Asymptotic freedom at zero temperature in free-standing crystalline membranes

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We investigate the effects of quantum (zero-temperature) long-wavelength fluctuations of free-standing crystalline membranes, which are two-dimensional objects embedded into three-dimensional space. The fluctuations produce logarithmic renormalization of elasticity and bending moduli of the membranes. We find one-loop RG equations to demonstrate that the system is in the "asymptotic freedom" regime; that is, the quantum fluctuations destabilize the flat membrane phase.

I. INTRODUCTION

Amazing electronic and mechanical properties of graphene [1] and other two-dimensional ($2d$) crystals [2] fuel continuously growing research in this area (see reviews [3,4], containing also numerous relevant references). This progress also brings important and new dimensions for research not only restricted to material science and its applications. It also shines some light and manifests some analogies with basic phenomena of quantum field theory. As is often stated in the immense graphene literature, graphene might serve as a tool for the realization or visualization of high-energy physics effects in the normal scale of a solid state physics laboratory. One could observe properties of charge carriers similar to ones of electrons in quantum electrodynamics, e.g., the carrier mass renormalization [3,5].

Naturally, after almost 10 years since the discovery of graphene and numerous publications, one may wonder, is there still something to be understood about graphene? It is the purpose of our article to demonstrate that this is indeed so. Our investigation concerns freely suspended graphene or other crystalline films. It turns out that different $2d$ crystals can be stable and maintain macroscopic continuity and high quality [6]. Being freely suspended (substrate-free) such objects ($2d$ crystals suspended in $3d$ space) can be considered as a new state of matter. In our paper, we examine quantum (zero-temperature) long-scale fluctuations in such films and their influence on macroscopic physical characteristics of the films.

Properties of the freely suspended crystalline films at finite temperatures are well investigated, especially theoretically. Nelson and Peliti [7] and Kantor and Nelson [8] first suggested that the anharmonic interaction produces a power dependence on the scale of the bending and elastic crystalline moduli of the films. The prediction was confirmed later by systematic renormalization group (RG) calculations for a four-dimensional crystalline membrane embedded into higher dimensional, $d > 4$, space [9–11]. The approach becomes exact in the limit $d \to \infty$; see [12–14]. The power behavior of the moduli was checked by various scaling and physical arguments and also atomistic simulations [15–17], and no essential quantitative or qualitative contradictions to the theoretical predictions were found.

Much less attention has been paid to quantum fluctuations in crystalline films. This is partially because the quantum ($T = 0$)

nonlinear effects lead to only logarithmic corrections, hence generally much smaller than the power-law renormalization produced by thermal fluctuations [18]. However, small does not necessary mean irrelevant. In this work we bring the attention of our readers to quantum fluctuations related to vibrational modes. We claim that a freely suspended crystalline film manifests the behavior known as "asymptotic freedom" in high-energy physics. The asymptotic freedom is a property that causes interactions between particles to become weaker as energy increases (or distance between particles decreases), and at increasing space scales the coupling constant grows. Note that the asymptotic freedom behavior is a feature of quantum chromodynamics. This is one more illustration of a remarkable peculiarity of the phenomenological Landau-like theoretical approach which is a powerful tool for describing different systems irrespective to their microscopic nature.

Our paper is organized as follows. In Sec. II we develop the low-energy (long-wavelength) nonlinear theory of interacting vibrations in freely suspended $2d$ crystalline membrane. In Sec. III one-loop renormalization group (RG) equations for the elastic and bending moduli are derived and solved. We end the paper with a discussion of possible physical consequences and interpretation of the results. We relegate to an Appendix technical steps of the RG calculations.

II. BASIC RELATIONS

We consider quantum fluctuations of a freely suspended crystalline film (membrane). The membrane is treated as $2d$ infinitesimally thin film embedded into $3d$ space. Below we assume that the membrane on average is perpendicular to the Z axis. We also assume that the membrane is symmetric; that is, both its sides are equivalent. Then in the main approximation the long-scale energy (Landau functional) of the membrane can be written as [7,8]

$$\mathcal{F} = \int dx\, dy \left\{ \mu w_{\alpha\beta}^2 + \frac{\lambda}{2} w_{\alpha\alpha}^2 + \frac{\kappa}{2} (\nabla^2 h)^2 \right\}. \tag{1}$$

Here the coefficients μ, λ are $2d$ Lamé (elastic) moduli, κ is the Helfrich (bending) module of the membrane, h is the membrane displacement along the Z direction, $w_{\alpha\beta}$ is the membrane in-plane distortion tensor, and the subscripts $\alpha, \beta, \ldots$ run over x, y. The distortion tensor can be written in

the main approximation as [19]

$$2w_{\alpha\beta} = \partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha h \partial_\beta h, \qquad (2)$$

where $\boldsymbol{u}$ is the in-plane displacement vector of the membrane.

Analyzing quantum fluctuations one should investigate dynamics of the investigated degrees of freedom. In the low-frequency limit the degrees of freedom (fields) to be taken into account are $\boldsymbol{u}$ and h. The corresponding modes are the longitudinal and the transverse sound modes with the acoustic dispersion laws $\omega = c_{l,t}k$ and also the bending mode with the quadratic dispersion law $\omega = (\kappa/\rho)k^2$, where ρ is the $2d$ mass density of the membrane, ω is frequency, and $\boldsymbol{k}$ is the (in-plane) wave vector. The sound velocities are expressed via the elasticity moduli as

$$c_l^2 = \frac{2\mu + \lambda}{\rho}, \quad c_t^2 = \frac{\mu}{\rho}. \qquad (3)$$

Note that optical vibrational modes may also exist in crystalline membranes (as, e.g., in graphene). However, since the optical modes have a finite frequency they are irrelevant for the low-frequency effects we are investigating.

We are interested in correlation functions of the macroscopic fields $\boldsymbol{u}$ and h, primarily in their pair correlation functions

$$\langle u_\alpha(t,\boldsymbol{r})u_\beta(0,0)\rangle = \int \frac{d\omega\, d^2k}{(2\pi)^3} e^{-i\omega t + i\boldsymbol{k}\boldsymbol{r}} F_{\alpha\beta}(\omega,\boldsymbol{k}), \qquad (4)$$

$$\langle h(t,\boldsymbol{r})h(0,0)\rangle = \int \frac{d\omega\, d^2k}{(2\pi)^3} e^{-i\omega t + i\boldsymbol{k}\boldsymbol{r}} G(\omega,\boldsymbol{k}). \qquad (5)$$

The above averages over quantum fluctuations (marked by angular brackets) can be calculated as functional integrals over the fields $\boldsymbol{u}$ and h with the weight $\exp(iI/\hbar)$, where I is the action

$$I = \int dt\, d^2r \left\{ \frac{\rho}{2}(\partial_t \boldsymbol{u})^2 + \frac{\rho}{2}(\partial_t h)^2 \right.$$
$$\left. - \left[\mu w_{\alpha\beta}w_{\alpha\beta} + \frac{\lambda}{2}w_{\alpha\alpha}^2 + \frac{\kappa}{2}(\nabla^2 h)^2 \right] \right\}, \qquad (6)$$

reflecting dynamics of the membrane. The first term in the action (6) is the kinetic energy of the membrane whereas the second term there is the membrane potential energy (1) (taken with the sign minus), both integrated over time. Note that the expression (6) is invariant under the transformation $h \to -h$ that reflects the assumed symmetry of the membrane.

There are terms of the second, third, and fourth order in the fields $\boldsymbol{u}$, h in Eq. (6). The second-order term describes the noninteracting modes, whereas the third-order and the fourth-order terms determine their interaction (coupling). The bare correlation functions (4) and (5) determined by the quadratic (harmonic) part of the action (6) read as

$$F_{\alpha\beta}(\omega,\boldsymbol{k}) = \frac{i\hbar}{\rho} \left[\frac{1}{\omega^2 - c_l^2 k^2 + i0} \frac{k_\alpha k_\beta}{k^2} \right.$$
$$\left. + \frac{1}{\omega^2 - c_t^2 k^2 + i0} \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2} \right) \right], \qquad (7)$$

$$G(\omega,\boldsymbol{k}) = \frac{i\hbar}{\rho} \frac{1}{\omega^2 - (\kappa/\rho)k^4 + i0}. \qquad (8)$$

Here the term $+i0$ implies the standard Feynman rule for the contour integration over frequency near the poles of the Green's functions. The positions of the poles correspond to the dispersion laws of the acoustic modes $\omega = c_{l,t}k$ and of the bending mode $\omega = (\kappa/\rho)^{1/2}k^2$.

Based on the interaction terms (of the third and fourth orders) in the action (6), one can develop a perturbation theory for the correlation functions of the fields $\boldsymbol{u}$ and h. Fluctuation corrections, say, to the bare correlation functions (7) and (8), can be presented by Feynman diagrams with lines corresponding to the bare correlation functions and vertices of the third and of the fourth order determined by the interaction terms. One can check by direct calculations that the perturbation theory produces logarithmic corrections to the parameters of the bare correlation functions (7) and (8). That is why below we use the renormalization group (RG) technique to examine long-scale behavior of the correlation functions of $\boldsymbol{u}$ and h.

One comment is in order here. Besides the above said long-wavelength logarithmic contributions into the self-energy functions, in our model as in any renormalizable quantum field theory, there is also a number of short-wavelength (ultraviolet, UV) divergent terms. They cannot be calculated in terms of the macroscopic theory. Therefore these terms should be included in the definitions of the observable values of the corresponding quantities. This is a very common situation, for example, for second-order phase transitions, where UV divergences in the self-energy function of the fluctuating order parameter have to be interpreted as contributions to the phase transition temperature (that cannot be calculated in terms of the macroscopic theory). For our case (quantum vibrations of freely suspended crystalline membranes) such UV divergent contribution to the out-of-plane correlation function (see, e.g., [21]) proportional to k^2 has a meaning of the crystalline membrane surface tension. The surface tension of a freely suspended film is zero. Therefore the sum of the UV contributions to the surface tension is zero. Quantum (and also thermal, classical) out-of-plane fluctuations in the prestrained crystalline membranes (or films with a finite bare surface tension, say σ) are considerably suppressed (see, e.g., [22], illustrating the point by atomic-scale numeric simulations for suspended nanoribbons). Nevertheless our RG equations (therefore their solutions, i.e., our results) in such a case hold up to a scale on the order of $(\kappa/\sigma)^{1/2}$.

III. RENORMALIZATION GROUP EQUATIONS

We use the renormalization group procedure in the Wilson formulation; see Ref. [20]. The idea of the procedure is to split the fluctuating fields into fast (short-scale) and slow (long-scale) parts and to integrate the distribution function (weight) $\exp(iI/\hbar)$ over the fast component. As a result, we obtain a distribution function for slow variables $\exp(iI'/\hbar)$ to be interpreted in terms of renormalized parameters of the slow action I'.

Note that the distortion tensor (2) is invariant under the transformation $\delta u_\alpha = -\theta_\alpha h$, $\delta h = \theta_x x + \theta_y y$ (where θ_α is an infinitesimally small rotation angle), which reflects the original rotational invariance of the film. This symmetry leads to the conclusion that the energy and the "potential" part of the action

have to be expressed in terms of the distortion tensor (2). Consequently, the action I' for the slow variables has the same form (6) but with renormalized parameters μ, λ, and κ.

The renormalization of the Lamé elastic moduli is determined by purely logarithmic integrals. Based on the results presented in the Appendix we end up with the following quantum ($T = 0$) one-loop RG equations:

$$\frac{d\mu}{d\xi} = -\frac{\hbar}{32\pi\rho^{1/2}\kappa^{3/2}}\mu^2, \tag{9}$$

$$\frac{d\lambda}{d\xi} = -\frac{\hbar}{32\pi\rho^{1/2}\kappa^{3/2}}(\mu^2 + 4\mu\lambda + 2\lambda^2), \tag{10}$$

where ξ is the logarithm of the scale. Renormalization of the bending modulus κ is more involved. We skip all the rather tedious algebra (those readers who are interested in mathematical details of the calculations can find all essential steps in the Appendix), and present the final result, the one-loop RG equation for the bending modulus κ:

$$\frac{d\kappa}{d\xi} = -\frac{\hbar}{8\pi\rho^{1/2}\kappa^{1/2}}\frac{3\mu^2 + 3\mu\lambda}{2\mu + \lambda}. \tag{11}$$

Thus bending oscillations become softer due to the quantum fluctuations (in contrast, thermal fluctuations lead to hardening of the oscillations).

One finds from Eqs. (9) and (10) the one-loop RG equation for the ratio of the Lamé moduli:

$$\frac{d}{d\xi}\frac{\lambda}{\mu} = -\frac{\hbar\mu}{32\pi\rho^{1/2}\kappa^{3/2}}\left(1 + 3\frac{\lambda}{\mu} + 2\frac{\lambda^2}{\mu^2}\right). \tag{12}$$

As follows from the equation, there are two fixed points of the ratio, $\lambda = -\mu/2$ and $\lambda = -\mu$. The fixed point $\lambda = -\mu$ (corresponding to zero bulk modulus) is unstable and we stay with the only stable fixed point $\lambda = -\mu/2$. It is worthwhile to compare this finding with the known result for renormalization of Lamé moduli by thermal fluctuations [9–17]. In the latter case the RG equations (formulated for a four-dimensional membrane) possess four different fixed points (including $\lambda = -\mu/2$) but the only stable fixed point is $\lambda = -\mu/3$. Since the stable fixed points are not identical for the two cases, one should not expect a sort of continuous matching of classical and quantum results.

Further we assume that the system (membrane) is in the state characterized by the ratio near the fixed point $\lambda = -\mu/2$. Substituting $\lambda = -\mu/2$ into Eq. (11) one obtains

$$\frac{d\kappa}{d\xi} = -\frac{\hbar\mu}{8\pi\rho^{1/2}\kappa^{1/2}}.$$

Then one finds from Eq. (9)

$$\frac{dg}{d\xi} = g^2, \qquad g = \frac{5\hbar\mu}{32\pi\rho^{1/2}\kappa^{3/2}}, \tag{13}$$

where we introduced the dimensionless coupling constant g. The above RG equations are correct provided $g \ll 1$. We see that the coupling constant increases as the scale grows. Therefore for large enough scales the system passes to the strong-coupling regime corresponding to strong fluctuations of the membrane shape.

Expanding the right-hand side of Eq. (12) near the fixed point $\lambda = -\mu/2$, we find

$$\frac{d}{dg}\left(\frac{\lambda}{\mu} + \frac{1}{2}\right) = -\frac{1}{5g}\left(\frac{\lambda}{\mu} + \frac{1}{2}\right).$$

Therefore the ratio λ/μ tends to $1/2$ as the coupling constant grows. This justifies our approach. However, the corresponding law is $\lambda/\mu + 1/2 \propto g^{-1/5}$; that is, the system approaches the fixed point not too fast. Therefore, in analyzing concrete experimental data, probably, it is worthwhile to consider μ and λ as independent variables.

We conclude from Eq. (13) that the coupling constant characterizing the quantum fluctuations of the crystalline membranes increases as the scale grows:

$$g = g_0/(1 - g_0\xi).$$

In other words, we encounter the "asymptotic freedom" behavior like in quantum chromodynamics. This scenario means that unlike thermal fluctuations, stabilizing the flat state of the crystalline membrane (hardening the classical bending rigidity), quantum fluctuations yield to a rough (crumpled) membrane state due to softening of quantum bending fluctuations.

IV. CONCLUSION

What we found looks a bit counterintuitive: at $T = 0$ crystalline membranes turn out to be rougher at large scales in contrast to finite temperatures. Although there is no theorem claiming that quantum fluctuations can be obtained as a sort of interpolation with $T \to 0$ from classical thermal fluctuations, our finding may shake some of the arguments used in analysis of thermal fluctuations by the very bold matching results obtained for four-dimensional membranes embedded in infinite-dimensional space [12–14].

A natural question appears concerning experimentally observable consequences of the qualitatively surprising but quantitatively rather modest (logarithmic) renormalization of the elastic moduli. For the graphene monoatomic films all physical parameters are known [3,4]. Namely, $\mu \simeq 9$ eV/Å^2, $\lambda \simeq 2$ eV/Å^2, $\kappa \simeq 0.7$ eV, and $2d$ density $\rho \simeq 7.6 \times 10^{-8}$ g/cm^2. Combining everything together and stretching experimental uncertainty, our estimations give $g \simeq 1/20$. The physical reason for the small value of g is quite transparent. A characteristic energy related to the elastic moduli (λ, μ, and κ) in the quantum limit is determined by the electronic mass m. On the other hand, the membrane vibrations are related to Debye energy, that is, are determined by the atomic mass M. One can see that in fact $g \sim (m/M)^{1/3}$ (apart of numeric factors).

Of course it is not very realistic to overcome the small bare coupling constant in graphene by a large logarithmic factor. However, the situation is not completely hopeless. We recall the situation with smectic liquid crystals, where logarithmic divergence of the layer displacements requires astronomic scales for its direct observation but could be routinely observed by the power-law tails in x-ray scattering of standard laboratory samples [23]. Similarly, one can think, say, about measurements of the simultaneous two-point correlation function $\langle\nabla h_1 \nabla h_2\rangle$. The bare correlation function is proportional to $\delta(r)$, where $r = r_1 - r_2$ is the separation between the points. However, the logarithmic renormalization

produces corrections to the correlation function proportional to r^{-2} (with some logarithmic factor). The small coefficient g in front of this correction makes its observation problematic but not impossible.

One more direction in which to think about observation of our findings is to get membranes with larger values of the bare coupling constant g. One can try to use, e.g., freely suspended crystalline smectic films with anomalously small bending elastic modulus κ. Such behavior is expected in the systems undergoing transition into the membrane ripple phase or into the so-called smectic $\tilde{A}$ or smectic $\tilde{C}$ structures with one-dimensional layer modulations. If the period of this modulation p is larger than the molecular size a, then the bare bending modulus acquires a small prefactor $(a/p)^2$. This factor for membrane ripple phase and for some modulated smectics can be as large as 10^2 (see, e.g., the survey [24], the papers [25,26], and the more recent discussion of modulated structures in nonchiral smectics [27]).

We did not touch the electron-phonon coupling (or, more precisely, coupling to the low-frequency modes we are investigating). The reason is that the Dirac electronic degrees of freedom, despite their softness, do not yield to any additional long-wavelength renormalization of quantum vibrations of the crystalline membrane. Indeed, at the Dirac point the electron-phonon coupling has the form of the so-called deformational potential [28,29]. The feedback influence of this deformation potential into the long-wavelength vibrational part of the Hamiltonian is irrelevant, and therefore does not change our main conclusion concerning asymptotic freedom behavior of quantum vibrational fluctuations in free-standing crystalline films.

We hope that our work will motivate further experimental and theoretical studies along this line—quantum fluctuations in freely suspended crystalline membranes.

ACKNOWLEDGMENTS

APPENDIX

Here we develop the RG procedure starting from splitting the fields $\boldsymbol{u}$ and h into the fast and slow parts: $\boldsymbol{u} \to \boldsymbol{u}' + \tilde{\boldsymbol{u}}$, $h \to h' + \tilde{h}$, where the prime designates slow fields and the tilde designates fast fields. One can assume that the fast fields $\tilde{\boldsymbol{u}}$ and $\tilde{h}$ are sums of spacial harmonics with wave vectors $\boldsymbol{q}$ in the interval $\Lambda' < q < \Lambda$, where Λ is the ultraviolet cutoff, and Λ' is a separation wave vector of slow and fast variables. Next, one should calculate a "slow" action $I'(\boldsymbol{u}', h')$ in accordance with the definition

$$\exp(i\,I'/\hbar) = \int \mathcal{D}\tilde{u}\,\mathcal{D}\tilde{h}\,\exp(i\,I/\hbar), \quad (A1)$$

where functional integration over fast fields is implied.

The rotational symmetry discussed in the main body of the paper guarantees that the "slow" action $I'(\boldsymbol{u}', h')$ is determined by the same expression (6) but with renormalized parameters.

Therefore one can calculate corrections only to the harmonic (quadratic) part of the action

$$I^{(2)} = \int dt\,d^2r\,\left\{ \frac{\rho}{2}(\partial_t \boldsymbol{u})^2 + \frac{\rho}{2}(\partial_t h)^2 \right.$$
$$\left. - \left[\mu(\partial_\alpha u_\beta)^2 + \frac{\lambda}{2}(\partial_\alpha u_\alpha)^2 + \frac{\kappa}{2}(\nabla^2 h)^2 \right] \right\}. \quad (A2)$$

Actually, there appear logarithmic corrections to the moduli μ, λ, κ whereas logarithmic corrections to the mass density ρ are absent.

If the coupling constant g (13) is small then the renormalization of the moduli μ, λ, κ can be calculated by a loop expansion. To find corrections to the harmonic action (A2) in the main one-loop approximation it is enough to use the third-order contribution to the action (6)

$$I^{(3)} = - \int dt\,d^2r \left[\mu \partial_\alpha u_\beta \partial_\alpha h \partial_\beta h + \frac{\lambda}{2} \nabla \boldsymbol{u} (\nabla h)^2 \right]. \quad (A3)$$

Substituting here $\boldsymbol{u} = \boldsymbol{u}' + \tilde{\boldsymbol{u}}$, $h = h' + \tilde{h}$ we find the following second order in the fast fields interaction term in the action

$$I_{\text{int}} = - \int dt\,d^2r \left[\mu \partial_\alpha u'_\beta \partial_\alpha \tilde{h} \partial_\beta \tilde{h} + \frac{\lambda}{2} \nabla \boldsymbol{u}' (\nabla \tilde{h})^2 \right]$$
$$- \int dt\,d^2r \left[\mu \partial_\alpha \tilde{u}_\beta \partial_\alpha \tilde{h} \partial_\beta h' + \mu \partial_\alpha \tilde{u}_\beta \partial_\alpha h' \partial_\beta \tilde{h} \right.$$
$$\left. + \lambda \nabla \tilde{\boldsymbol{u}} \nabla h' \nabla \tilde{h} \right], \quad (A4)$$

needed to calculate the one-loop contribution to the harmonic action (A2).

Let us first consider the one-loop correction to the slow harmonic action produced by the first term in Eq. (A4),

$$\Delta_1 I^{(2)} = \frac{i}{2\hbar} \int dt\,d^2r \left(\mu \partial_\alpha u'_\beta + \frac{\lambda}{2} \nabla \cdot \boldsymbol{u}' \delta_{\alpha\beta} \right)$$
$$\times \left(\mu \partial_\mu u'_\nu + \frac{\lambda}{2} \nabla \cdot \boldsymbol{u}' \delta_{\mu\nu} \right)$$
$$\times \int dt_1\,d^2r_1\,\langle \partial_\alpha \tilde{h} \partial_\beta \tilde{h} \partial_\mu \tilde{h}_1 \partial_\nu \tilde{h}_1 \rangle_0, \quad (A5)$$

that determines renormalization of the Lamé coefficients μ and λ. Here the subscript 0 marks correlation functions found by averaging with the harmonic action (A2). First we have to calculate the integral entering the correction (A5). It is simpler to calculate it in the Fourier representation

$$\int dt_1\,d^2r_1\,\langle \partial_\alpha \tilde{h} \partial_\beta \tilde{h} \partial_\mu \tilde{h}_1 \partial_\nu \tilde{h}_1 \rangle_0$$

$$= -\frac{2\hbar^2}{\rho^2} \int \frac{d\omega\,d^2q}{(2\pi)^3} \frac{q_\alpha q_\beta q_\mu q_\nu}{[\omega^2 - (\kappa/\rho)q^4 + i0]^2}$$

$$= -\frac{i\hbar^2}{32\pi\rho^{1/2}\kappa^{3/2}} \ln \frac{\Lambda}{\Lambda'} (\delta_{\alpha\beta}\delta_{\mu\nu} + \delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\alpha\nu}\delta_{\beta\mu}).$$

Substituting the result into Eq. (A5) one finds corrections to μ and λ leading to RG equations (9) and (10) presented in the main body of the paper.

Now we pass to calculation of the one-loop renormalization of κ determined by the second term in Eq. (A4). The

corresponding correction to the slow harmonic action is

$$\Delta_2 I^{(2)} = \frac{i}{2\hbar} \int dt_1 \, dt_2 \, d^2 r_1 \, d^2 r_2 \, \partial_\alpha h_1' \partial_\mu h_2' \langle \partial_\beta \tilde{h}_1 \partial_\nu \tilde{h}_2 \rangle_0$$
$$\times \langle (\mu \partial_\alpha \tilde{u}_{1\beta} + \mu \partial_\beta \tilde{u}_{1\alpha} + \lambda \boldsymbol{\nabla} \cdot \tilde{\boldsymbol{u}}_1 \delta_{\alpha\beta})(\mu \partial_\mu \tilde{u}_{2\nu}$$
$$+ \mu \partial_\nu \tilde{u}_{2\mu} + \lambda \boldsymbol{\nabla} \cdot \tilde{\boldsymbol{u}}_2 \delta_{\mu\nu}) \rangle_0. \qquad \text{(A6)}$$

The principal technical problem here is that the main contribution related to fast variables is ultraviolet. Therefore one has to extract the logarithmic term on top of the ultraviolet contribution. To be confident about results of this rather involved calculation we do it both in real space and in Fourier space.

1. Real-space calculations

A characteristic time $t = t_1 - t_2$ in Eq. (A6) is determined by the $\langle \tilde{u} \tilde{u} \rangle$ correlation time and is, consequently, much less than one of the $\langle \tilde{h} \tilde{h} \rangle$ correlation time. Therefore one can substitute there the simultaneous $\langle \tilde{h} \tilde{h} \rangle$ correlation function. Changing variables as $\tau = (t_1 + t_2)/2$, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $\boldsymbol{R} =$

$(\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ and expanding $\boldsymbol{\nabla} h$ over $\boldsymbol{r}$, one finds

$$\Delta_2 I^{(2)} = -\frac{i}{4\hbar} \int d\tau \, d^2 R \, \partial_\alpha \partial_\gamma h \partial_\mu \partial_\delta h \int d^2 r \, r_\gamma r_\delta \langle \partial_\beta \tilde{h}$$
$$\times (0, \boldsymbol{r}_1) \partial_\nu \tilde{h}(0, \boldsymbol{r}_2) \rangle_0 \int dt \, \langle (\mu \partial_\alpha \tilde{u}_{1\beta} + \mu \partial_\beta \tilde{u}_{1\alpha}$$
$$+ \lambda \boldsymbol{\nabla} \cdot \tilde{\boldsymbol{u}}_1 \delta_{\alpha\beta})(\mu \partial_\mu \tilde{u}_{2\nu} + \mu \partial_\nu \tilde{u}_{2\mu} + \lambda \boldsymbol{\nabla} \cdot \tilde{\boldsymbol{u}}_2 \delta_{\mu\nu}) \rangle_0,$$
$$\qquad \text{(A7)}$$

where $h = h(\tau, \boldsymbol{R})$.

The correlation functions entering (A7) can be extracted from

$$\langle \tilde{h}(0, \boldsymbol{r}) \tilde{h}(0, 0) \rangle_0 = \frac{i\hbar}{\rho} \int \frac{d\omega \, d^2 k}{(2\pi)^3} \frac{\exp(i \boldsymbol{k} \boldsymbol{r})}{\omega^2 - (\kappa/\rho)k^4 + i0}$$
$$= \frac{\hbar}{2\rho^{1/2}\kappa^{1/2}} \int \frac{d^2 k}{(2\pi)^2} \frac{\exp(i \boldsymbol{k} \boldsymbol{r})}{k^2}$$
$$= \frac{\hbar}{4\pi\rho^{1/2}\kappa^{1/2}} \ln(L/r)$$

and

$$\int dt \, \langle \tilde{u}_\alpha(t, \boldsymbol{r}) \tilde{u}_\beta(0, 0) \rangle_0 = -\frac{i\hbar}{\rho} \int \frac{d^2 k}{(2\pi)^2} \exp(i \boldsymbol{k} \boldsymbol{r}) \left[\frac{k_\alpha k_\beta}{k^2} \frac{1}{c_l^2 k^2} + \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2} \right) \frac{1}{c_t^2 k^2} \right]$$
$$= -\frac{i\hbar}{4\pi(2\mu + \lambda)} \left\{ [\ln(L/r) + 1/2] \, \delta_{\alpha\beta} - \frac{r_\alpha r_\beta}{r^2} \right\} - \frac{i\hbar}{4\pi\mu} \left\{ [\ln(L/r) - 1/2] \, \delta_{\alpha\beta} + \frac{r_\alpha r_\beta}{r^2} \right\}.$$

Taking space derivatives one finds

$$\langle \partial_\beta \tilde{h}(0, \boldsymbol{r}_1) \partial_\nu \tilde{h}(0, \boldsymbol{r}_2) \rangle_0 = \frac{\hbar}{4\pi\rho^{1/2}\kappa^{1/2}r^2} \left(\delta_{\beta\nu} - 2\frac{r_\beta r_\nu}{r^2} \right)$$

and

$$\int dt \, \langle \partial_\mu \tilde{u}_\alpha(t, \boldsymbol{r}) \partial_\nu \tilde{u}_\beta(0, 0) \rangle_0 = \frac{i\hbar}{4\pi(2\mu + \lambda)r^2} \bigg\{ -(\delta_{\mu\nu}\delta_{\alpha\beta} + \delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha})$$
$$+ 2\bigg(\delta_{\alpha\beta} \frac{r_\mu r_\nu}{r^2} + \delta_{\alpha\mu} \frac{r_\beta r_\nu}{r^2} + \delta_{\alpha\nu} \frac{r_\mu r_\beta}{r^2} + \delta_{\mu\beta} \frac{r_\alpha r_\nu}{r^2}$$
$$+ \delta_{\nu\beta} \frac{r_\alpha r_\mu}{r^2} + \delta_{\mu\nu} \frac{r_\alpha r_\beta}{r^2} \bigg) - 8\frac{r_\alpha r_\beta r_\mu r_\nu}{r^4} \bigg\} + \frac{i\hbar}{4\pi\mu r^2} \bigg\{ -\big(\delta_{\mu\nu}\delta_{\alpha\beta} - \delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha} \big)$$
$$+ 2\bigg(\delta_{\alpha\beta} \frac{r_\mu r_\nu}{r^2} - \delta_{\alpha\mu} \frac{r_\beta r_\nu}{r^2} - \delta_{\alpha\nu} \frac{r_\mu r_\beta}{r^2} - \delta_{\mu\beta} \frac{r_\alpha r_\nu}{r^2} - \delta_{\nu\beta} \frac{r_\alpha r_\mu}{r^2} - \delta_{\mu\nu} \frac{r_\alpha r_\beta}{r^2} \bigg) + 8\frac{r_\alpha r_\beta r_\mu r_\nu}{r^4} \bigg\}.$$

Substituting the above expressions into Eq. (A7) one obtains

$$\Delta_2 I^{(2)} = \frac{\hbar}{32\pi\rho^{1/2}\kappa^{1/2}} \int d\tau \, d^2 R \, \partial_\alpha \partial_\gamma h \partial_\mu \partial_\delta h \int d^2 r \, \frac{r_\gamma r_\delta}{r^2} \frac{1}{2\pi r^2} \left[\frac{16\mu^2}{2\mu + \lambda} \left(-\delta_{\alpha\mu} + \frac{r_\alpha r_\mu}{r^2} \right) - \frac{8\mu\lambda}{2\mu + \lambda} \delta_{\alpha\mu} - 4\mu\delta_{\alpha\mu} \right]$$
$$= -\frac{\hbar}{16\pi\rho^{1/2}\kappa^{1/2}} \int \frac{d\tau \, d^2 R}{2\pi} (\nabla^2 h)^2 \ln \frac{\Lambda}{\Lambda'} \left[\frac{\mu^2}{2\mu + \lambda} + \frac{2\mu\lambda}{2\mu + \lambda} + \mu \right]. \qquad \text{(A8)}$$

The contribution (A8) to the effective action gives a correction to κ that leads to the RG equation (11).

2. Fourier-space calculations

In the subsection we start from the same correction to the action (A6) that is rewritten in the Fourier representation as

$$\Delta_2 I^{(2)} = \frac{i}{2\hbar} \int \frac{d\omega \, d^2 k}{(2\pi)^3} k_\alpha k_\mu h(\omega, \boldsymbol{k}) h(-\omega, -\boldsymbol{k}) \int \frac{d\nu \, d^2 q}{(2\pi)^3} (q_\beta + k_\beta)(q_\nu + k_\nu) G(\omega + \nu, \boldsymbol{k} + \boldsymbol{q})(\mu q_\alpha \delta_{\beta\kappa} + \mu q_\beta \delta_{\alpha\kappa} + \lambda q_\kappa \delta_{\alpha\beta})$$
$$\times (\mu q_\mu \delta_{\nu\lambda} + \mu q_\nu \delta_{\mu\lambda} + \lambda q_\lambda \delta_{\mu\nu}) F_{\kappa\lambda}(\nu, \boldsymbol{q}). \qquad \text{(A9)}$$

Since the bending mode is slower than the acoustic ones, we may substitute $F(\nu,\boldsymbol{q}) \to F(0,\boldsymbol{q})$ in Eq. (A9). Integrating then over ν, one obtains

$$\Delta_2 I^{(2)} = \frac{i}{4\rho^{1/2}\kappa^{1/2}} \int \frac{d\omega \, d^2k}{(2\pi)^3} k_\alpha k_\mu h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}) \int \frac{d^2q}{(2\pi)^2} (q_\beta + k_\beta)(q_\nu + k_\nu)(\boldsymbol{q}+\boldsymbol{k})^{-2} (\mu q_\alpha \delta_{\beta\kappa} + \mu q_\beta \delta_{\alpha\kappa} + \lambda q_\kappa \delta_{\alpha\beta})$$
$$\times (\mu q_\mu \delta_{\nu\lambda} + \mu q_\nu \delta_{\mu\lambda} + \lambda q_\lambda \delta_{\mu\nu}) F_{\kappa\lambda}(0,\boldsymbol{q}).$$

Here $\boldsymbol{k}$ is the wave vector of the slow variables whereas $\boldsymbol{q}$ is the wave vector of fast variables; therefore $q \gg k$.

In the main order in k/q we obtain an ultraviolet integral for $\Delta_2 I^{(2)}$. To extract the renormalization of κ one has to expand the above expression for $\Delta_2 I^{(2)}$ up to the second order in $\boldsymbol{k}$ to obtain

$$\Delta_2 I^{(2)} = \frac{\hbar}{4\rho^{1/2}\kappa^{1/2}} \int \frac{d\omega \, d^2k}{(2\pi)^3} k_\alpha k_\mu h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}) \int \frac{d^2q}{(2\pi)^2} \left[\frac{k_\beta k_\nu}{q^2} - 2\frac{q_\beta k_\nu + k_\beta q_\nu}{q^2}\frac{\boldsymbol{q}\boldsymbol{k}}{q^2} + 4\frac{q_\beta q_\nu}{q^2}\frac{(\boldsymbol{q}\boldsymbol{k})^2}{q^4} - \frac{q_\beta q_\nu}{q^2}\frac{k^2}{q^2} \right]$$
$$\times (\mu q_\alpha \delta_{\beta\kappa} + \mu q_\beta \delta_{\alpha\kappa} + \lambda q_\kappa \delta_{\alpha\beta})(\mu q_\mu \delta_{\nu\lambda} + \mu q_\nu \delta_{\mu\lambda} + \lambda q_\lambda \delta_{\mu\nu})\frac{1}{q^2}\left[\frac{1}{2\mu+\lambda}\frac{q_\kappa q_\lambda}{q^2} + \frac{1}{\mu}\left(\delta_{\kappa\lambda} - \frac{q_\kappa q_\lambda}{q^2} \right) \right]. \quad (A10)$$

Below, we separately calculate contributions to $\Delta_2 I^{(2)}$ related to the longitudinal factor $q_\kappa q_\lambda/q^2$ and to the isotropic factor $\delta_{\kappa\lambda}$ in the last line of Eq. (A10). The "longitudinal" contribution to the slow action is

$$\Delta_{21} I^{(2)} = \frac{\hbar}{4\rho^{1/2}\kappa^{1/2}}\left(\frac{1}{2\mu+\lambda} - \frac{1}{\mu} \right) \int \frac{d\omega \, d^2k}{(2\pi)^3} k_\alpha k_\mu k_\kappa k_\lambda h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}) \int \frac{d^2q}{(2\pi)^2 q^4}$$
$$\times \left[\frac{\delta_{\beta\kappa}\delta_{\nu\lambda}}{q^2} - 2\frac{\delta_{\nu\kappa}q_\beta q_\lambda}{q^4} - 2\frac{\delta_{\beta\kappa}q_\nu q_\lambda}{q^4} - \frac{q_\beta q_\nu \delta_{\kappa\lambda}}{q^4} + 4\frac{q_\beta q_\nu q_\kappa q_\lambda}{q^6} \right](2\mu q_\alpha q_\beta + \lambda q^2 \delta_{\alpha\beta})(2\mu q_\mu q_\nu + \lambda q^2 \delta_{\mu\nu})$$
$$= \frac{\hbar}{16\pi\rho^{1/2}\kappa^{1/2}}\frac{(\mu+\lambda)(\mu+2\lambda)}{2\mu+\lambda} \ln\frac{\Lambda}{\Lambda'} \int \frac{d\omega \, d^2k}{(2\pi)^3} k^4 h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}).$$

The "isotropic" contribution to the slow action is written as

$$\Delta_{22} I^{(2)} = \frac{\hbar}{4\rho^{1/2}\kappa^{1/2}} \int \frac{d\omega \, d^2k}{(2\pi)^3} k_\alpha k_\mu h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}) \int \frac{d^2q}{(2\pi)^2 q^2}\left[\frac{k_\beta k_\nu}{q^2} - 2\frac{q_\beta k_\nu}{q^2}\frac{\boldsymbol{q}\boldsymbol{k}}{q^2} - 2\frac{k_\beta q_\nu}{q^2}\frac{\boldsymbol{q}\boldsymbol{k}}{q^2} + 4\frac{q_\beta q_\nu}{q^2}\frac{(\boldsymbol{q}\boldsymbol{k})^2}{q^4} \right.$$
$$\left. - \frac{q_\beta q_\nu}{q^2}\frac{k^2}{q^2} \right]\left[\mu(q_\alpha q_\mu \delta_{\beta\nu} + q_\beta q_\mu \delta_{\alpha\nu} + q_\alpha q_\nu \delta_{\beta\mu} + q_\beta q_\nu \delta_{\alpha\mu}) + 2\lambda(q_\alpha q_\beta \delta_{\mu\nu} + q_\mu q_\nu \delta_{\alpha\beta}) + \frac{\lambda^2}{\mu}\delta_{\alpha\beta}\delta_{\mu\nu} \right]$$
$$= \frac{\hbar}{16\pi\rho^{1/2}\kappa^{1/2}} (-2\mu - 2\lambda) \ln\frac{\Lambda}{\Lambda'} \int \frac{d\omega \, d^2k}{(2\pi)^3} k^4 h(\omega,\boldsymbol{k}) h(-\omega,-\boldsymbol{k}).$$

Summing up the "longitudinal" and the "isotropic" contributions we find the same correction to κ as in real-space calculations, leading to the RG equation (11).

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