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Three-point correlation function of a scalar mixed by an almost smooth random velocity field

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We demonstrate that if the exponent γ that measures nonsmoothness of the velocity field is small then the isotropic zero modes of the scalar's triple correlation function have the scaling exponents proportional to $\sqrt{\gamma}$. Therefore, zero modes are subleading with respect to the forced solution that has normal scaling with the exponent γ. [S1063-651X(97)51605-9]

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Kraichnan's model of passive scalar advection by a δ-function-correlated velocity field [1] has become a paradigm within which an analytical theory of anomalous scaling in turbulence starts to appear [2–10]. The instrument is the perturbation theory around three limiting cases where scalar statistics are Gaussian: (i) infinite space dimensionality $d=\infty$ [1,5,6,10], (ii) an extremely irregular velocity field $\gamma=2$ which corresponds to a smooth scalar field [3,7,9], and (iii) a smooth velocity field $\gamma=0$ (the Batchelor-Kraichnan limit) [1,2,4,11–14,17]. The perturbation theory is regular in the first two cases and the sets of the exponents obtained agree when the limits intersect [6,7]. The perturbation theory around the Batchelor-Kraichnan limit is singular [4], only the dipole part of the three-point correlation function has been found so far [15]. In this paper, motivated by [4,15], we find the isotropic part of the triple correlation function and show that the leading term has a normal scaling in the convective interval.

It is instructive to first discuss the physics involved in understanding the significant difference between the first two limits on the one hand and the third limit on the other. Since the scalar field, at any point, is the superposition of fields brought from d directions, then it follows from a central limit theorem that the scalar's statistics approach Gaussian when space dimensionality d increases. In the case $\gamma=2$, an irregular velocity field acts like Brownian motion so that turbulent diffusion is much like linear diffusion: statistic is Gaussian provided the input is Gaussian. What is general in both limits $d=\infty$ and $\gamma=2$ is that the degree of Gaussianity (say, flatness) is independent of the ratio r/L, where r is a typical distance in the correlation function and L is an input scale. Quite contrary, $\ln(L/r)$ is the parameter of Gaussianity in the Batchelor-Kraichnan limit [14,16] so that statistic is getting Gaussian at small scales whatever the input statistics. At $\gamma=0$ the mechanism of Gaussianity is temporal rather than spatial: since the stretching is exponential in a smooth velocity field then the cascade time grows logarithmically as the scale decreases. That leads to the essential difference: at small, yet nonzero, $1/d$ and $2-\gamma$ the degree of non-Gaussianity increases downscales as one expects from intermittency and anomalous scaling, while at small γ the degree of non-Gaussianity first decreases downscales until $\ln(L/r)\simeq 1/\gamma$, and then starts to increase, the first region grows with diminishing γ. Already that simple reasoning shows that the way from the Batchelor-Kraichnan limit towards an anomalous scaling at a nonsmooth velocity field is not to be easy. The formal reason for this perturbation theory to be singular is that, at the limit $\gamma=0$, the many-point correlation functions have singularity (smeared by molecular diffusion only) at the collinear geometry—smooth velocity provides for homothetic transformation that does not break collinearity [17]. Even weak nonsmoothness of the velocity smears the singularity, i.e., strongly influences the solution in the narrow region near collinearity; such a situation calls for a boundary layer approach introduced into this problem by Shraiman and Siggia [4,11].

The three-point correlation function of the scalar $F(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ advected by δ-function-correlated velocity field satisfies the closed balance equation [2]

$$(\hat{\mathcal{L}}+\hat{\mathcal{L}}_d)F_3=-\chi_3. \tag{1}$$

Here, $\chi_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ is the triple correlation function of the (non-Gaussian) pumping which depends on differences $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$. If $|\mathbf{r}_{ij}|\ll L$ (where L is the pumping length) then $\chi_3\simeq P_3$, where P_3 is the third-order flux. At growing $|\mathbf{r}_{ij}|$ the function χ_3 tends to zero on distances larger than L. The operator of molecular diffusion $\hat{\mathcal{L}}_d=\kappa(\nabla_1^2+\nabla_2^2+\nabla_3^2)$ is expressed via the diffusivity κ, and the operator of turbulent diffusion

$$\hat{\mathcal{L}}=-(1/2)\sum_{i,j=1}^{3}\mathcal{K}_{\alpha\beta}(\mathbf{r}_{ij})\nabla_i^\alpha\nabla_j^\beta \tag{2}$$

is expressed via the eddy diffusivity [1]

$$\mathcal{K}^{\alpha\beta}(\mathbf{r})=Dr^{-\gamma}\left[r^2\delta^{\alpha\beta}-r^\alpha r^\beta+\frac{d-1}{2-\gamma}r^2\delta^{\alpha\beta}\right]. \tag{3}$$

Parameter γ is a measure of velocity nonsmoothness $0\leqslant\gamma\leqslant 2$.

At $\gamma=0$, the operator $\hat{\mathcal{L}}$ is singular for collinear geometry—see Eq. (8) below. That leads to an angular singularity in the correlation functions which is smoothed only by diffusion, which is therefore relevant at all scales [17]. Contrarily, at $\gamma>0$ the operator $\hat{\mathcal{L}}$ is not singular at the collinear geometry and therefore the angular singularity is absent, as was pointed out in [15]. Therefore we can omit the diffusive term $\hat{\mathcal{L}}_d$ in Eq. (1) in comparison with $\hat{\mathcal{L}}$. This is possible as long as $\kappa\ll\gamma Dr^2$.

In the following we believe χ_3 to be an isotropic function of $\mathbf{r}_{ij}$ which dictates the symmetry of the solution of Eq. (1). In this case, F_3 can be treated as a function of three distances r_{12}, r_{13}, and r_{23} only. Then the operator $\hat{\mathcal{L}}$ can also be rewritten in terms of the separations [14]

$$\hat{\mathcal{L}}=\frac{D(d-1)}{2-\gamma}\sum_{i>j} r_{ij}^{1-d}\partial_{ij}r_{ij}^{1+d-\gamma}\partial_{ij}+\cdots \tag{4}$$

where the dots stand for the terms with cross derivatives $\partial_{ij}\partial_{kl}$. Since $\chi_3=P_3$ at $r_{ij}\ll L$ we can easily find a solution of Eq. (1) in the region $r_{ij}\ll L$ (cf. [14]). Using Eq. (4) we get

$$F_{\rm forc}=\frac{(2-\gamma)P_3L^\gamma}{3D(d-1)d\gamma}\left(C-\frac{r_{12}^\gamma+r_{13}^\gamma+r_{23}^\gamma}{L^\gamma}\right), \tag{5}$$

where C is an arbitrary constant. We call Eq. (5) the forced solution, it satisfies the equation $r_{ij}\ll L$ but not necessarily matching conditions at $r_{ij}\gtrsim L$. The solution that satisfies the conditions can be written at $r_{ij}\ll L$ as $F=F_{\rm forc}+Z_0$, where Z_0 is a zero mode of the operator of the turbulent diffusion: $\hat{\mathcal{L}}Z_0=0$. We examine here different solutions of the equation $\hat{\mathcal{L}}Z=0$. Whether the given mode Z contributes the correlation function has to be determined from the matching at $r_{ij}\sim L$ which is beyond the scope of our paper. Note that we consider isotropic zero modes while anisotropic (dipole) zero modes for the more physical problem with an imposed mean gradient were treated in [15].

We introduce instead of r_{ij} the new set of variables

$$x_1=\frac{r_{13}}{r_{12}}\cos\theta,\quad x_2=\frac{r_{13}}{r_{12}}\sin\theta,\quad s=r_{12}r_{13}\sin\theta, \tag{6}$$

where θ is the angle between $\mathbf{r}_{12}$ and $\mathbf{r}_{13}$ and $-\infty<x_1<\infty$, $0<x_2<\infty$, $0<s<\infty$. Note that s is the only dimensional parameter among s, x_1, and x_2. The operator $\hat{\mathcal{L}}$ and both correlation functions χ_3 and F_3 should be invariant under permutations of r_{12}, r_{13}, and r_{23}. In terms of the variable $z=x_1+ix_2$ these transformations can be written as follows:

$$1\leftrightarrow 2:\quad z\to 1-z^*,\quad 2\leftrightarrow 3:\quad z\to\frac{1}{z^*},$$

$$1\leftrightarrow 3:\quad z\to 1+\frac{1}{z^*-1},\quad 1\to 2\to 3:\quad z\to\frac{1}{1-z},$$

$$1\to 3\to 2:\quad z\to 1-\frac{1}{z}, \tag{7}$$

where z^* is complex conjugated to z. The variable s (which is the doubled area of the triangle) is obviously invariant under the permutations.

Below we treat the dimensionality $d=2$. We start with the case $\gamma=0$. Then the operator (2) is rewritten in terms of the variables (6) as follows [17]:

$$\hat{\mathcal{L}}_0=2Dx_2^2(\partial_1^2+\partial_2^2). \tag{8}$$

Then a solution of the equation $\hat{\mathcal{L}}_0F_3=-\chi_3$ can be written by using the explicit expression for the resolvent of Laplacian (cf. [17]):

$$F_3(s,x_1,x_2)=\frac{1}{8\pi D}\int_{-\infty}^{+\infty}\int_0^\infty\frac{dx_1'dx_2'}{x_2'^2}\times\ln\left[\frac{(x_1'-x_1)^2+(x_2'+x_2)^2}{(x_1'-x_1)^2+(x_2'-x_2)^2}\right]\chi_3(s,x_1',x_2'). \tag{9}$$

We are interested in the behavior of F_3 at $r_{ij}\ll L$ which is not sensitive to the particular form of the pumping χ_3. Thus we can choose any convenient form of χ_3 supplying the convergence of the integral in Eq. (9). We take the Lorentzian $\chi_3=P_3/[1+(r_{12}^2+r_{13}^2+r_{23}^2)/L^2]$, perform the integration over x_1' and obtain

$$F_3=\frac{L^2P_3}{16sD}\int_0^\infty\frac{dt}{tw}\ln\left[\frac{[x_2(1+t)+w]^2+(x_1-1/2)^2}{(x_2|1-t|+w)^2+(x_1-1/2)^2}\right],$$

where $w=\sqrt{3/4+x_2^2t^2+L^2x_2t/(2s)}$. The asymptotic at $r_{ij}\ll L$ is $F_3=(P_3/2D)\ln[2L^2/(r_{12}^2+r_{13}^2+r_{23}^2+2\sqrt{3}s)]+\text{const}$, which can be rewritten as

$$F_3=\frac{P_3}{3D}\ln\frac{L^3}{r_{12}r_{13}r_{23}}+\frac{P_3}{6D}\ln\frac{(x_1^2+x_2^2)[(x_1-1)^2+x_2^2]}{[(x_1-1/2)^2+(x_2+\sqrt{3}/2)^2]^3}+\text{const}. \tag{10}$$

Here, the first term is the forced solution $F_{\rm forc}$ which can be obtained from Eq. (5) at $\gamma\to 0$, $d=2$, $C=3$. The second term in Eq. (10) is the zero mode Z_0, which has logarithmic singularities in the points $z=0$, $z=1$, and $z=\infty$ that is where one of the r_{ij} tends to zero. Note that the whole function F_3 has no singularity where one of the r_{ij} tends to zero. As follows from Eq. (10) the zero mode Z_0 has the linear term in the expansion over x_2 which, by virtue of Eq. (6), corresponds to $|\theta|$ term at small angles θ. That is just the angular singularity mentioned above. Let us emphasize again that the singularity is smoothed only by diffusion at $\gamma=0$ [17].

We see that the zero mode Z_0 at $\gamma=0$ does not depend on the dimensional parameter s. Besides, any function of s is a zero mode of the operator since $\hat{\mathcal{L}}_0$ does not contain the derivative over s, which is a remarkable property of the case $d=2$ and $\gamma=0$. Nevertheless, all those s-dependent zero modes do not contribute to Z_0.

Because of the scaling properties of $\hat{\mathcal{L}}$, it is possible to seek the zero mode in the scale-invariant form

$$Z=\left(\frac{s}{x_2}\right)^\Delta\{1+(x_1^2+x_2^2)^\Delta+[(1-x_1)^2+x_2^2]^\Delta\}X(x_1,x_2).$$

The function $X(x_1,x_2)$ should be invariant under all transformations (7) and have no angular singularities since the proportionality coefficient between Z and X is equal to $r_{12}^{2\Delta}+r_{13}^{2\Delta}+r_{23}^{2\Delta}$, as follows from Eq. (6).

The equation $\hat{\mathcal{L}}Z=0$ can be rewritten as $\hat{\mathcal{L}}_XX=0$, where $\hat{\mathcal{L}}_X$ is a differential operator of the second order over $\partial_1\equiv\partial/\partial x_1$ and $\partial_2\equiv\partial/\partial x_2$. Coefficients at the derivatives are quite complicated functions of x_1, x_2, Δ which can be found from Eq. (2). Fortunately only particular parts of the operator $\hat{\mathcal{L}}_X$ will be needed by us.

At $\gamma=0$ the operator $\hat{\mathcal{L}}_X$ is determined by Eq. (8). The operator tends to zero at $x_2\to 0$. Therefore, at small x_2, besides Eq. (8), we should also take into account the residue. The term leading to small x_2 can be written as

$$\hat{\mathcal{L}}_X \propto \hat{\mathcal{L}}_2 = [2x_2^2 + c_0(x_1)]\partial_2^2 - 4\Delta x_2 \partial_2 + 2\Delta(\Delta+1),$$

$$c_0(x) = -\frac{3\gamma}{4}(1-x)x[x\ln|x| + (1-x)\ln|1-x|]. \quad (11)$$

Note that $c_0>0$. Expression (11) is correct if $x_2 \ll |x_1|, |x_1-1|$; $|x_1|, |x_1-1| \gg \exp(-1/\gamma)$; $|x_1|, |x_1-1| \ll \exp(1/\gamma)$. The asymptotic behavior of a solution of $\hat{\mathcal{L}}_2 X=0$ at $x_2 \ll \sqrt{c_0}$ is

$$X = A_1(x_1) + A_2(x_1)x_2, \quad (12)$$

where $A_1(x)$ and $A_2(x)$ are arbitrary functions. The analyticity of X at small angles excludes the second term in Eq. (12) since it would supply the contribution to X which behaves $\propto|\theta|$. The equation $\hat{\mathcal{L}}_2 X=0$ can be solved explicitly, a solution having the asymptotic (12) with $A_2=0$ is expressed via the hypergeometric function

$$X_0 = A_1(x_1) F\left(-\frac{1+\Delta}{2}, -\frac{\Delta}{2}; \frac{1}{2}; -\frac{2x_2^2}{c_0(x_1)}\right). \quad (13)$$

Expression (13) gives the behavior of the zero mode in the vicinity of the boundary layer $x_2 \sim \sqrt{c_0}$. To describe the zero mode outside the boundary layer it is more convenient to return to Z which is a harmonic function there since $\hat{\mathcal{L}}$ can be approximated by Eq. (8). The asymptotic of Eq. (13), valid at $x_2 \gg \sqrt{c_0}$, gives $Z \propto (\Delta+1)\cos(\pi\Delta/2) - x_2 \sin(\pi\Delta/2)[c_0(x_1)/2]^{-1/2}$. That behavior occurs outside the boundary layer but at small x_2. We are interested in $\Delta \ll 1$. Thus we come to the following problem: find the harmonic function $Z(x_1,x_2)$ in the upper half-plane $x_2>0$ at the boundary condition

$$\sqrt{2c_0}\,\partial_2 Z + \pi\Delta Z = 0, \quad (14)$$

which is imposed on the function Z at $x_2=0$ since at small γ the width of the boundary layer is negligible.

Let us show that there is no zero mode with $\Delta \ll \sqrt{\gamma}$. A harmonic function $Z(x_1,x_2)$ inside the region can be presented as an integral of its normal derivative $\partial Z/\partial n$ along the contour which is the boundary of the region

$$Z(z) = \frac{1}{\pi}\oint |dt| \frac{\partial Z(t)}{\partial n} \ln|z-t|, \quad (15)$$

where $z = x_1 + ix_2$ and t is the complex variable going along the contour. Let us consider the contour consisting of the semicircles around the singular points $0,1,\infty$ and the parts near the real axis (outside the boundary layer but at small x_2) that link the semicircles. If $\Delta \ll \sqrt{\gamma}$ then Eq. (14) tells us that only contributions to Eq. (15) from the semicirles are relevant, since the contributions from the parts of the real axis are negligible in this case. Separate consideration of the vicinities of the singular points $0,1,\infty$ (see below) shows that the logarithmic derivative of Z has to be bounded there. Thus the only possible contribution to the zero mode associated, say, with the singular point $z=0$ is $\propto \mathrm{Re}\, \ln z = \ln\sqrt{x_1^2+x_2^2}$. The complete zero mode should be symmetric under transformations (7). Performing all the transformations to $\ln\sqrt{x_1^2+x_2^2}$ and summing the results we obtain zero. That means that the function possessing the required symmetry does not exist.

Another (equivalent) way of showing that there is no zero mode with $\Delta \ll \sqrt{\gamma}$ is to continue Z to negative x_2 by $Z(x_1,x_2) = Z(x_1,-x_2)$. Because in our case we can believe $\partial_2 Z(x_1,x_2)=0$ the function should be harmonic in semicircles surrounding the singular points $0,1,\infty$. Then one can use the properties of the analytical functions in the circles to exclude the existence of zero modes with bounded logarithmic derivatives near the singular points. Note the difference with the dipole case where such a mode has been found [15].

Here, we describe the set of zero modes that do not have additional smallness of Δ relative to $\sqrt{\gamma}$ so that the whole boundary condition (14) is to be taken into account. Necessary information about the structure of the modes can be extracted from the analysis of the vicinities of the singular points $z=0$, $z=1$, and $z=\infty$, where one needs a separate consideration. Using the symmetry properties (7) we can reduce the consideration to the vicinity of one of the points, say $z=1$. At $x_2 \ll 1$ and $|x_1-1| \ll 1$ the operator $\hat{\mathcal{L}}_X$ acquires the following form:

$$\hat{\mathcal{L}}_X = \mu[\rho^2\partial_\rho^2 + 3\rho\partial_\rho + 3\partial_\varphi^2] + 2\sin^2\varphi[\rho^2\partial_\rho^2 + \rho\partial_\rho + \partial_\varphi^2]$$
$$- 4\Delta[\sin^2\varphi\,\rho\partial_\rho + \cos\varphi\,\sin\varphi\,\partial_\varphi] + 2\Delta(\Delta+1), \quad (16)$$

$$x_1 - 1 = \rho\cos\varphi, \quad x_2 = \rho\sin\varphi, \quad \mu = \tfrac{1}{2}(\rho^{-\gamma}-1). \quad (17)$$

In the exponentially narrow vicinity of the singular point, where $\rho \ll \exp(-1/\gamma)$ one has $\mu \gg 1$ and the equation $\hat{\mathcal{L}}_X X=0$ is reduced to $[\rho^2\partial_\rho^2 + 3\rho\partial_\rho + 3\partial_\varphi^2]X=0$. Solutions of that equation can be expanded into the Fourier series $X_m \propto \sin(m\varphi)\rho^{\lambda_m}$, with $\lambda_m = -1 + \sqrt{1+3m^2}$ where only non-negative λ_m are taken since X should remain finite at $\rho\to 0$. We thus come to the conclusion that the matching condition on the boundary of the vicinity $\rho \sim \exp(-1/\gamma)$ should be imposed on the logarithmic derivative of X (or Z) which remains constant there.

Now, let us consider the region $1 \gg \rho \gg \exp(-1/\gamma)$, where $2\mu = \gamma\ln(1/\rho) \ll 1$. We can consider separately small angles $\varphi \ll 1$ where $\hat{\mathcal{L}}_X X=0$ is reduced to

$$[(3\mu + 2\varphi^2)\partial_\varphi^2 - 4\Delta\varphi\partial_\varphi + 2\Delta(\Delta+1)]X = 0, \quad (18)$$

which exactly corresponds to $\hat{\mathcal{L}}_2 X=0$. Again an appropriate solution of Eq. (18) is as follows:

$$X \propto F\left(-\frac{1+\Delta}{2}, -\frac{\Delta}{2}; \frac{1}{2}; -\frac{2\varphi^2}{3\mu}\right), \quad (19)$$

that gives the asymptotic $Z \propto 1 - \pi\Delta\varphi/\sqrt{6\mu}$ at $1 \gg \varphi \gg \sqrt{\mu}$. Now substituting μ we come to the boundary condition

$$\sqrt{3\gamma \ln(1/\rho)}\,\partial_\varphi Z = -\pi\Delta Z, \tag{20}$$

imposed on the harmonic function Z at $\varphi=0$. Note that Eq. (20) is nothing but the limit of Eq. (14) at $\rho \ll 1$. That boundary condition is simple enough and permits explicit expression for Z near the singularity. Let us represent Z in the following form:

$$Z = \mathrm{Re}\{f(\ln\rho^{-1} + i\varphi) + f(\ln\rho^{-1} + i\pi - i\varphi)\}, \tag{21}$$

which is harmonic and invariant under $\varphi \to \pi - \varphi$. Taking into account $\ln(1/\rho) \gg 1$, we obtain $Z = 2\,\mathrm{Re}\{f(\ln\ln\rho^{-1})\}$, $\partial Z/\partial\varphi = \pi\,\mathrm{Re}\{f''(\ln\rho^{-1})\}$ at $\varphi=0$. Thus we see that $f(x)$ satisfies the equation $f''(x) + 2\Delta f(x)/\sqrt{3\gamma x} = 0$ with the asymptotic behavior of the solution at $x \gg 1$

$$f(x) = \exp\left[\pm\frac{4}{3}\,i\left(\frac{4\Delta^2}{3\gamma}\right)^{1/4} x^{3/4}\right]. \tag{22}$$

Expanding Eq. (21) with Eq. (22) over $\ln^{-1}(1/\rho)$ we obtain

$$Z \propto \cos\left[\frac{4}{3}\left(\frac{4\Delta^2}{3\gamma}\right)^{1/4}\left(\ln\frac{1}{\rho}\right)^{3/4} + \phi_0\right] \times \left\{1 - \frac{\Delta}{\sqrt{3\gamma}}\frac{\varphi(\pi-\varphi)}{\sqrt{\ln(1/\rho)}}\right\}, \tag{23}$$

where ϕ_0 is some phase. We can believe $|\phi_0| < \pi$; its actual value has to be determined by the matching at $-\ln\rho \sim \gamma$.

Symmetry requirement, with respect to $x_1 \to 1 - x_1$, leads to the condition $\partial_1 Z(1/2, x_2) = 0$ which can be used as the quantization rule for the zero modes having the asymptotic (23). They can be classified in accordance with the number of zeros n which the function Z has where x_1 goes from 1/2 to $1 - \exp(-\gamma)$. Using expression (23) we conclude that $\Delta_{\min} = \alpha\sqrt{\gamma}$ and $\Delta_n = \beta\sqrt{\gamma}n^2$ for $n \gg 1$ with yet unknown numerical factors α and β, which are of order unity. Note that nonsymmetric zero modes Z (with another values of α and β) may exist, yet they cannot contribute to Z_0. For all the modes, the dependence $\Delta(\gamma)$ obtained here has an infinite slope at zero which has also been observed in numerics [18]. Phenomenological arguments in favor of $\Delta \propto \sqrt{\gamma}$ were given before in [11]. We conclude that the set of zero modes thus found at small γ has exponents larger than the exponent γ of the forced solution. Therefore, the isotropic part of the triple correlation function is shown here to have a normal scaling for sufficiently small γ. Since at $\gamma=2$ the lowest zero mode has $\Delta=4$, it is likely that the scaling of the isotropic part of the triple correlation function is normal for all γ.

Note that forced solution does not contribute to the structure function $3 = \langle(\Theta_1/\Theta_2)^3\rangle$; therefore, it scales with the exponent which is $\propto\sqrt{\gamma}$.

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