarithm increase with the order of perturbation theory. This suggests that a substantial renormalization of the spectrum might occur. By considering a one-loop approximation, Kraichnan found the spectrum $k^{-3}\ln^{-1/3}(kL)$ which corresponds to the vorticity correlator $\langle\omega(\mathbf{r}_1)\omega(\mathbf{r}_2)\rangle \propto \ln^{2/3}(L/\mid \mathbf{r}_1 - \mathbf{r}_2 \mid)$ depending explicity on the pumping scale L [3]. This estimate can be obtained also by different uncontrollable closures assuming weak phase coherence (see [4] and references therein). A natural question arises: does the account of higher orders and of (at least substantial) time correlations destroy this spectrum? Alternative predictions for the exponent have been suggested: -4 [5], $-11/3$ [6], and those of conformal models [7]. The way to solve the problem of accounting for nonlocal interaction in the vorticity cascade is suggested in the present paper.

"Logarithm" is the keyword that explains how it is possible to find a statistical solution of the Euler equation. We show that the simultaneous vorticity correlators are solely determined by the influence of larger scales, which can be described in terms of the tensor of velocity derivatives with a symmetric part (strain) and an antisymmetric one (vorticity). This confirms the generally accepted physical picture of the vorticity cascade: a fluid blob embedded into a larger scale velocity shear is extended along the direction of a positive strain value and compressed along the direction of a negative one; such stretching provides for the vorticity flux into the small scales with the rate of transfer proportional to the strain. Vorticity rotates the fluid blob decelerating stretching due to interchange of the axis of a positive and negative strain. The problem of determining the vorticity spectrum in the inertial interval turns thus to be the problem of a passive scalar [8,9] advected by the velocity field produced by the previous (larger) scales. As one passes into smaller scales, the effective vorticity and strain that act on the scalar are renormalized. To find the law of the renormalization one should take into account time correlations between the velocity gradients produced by different spectral intervals. By direct calculation of the different-time correlation function, we show that, contrary to previous assumptions, the correlations are substantial. Surprisingly, that does not affect the scaling of simultaneous correlation functions which are determined solely by the flux of the squared vorticity and the mean stretching rate (average strain) $\bar{s}$. The control parameter of the theory is the product $\bar{s}\tau_s$ where τ_s is the strain correlation time. We can solve the problem in two limits assuming this parameter to be either small or large. The forms of vorticity correlation functions are the same in both limits. We find $\bar{s}$ self-consistently in the following way: The vorticity correlation function is expressed via the mean strain, then the strain correlation function is expressed via the vorticity one by inverting the *curl* operator. All calculations are done in the locally comoving reference frame (in the so-called quasi-Lagrangian variables) so that different-time correlation functions are spatially nonuniform while the final answers for simultaneous correlation functions possess spatial homogeneity. Solving the integral equations for the average strain we get a solution $\bar{s} \propto \ln^{1/3}(kL)$. We can calculate the correlation time of the vorticity (which is logarithmically large in the comoving reference frame) but not of the strain. This prevents our formalism from being quantitative: we cannot calculate the numerical factors but only the powers of logarithms.

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The fact that the renormalization is only logarithmic and anomalous exponents are absent for the vorticity cascade seems quite natural. The point is that all powers of the vorticity are the integrals of motion in the 2D Euler equation. The solution found corresponds to a dimensionless (logarithmic) vorticity thus allowing one to have nonzero fluxes of all vorticity integrals. The same reasoning explains why a conformal minimal model [7] cannot be a general turbulent solution [10].

Let us turn to the calculations. The Euler equation for the vorticity field $\omega = \operatorname{curl} \mathbf{u}$

$$\dot{\omega} + u_\alpha \nabla_\alpha \omega = \phi \tag{1}$$

contains the random external source ϕ. To eliminate homogeneous sweeping we pass to the locally comoving reference frame introducing the quasi-Lagrangian (QL) velocity $\mathbf{v}(t,\mathbf{r})$ related to the Eulerian velocity:

$$\mathbf{u}(t,\mathbf{r}) = \mathbf{v}\left(t, \mathbf{r} - \int^t \mathbf{v}(\tau,0)\, d\tau\right) . \tag{2}$$

The presence of a marked point ($\mathbf{r}=0$) makes the theory spatially nonuniform in QL variables. This is the price one should pay for sweeping elimination [2]. Equation (1) takes the form

$$\dot{\omega} + (v_\alpha - v_{0\alpha})\nabla_\alpha \omega = \phi , \tag{3}$$

where $v_{0\alpha} = v_\alpha(t,0)$. We aim at finding simultaneous correlators that are the same for both sets of variables.

As a first step, we take in (3) only the long-range velocity with the wave vectors $q \lesssim L^{-1}$, temporarily omitting the short-range part of v. We are going to describe thus the correlation functions of the passive scalar ω advected by a large-scale velocity field $V(t,\mathbf{r})$. We do this in the spirit of Kraichnan's approach [9]. For the points with $|\,\mathbf{r}\,| < L$ one can expand $V_\alpha(\mathbf{r}) - V_\alpha(0) = \sigma_{\alpha\beta} r_\beta$. Solving the resulting equation

$$\dot{\omega} + \sigma_{\alpha\beta} r_\beta \nabla_\alpha \omega = \phi , \tag{4}$$

one gets

$$\omega(t,\mathbf{r}) = \int_{-\infty}^{t} d\tau\, \phi(\tau, w(t,\tau)\mathbf{r}) . \tag{5}$$

Here w must satisfy $\dot{w} + w\sigma = 0$. The formal solution of this equation can be written as

$$w(t,t_0) = \tilde{T}\exp\left(-\int_{t_0}^{t} dt\, \sigma(t)\right) , \tag{6}$$

where $\tilde{T}$ designates the antichronological ordering. It is useful to divide $2\sigma_{\alpha\beta} = s_{\alpha\beta} - a_{\alpha\beta}$ where the symmetric tensor s is the strain of the velocity V and the antisymmetric part $a_{\alpha\beta} = \Omega\epsilon_{\alpha\beta}$ is expressed through its vorticity $\Omega = \operatorname{curl} \mathbf{V}$. The matrix w can be represented in the form of the product $w = w_s w_a$:

$$\dot{w}_a - w_a a/2 = 0, \quad \dot{w}_s + w_s \hat{s}/2 = 0 , \tag{7}$$

where $\hat{s} = w_a s w_a^T$. The transfer matrix w_s describes stretching while w_a describes rotation of a fluid element.

The pair correlation function of ω

$$\langle \omega(t_1,\mathbf{r}_1)\omega(t_2,\mathbf{r}_2)\rangle = \int_{-\infty}^{t_1} d\tau_1 \int_{-\infty}^{t_2} d\tau_2 \Xi(\tau_1 - \tau_2, w(t_1,\tau_1)\mathbf{r}_1 - w(t_2,\tau_2)\mathbf{r}_2) \tag{8}$$

is expressed via $\langle \phi(t_1,\mathbf{r}_1)\phi(t_2,\mathbf{r}_2)\rangle = \Xi(t_1-t_2, \mathbf{r}_1-\mathbf{r}_2)$. Averaging with respect to both external velocity and external source is implied. The simultaneous correlation function in the inertial interval (for $r_1, r_2 \ll L$) can be extracted from (8). The main contribution to the integral

$$\langle \omega(t,\mathbf{r}_1)\omega(t,\mathbf{r}_2)\rangle = \int_{-\infty}^{t} d\tau \int_{2(\tau-t)}^{2(t-\tau)} d\tau' \Xi(\tau', w(t,\tau + \tau'/2)\mathbf{r}_1 - w(t,\tau-\tau'/2)\mathbf{r}_2) \tag{9}$$

is determined by the intervals of time τ, τ' when it is possible to neglect the dependence of Ξ on the space argument. Then we get the estimate

$$\langle \omega(t,\mathbf{r}_1)\omega(t,\mathbf{r}_2)\rangle \simeq \int_{t-\tau_*}^{t} d\tau \int_{2(\tau-t)}^{2(t-\tau)} d\tau' \Xi(\tau',0) \simeq \tau_* P_2 . \tag{10}$$

The integral $P_2 = \int_{-\infty}^{\infty} dt\, \Xi(t,0)$ is the pumping rate of the squared vorticity (enstrophy flux) and τ_* is the time necessary for the mean distance between two points to increase from r_{12} to L: $\exp(\bar{s}\tau_*) r_{12} \sim L$. It is correct if the value τ_* is larger than both τ_s and the characteristic time of Ξ, which is true for sufficiently large values of $\ln(L/r_{12})$. We thus come to the Batchelor-Kraichnan expression for the simultaneous pair correlation function

$$\langle \omega(\mathbf{r}_1)\omega(\mathbf{r}_2)\rangle \simeq (P_2/\bar{s}) \ln(L/\,|\,\mathbf{r}_1 - \mathbf{r}_2\,|) . \tag{11}$$

The correlator (11) corresponds to $F(k) = P_2/\bar{s}k^2$ in the $\mathbf{k}$ space. This gives $E(k) \propto k^{-3}$.

The mean stretching rate $\bar{s}$ can be easily found in the cases of a rapid and slow strain. We start from considering the rapid case (assuming $\bar{s}\tau_s \ll 1$):

$$\begin{aligned} \bar{s} &= \frac{1}{16}\int dt_1\, \mathrm{tr}\langle[\hat{s}(t_1)\hat{s}(t_2)]\rangle \\ &= \frac{1}{16}\int dt_1 \mathrm{tr}\langle w_a^T(t_1,t_2) s(t_1) w_a(t_1,t_2) s(t_2)\rangle . \end{aligned} \tag{12}$$

The value $\bar{s}$ is related to w_s (determined by $\hat{s}$) but not to w_a since $|\, w_a \mathbf{r}\,| = |\,\mathbf{r}\,|$ by virtue of $w_a w_a^T = 1$. As one can see, the value of the mean strain $\bar{s}$ strongly depends on the relation between the time τ_s of the strain correlation and the time $\tau_a \simeq \Omega^{-1}$. If $\tau_s < \tau_a$ then the presence of the vorticity is irrelevant [one can put $w_a(t_1,t_2) \approx 1$]. If, in contrast, $\tau_s > \tau_a$, then $\bar{s}$ is suppressed [11].

To find the vorticity correlation time, we calculate

$$\int_{-\infty}^{\infty} dt_1 \langle \omega(t_1, \mathbf{r}_1)\omega(0, \mathbf{r}_2)\rangle = \int_0^{\infty} d\tau_1 \int_0^{\infty} d\tau_2 \int_{-\infty}^{\infty} dt_1 \Xi(t_1 - \tau_1 + \tau_2, w(t_1, t_1 - \tau_1)\mathbf{r}_1 - w(0, -\tau_2)\mathbf{r}_2)$$
$$\simeq (P_2/\bar{s}^2)[\ln^2(L/r_1) + \ln^2(L/r_2) + \cdots] \,. \tag{13}$$

The main contribution to (13) stems from the region where $\exp(\tau_1 \bar{s}) \lesssim L/r_1$ and $\exp(\tau_2 \bar{s}) \lesssim L/r_2$. This gives the logarithm-squared terms that are space inhomogeneous as it is natural for different-time correlation functions in QL variables. The double logarithm here manifests an anomalously large correlation time of ω which is a consequence of the QL choice of variables. At $r_1 = 0$ or $r_2 = 0$, the vorticity correlation time turns into infinity which corresponds to infinite correlation time in Lagrangian variables.

Many-point correlation functions can be extracted from the same relation (5). For example, the four-point correlation function is as follows:

$$\langle \omega_1\omega_2\omega_3\omega_4\rangle = \int^{t_1} d\tau_1 \int^{t_2} d\tau_2 \int^{t_3} d\tau_3 \int^{t_4} d\tau_4 \langle \phi_1\phi_2\phi_3\phi_4\rangle \,.$$

The reducible part of $\langle\phi_1\phi_2\phi_3\phi_4\rangle$ gives the contribution to $\langle\omega_1\omega_2\omega_3\omega_4\rangle$ which is a product of the pair correlation functions (8). In the contribution supplied by the irreducible part of $\langle\phi_1\phi_2\phi_3\phi_4\rangle$, there will be only one integration giving a logarithmic factor so that in the inertial interval it is small in comparison with the product of the pair correlation functions proportional to the squared logarithm. The same is true for many-point correlation functions of the order $n < \ln(L/r)$: in the limit of the large logarithm when $\tau_* \gg \tau_s$, the main contribution to the correlation functions is supplied by their reducible parts. Therefore the statistics of the passive scalar ω advected by a large-scale velocity field appears to be asymptotically Gaussian irrespective of the statistics of the external influence ϕ [see [11] for the proof and for the analysis of the non-Gaussian tails of the probability distribution at finite values of $\ln(L/r)$].

Let us emphasize that this statement on the Gaussian property of a steady forced turbulence of a passive scalar is new compared to Kraichnan's theory [9] where the statistics of ω were claimed to depend on the statistics of ϕ. Such a dependence was thought of as the result of the conservation of arbitrary power of vorticity so that the $(2n)$th correlation function should be determined by the input of the $(2n)$th integral of motion and be independent of lower moments. One can show that this is not the case since only the flux of squared vorticity is constant in the inertial interval while higher fluxes grow with k due to a contribution to their pumping from lower moments:

$$\langle(\mathbf{V}_1\cdot\nabla_1 + \mathbf{V}_2\cdot\nabla_2)\omega_1^n\omega_2^n\rangle \propto \ln^{(n-1)}(L/r_{12}). \tag{14}$$

Now let us turn to the complete problem of describing vorticity cascade taking into account the contributions of different scales. The velocity is now connected to the vorticity: $v_\alpha(r) = \epsilon_{\alpha\gamma}\nabla_\gamma \int d^2r'\omega(r')\ln(L/R)/2\pi$ with $\mathbf{R} = \mathbf{r}' - \mathbf{r}$. The velocity difference that enters Eq. (3) can be divided into three parts:

$$v_\alpha(r) - v_{0\alpha} = -\tfrac{1}{2}\omega(r)\epsilon_{\alpha\beta}r_\beta + \tfrac{1}{2}s_{\alpha\beta}(r)r_\beta + \frac{\epsilon_{\alpha\gamma}}{2\pi}\int_{r'\le r} d^2r' \left(\frac{R_\gamma}{R^2} - \frac{r'_\gamma}{r'^2}\right) \times[\omega(r') - \omega(r)] \,. \tag{15}$$

Here we have introduced the scale-dependent strain that is determined by the scales larger than r:

$$s_{\alpha\beta} = \frac{\epsilon_{\alpha\gamma}}{2\pi}\int_{r'>r} d^2r' \left(\frac{2r'_\gamma r'_\beta}{r'^4} - \frac{\delta_{\gamma\beta}}{r'^2}\right)\omega(r') + (\alpha \leftrightarrow \beta). \tag{16}$$

The crucial point in the consideration is to drop the last term in (15). By doing so, one neglects in (3) both the contributions from smaller scales and an interaction of the scales of the same order. The formal reasons for this are the small region of integration and the presence of the difference $\omega(r') - \omega(r)$ in this term. As a result, it gives smaller powers of the logarithm in the renormalization of the correlation functions than (16) gives. The physical reason for such a neglect is that small-scale influence is averaged while the spectral transfer due to interaction of comparable scales can be neglected in comparison with the spectral transfer by the renormalized strain. After neglecting this term, we come to Eq. (4) where Ω is replaced by $\omega(r)$ and where the strain $\bar{s}(r)$ should be found self-consistently as to provide the vorticity distribution which gives that very strain according to (16).

If one presumes $\bar{s}\tau_s \ll 1$, then the averaged strain that determines the behavior of the function w in (8) and (9) is expressed via the different-time correlation function according to (12):

$$\int_{-\infty}^{\infty} dt_1 \langle s_{\beta\alpha}(t_1, \mathbf{r}_1)s_{\mu\nu}(t_2, \mathbf{r}_2)\rangle = \frac{4}{(2\pi)^2}\int d^2R_1 \int d^2R_2 \left(\epsilon_{\alpha\gamma}\frac{R_{1\gamma}R_{1\beta}}{R_1^4} + \epsilon_{\beta\gamma}\frac{R_{1\gamma}R_{1\alpha}}{R_1^4}\right)\left(\epsilon_{\nu\lambda}\frac{R_{2\lambda}R_{2\mu}}{R_2^4} + \epsilon_{\mu\lambda}\frac{R_{2\lambda}R_{2\nu}}{R_2^4}\right)$$
$$\times \int_{-\infty}^{\infty} dt_1 \langle \omega(t_1, \mathbf{r}'_1)\omega(t_2, \mathbf{r}'_2)\rangle \,. \tag{17}$$

One should substitute here the expression that is a generalization of (13) for r-dependent $\bar{s}(r)$. The function w is now assumed to be expressed via the current strain $\bar{s}(r)$ which we are going to find. We are interested in logarithmic terms at $r \simeq r'_1, r'_2$. It can be readily checked that terms depending only on r_1 or on r_2 [like double logarithmic terms in (13)] do not give any logarithmic contribution to (17) due to averaging over angles. The same is true for any term depending separately on r_1, r_2. The main contribution to (17) stems from the region $r_1 \simeq r_2$ and gives thus the logarithm in the first power:

$$\int_{-\infty}^{\infty} dt_1 \langle \omega(t_1,\mathbf{r}_1')\omega(t_2,\mathbf{r}_2')\rangle \to \int d\xi \frac{P_2}{\bar{s}^2(\xi)} \ . \tag{18}$$

The strain here is considered as a function of the logarithmic variable $\xi = \ln(L/r)$. The contribution to (17) due to (18) can be calculated explicitly. Up to a logarithmic accuracy it is

$$\int_{-\infty}^{\infty} dt_1 \langle s_{\beta\alpha}(t_1,\mathbf{r}_1) s_{\mu\nu}(t_2,\mathbf{r}_2)\rangle$$

$$= \int d\xi \frac{P_2}{2\bar{s}^2(\xi)} (\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\beta}\delta_{\mu\nu} + \delta_{\alpha\nu}\delta_{\beta\mu}) \ . \tag{19}$$

We thus come to a self-consistent equation for $\bar{s}(\xi)$ which is valid at $\xi \gg 1$. This equation follows from (19)

$$\bar{s}(\xi) \simeq P_2 \int^{\xi} \frac{d\xi'}{\bar{s}^2(\xi')} \ . \tag{20}$$

The solution of Eq. (20) for large ξ is

$$\bar{s} \simeq (P_2\xi)^{1/3} \ . \tag{21}$$

Now instead of (11) we have $\langle \omega(\mathbf{r}_1)\omega(\mathbf{r}_2)\rangle \simeq P_2 \int d\xi/\bar{s} \propto \xi^{2/3}$. It gives $E(k) \simeq P_2^{2/3} k^{-3} \ln^{-1/3}(kL)$.

The logarithmic renormalization with a rapid strain would be self-consistent if $\bar{s}\tau_s \to 0$ as $\xi \to \infty$. Unfortunately, if one estimates the correlation time $\tau_s \simeq \bar{s}/\langle s^2\rangle \propto \xi^{-1/3}$ one can see that $\bar{s}\tau_s$ does not tend to zero. This means that the renormalized strain is not rapid. Fortunately, one can make all the above steps (solving the passive scalar problem and then renormalization) also in the opposite limit of a slow strain. Differentiating the equation for w and neglecting $\dot{s}$ comparing to s^2 one obtains the equation $d^2 w_s/dt^2 + w_s s^2/4 = 0$. In 2D, s^2 is proportional to a unit matrix so that the answer is given by the central limit theorem as in a scalar case: as $t \to \infty$, $\ln|wr| \propto \int^t \sqrt{\mathrm{tr}s^2(t')}dt' \propto \bar{s}t$ where $\bar{s} = \langle\sqrt{\mathrm{tr}s^2}\rangle$. The law of renormalization is now as follows: $s^2(\xi) \simeq \int^{\xi} d\xi'/s(\xi')$ which gives again (21) and the same energy spectrum.

Note that the simultaneous strain correlator $\langle s^2\rangle \propto \xi^{2/3}$ grows with ξ by the same law as $\langle\omega^2\rangle$. This means that whatever be the relation between the strain and the vorticity produced by the external pump at large scales, that relation tends to the universal limit in the inertial interval. The independence of the ratio $\langle s^2\rangle/\langle\omega^2\rangle$ of k in the inertial interval justifies the use of s instead of $\hat{s}$ in (17) and guarantees that local suppression of the vorticity cascade which happens in the regions of large vorticity does not change the form of our solution. This phenomenon should have strong influence on the value of Kolmogorov's constant which we cannot calculate.

The logarithmic renormalization of the strain corroborates neglecting the last term in (15). By using the solution found, one can estimate the contribution from this term into the correlation functions and show that it has one power of ξ less than those taken into account.

This allows us to conclude that the shape of the energy spectrum is found correctly and it does not depend on a particular value of the unknown numerical constants. The same is true for the high-order correlation function: we can find k-dependencies but not constant factors. For example, the fluxes of the vorticity integrals of motion are as follows: $\langle(v_1-v_2)\omega_1^n\omega_2^n\rangle \propto \ln^{2(n-1)/3}(L/r_{12})$, to be compared with (14). We thus found the universal set of the simultaneous correlation functions which corresponds to small-scale dissipation of all vorticity integrals of motion. Please note that we have found the whole set of simultaneous correlation functions for a nonlinear turbulent problem directly from the Euler equation. The fact that the true form of the pair correlation function coincides with the result of the one-loop calculation means that there is no renormalization of the vertex index. Note that this coincidence does not mean that the approximation [3] is correct (see also [12]); what this does mean is that time correlations do not influence the structure of simultaneous correlation functions. Probably, there can exist other steady or quasisteady distributions [5,7] but they do not correspond to the dissipation rates of all powers of the vorticity nonzero in the inviscid limit. Unlike conformal solutions, our logarithmic solution is universal, i.e., the scaling does not depend on the statistics of the pumping. Recent numeric analyses confirm our predictions [13].

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