Письма в ЖЭТФ, том 58, вып.8, стр.672 - 676 ©1993 г. 25 октября

DYNAMIC FLUCTUATIONS EFFECTS IN SOAP FILMS

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Submitted 16 September 1993

Dynamic fluctuation phenomena in free standing soap films are investigated. It is shown that the squeezing mode (where the water is pumped back and forth) induces divergent at the intermediate scales contributions to kinetic coefficients (viscosity and diffusion), leading to anomalous scaling behavior of the coefficients. The corresponding exponents are calculated using RG-methods in the one-loop approximation. They are $\epsilon/19$ and $18\epsilon/19$, where $\epsilon = 4-d$ and d is the dimensionality of the space (the physical value being $d = 2$). The role of bending fluctuations is also discussed.

We will consider dynamics of thin liquid films which may be prepared by using special dopes to conventional soap films. Such films have been investigated experimentally (mainly using a light scattering methodic) for thicknesses h in the region $10^2 - 10^4$ Å[1].

As was demonstrated by two authors of this paper (E.K. and V.L.) [2, 3] thermal fluctuation effects are relevant in the long wavelength dynamics of free suspended films. Since for soap films the thickness $h \gg a_m$ (a_m is the molecular size) the effects investigated in [2, 3] are important on very large scales. Nevertheless for soap films the dynamics on intermediate scales (exceeding h but not very large) is also sensitive to fluctuation effects. The peculiarity of these effects in soap films is related to the fact that the film is at least a two-component solution and therefore there is the so-called squeezing mode [1, 4] where the water is pumped back and forth through a slab with thickness h. The dispersion law of the squeezing mode in the linear approximation was found in [4], in the region $qh \ll 1$ (q is the wave vector) it is

$$\omega = -i\zeta q^2(\alpha q^2 + \alpha_1) \quad . \tag{1}$$

Here ζ is the kinetic coefficient (which may be called a diffusion one) and α, α_1 are the modules characterizing the elasticity of the soap film.

Since a soap film consists of a volume part containing water and of surface layers where soap molecules are concentrated, the coefficients entering (1) can be estimated through the parameters of the layers and water. Namely, $\zeta \sim (\eta h)^{-1}$ (η is the viscosity of water), $\alpha \sim \gamma h^2$ (γ is the surface tension of the soap layers) and $\alpha_1 = d^2V/dh^2$, where $V(h)$ is the interaction potential between the soap layers [4]. Note that we assume a natural estimation $\varepsilon_s \sim \gamma$, where $\varepsilon_s = -n_s \partial\gamma/\partial n_s$ and n_s is the surface concentration of the soap molecules. The interaction between the soap layers is mainly associated with the Van der Waals forces which give [5]

$$V = \Theta^2/\gamma h^2 \quad , \tag{2}$$

where the parameter Θ has the dimension of the energy. Therefore there exists the region of wave vectors

$$qh \gg \Theta/\gamma h^2 \quad , \tag{3}$$

where we may neglect α_1 in the dispersion law (1). We see that the squeezing mode is a soft one which explains the essential role of the squeezing fluctuations in the dynamics of soap films.

To investigate the dynamic fluctuation effects we will utilize a diagram technique of the type first developed by Wyld [6] for the problem of hydrodynamical turbulence and extended on a wide class of physical systems by Martin, Siggia and Rose [7]. A textbook description of the diagram technique can be found in the book by Ma [8] (see also the monograph [3]). Note that this diagram technique is a classical limit of the Keldysh diagram technique [9] applicable to any physical system. As was demonstrated by de Dominicis [10] and Janssen [11] (see also [12] and [13]) Wyld's diagrammatic technique is generated by the conventional quantum field theory fashion starting from an effective action I. The corresponding methods can be found in the monograph by Popov [14].

To use Wyld diagram technique we should first derive a system of nonlinear equations describing a soap film. In comparison with the system of equations of a freely suspended film constructed in [2, 3] it includes the equation for the variable ψ determining the $2d$ density of the water in the film. It can be derived as the equation for the concentration in a two-component solutions. After elimination of all hard degrees of freedom we come to the closed system of nonlinear equations for the variable ψ and the transversal with respect to a wave vector component of the velocity $\mathbf{v}_\perp$.

This system enables us to construct the effective action I for the mentioned degrees of freedom. In the main approximation it has the form

$$I = \int dt\, dx\, dy \left(p_\psi(\partial\psi/\partial t + \mathbf{v}_\perp \nabla\psi) + \zeta\alpha\nabla^2 p_\psi \nabla^2\psi + iT\zeta(\nabla p_\psi)^2 \right.$$
$$\left. -\alpha\nabla_\beta p_\alpha \nabla_\alpha\psi\nabla_\beta\psi + \eta_1\nabla_\alpha p_\beta \nabla_\alpha v_\beta + iT\eta_1(\nabla_\alpha p_\beta)^2 \right) \quad , \tag{4}$$

where T is the temperature and p_ψ and p_α are supplementary Bose fields [3, 7, 10, 11] conjugated to the fields ψ and v_α. Here we have implied that in equilibrium the film is arranged along the $X-Y$ plane, all variables characterizing the film are believed to be functions of the time t and coordinates x, y and we have omitted a α_1-proportional term in (4).

It is easy to find from the second-order part of (4) the bare expressions for the correlation functions

$$D_\psi = \langle\psi\psi\rangle_{\omega,q} = \frac{2T\zeta q^2}{\omega^2 + \zeta^2\alpha^2 q^8} \quad , \tag{5}$$

$$D_{\alpha\beta} = \langle v_\alpha v_\beta\rangle_{\omega,q} = \frac{2T}{\eta_1 q^2}(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}) \quad . \tag{6}$$

Fluctuation corrections to the bare values (5, 6) are determined by the interaction term

$$p_\psi \mathbf{v}_\perp \nabla\psi - \alpha\nabla_\beta p_\alpha \nabla_\alpha\psi\nabla_\beta\psi \tag{7}$$

of (4) which generates two third-order vertices in diagrams. Note that both vertices are originated from the reactive part of the dynamic equations whereas we are interesting in fluctuation contributions to the kinetic coefficients ζ and η_1.

Fig. 1. The first contributions to the polarization operator.

The one-loop contributions to polarization operators are determined by the diagrams presented in Fig.1 where a solid line designates the correlation function D_ψ, a dashed line designates the correlation function $D_{\alpha\beta}$, black and white circles designate the third order vertices entering (7). Comparing these contributions with (5, 6) we conclude that the corrections

$$\zeta_{fl} \simeq \frac{T}{\alpha\eta_1 q^2}, \qquad \eta_{1fl} \simeq \frac{T}{\zeta\alpha q^2} \tag{8}$$

to the coefficients ζ and η_1 arise. Both contributions diverge at large scales and exceeds the bare values at wave vectors

$$q < \left(\frac{T}{\alpha\zeta\eta_1}\right)^{1/2} . \tag{9}$$

In this region we cannot restrict ourselves only to the first corrections and should take into account higher-order contributions to the self-energy functions. This is the same situation as near a second-order phase transition and therefore we can expect a scaling behavior of the coefficients characterizing the "dressed" correlation functions (5, 6). Let us introduce scaling exponents Δ_η and Δ_ζ which determine the long wavelength behavior of the coefficients ζ and η_1

$$\zeta \propto q^{-\Delta_\zeta}, \qquad \eta_1 \propto q^{-\Delta_\eta} . \tag{10}$$

To estimate the values of these exponents one can use renormalization-group (RG) methods. The marginal dimension for the effective action (4) is $1+4$ (time + 4d space) It is not very difficult to check that the effective action (4) is renormalizable and that there are no corrections to the coefficients T and α which is accounted for by the fluctuation-dissipation theorem and by the fact that α is the static module (in statics fluctuations are not relevant). In the dimension $d = 4 - \epsilon$ the one-loop RG equations for the coefficients ζ and η_1 are

$$\frac{d\zeta}{dL} = (d-1)g\zeta, \qquad \frac{d\eta_1}{dL} = \frac{1}{d+2}g\eta_1 . \tag{11}$$

Here

$$g = \frac{TS_d}{(2\pi)^d \eta_1 \zeta\alpha d}\Lambda^{-\epsilon}$$

is an invariant charge, S_d is the area of the d-dimensional sphere, $L = ln(\Lambda/k)$, Λ is a cutoff. For $\epsilon \ll 1$ the fixed point of (11) is

$$g^* = \frac{6}{19}\epsilon.$$

Thus the exponents determined by (11) are

$$\Delta_\zeta = 18\epsilon/19, \qquad \Delta_\eta = \epsilon/19 . \tag{12}$$

For the two-dimensional soap films $\epsilon = 2$ which is not a small parameter. Nevertheless we may hope that (12) give a reasonable estimation for the exponents Δ_η and Δ_ζ at $\epsilon = 2$. Therefore we may expect that the viscosity coefficient η_1 only weakly depends on the scale whereas ζ diverges with the exponent close to 2 and therefore the dispersion law for the squeezing mode (with the α_1-proportional term neglected!) only slightly differs from a diffusion one. Let us note that the behavior of the second viscosity coefficient η_2 is determined by the same exponent Δ_η and therefore the attenuation of the longitudinal sound will be proportional to $\omega^{2-\Delta_\eta}$.

Let us give the general picture of fluctuation effects in the soap films. At shortest scales the dispersion laws of all modes are determined by the linear dynamic equations. At scales determined by (3), (9) squeezing fluctuations lead to the scaling behavior discussed above.

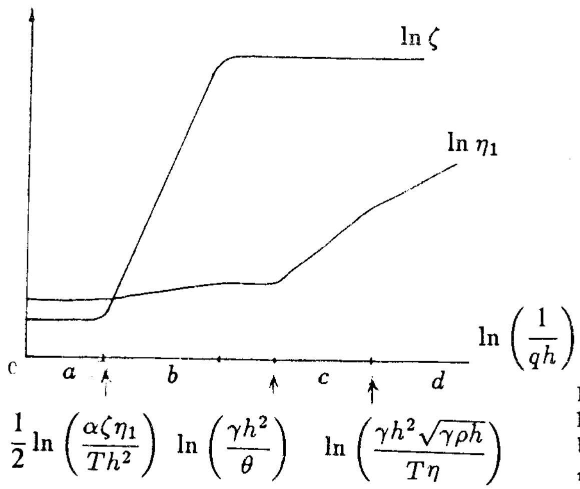

Fig. 2. The schematical dependence of the kinetic coefficients η_1 and ζ on scales: a) bare values, b) scaling with Δ_ζ, Δ_η, c) $\eta_1 \propto q^{-1/2}$, d) $\eta_1 \propto q^{-1/3}$.

At larger scales a new phenomenon should be taken into account. A feature of a freely suspended film is the possibility of its bending motion. Acoustic oscillations associated with this bending motion have an anomalously weak attenuation. Namely, in the linear approximation the dispersion law of the bending mode is

$$\omega = \pm c_s q - i\mu q^4 \quad , \tag{13}$$

where ω is the frequency and q is the wave vector. The mode with the dispersion law (13) may be called the shear sound, c_s being the velocity of the sound. The viscous damping of the sound proportional to q^2 is absent due to the rotational invariance of the film [2, 3]. For the soap film the coefficients in (13) are of the order of [4]

$$c_s \sim (\gamma/\rho h)^{1/2}, \quad \mu \sim \gamma h^3/\eta,$$

where ρ is the $3d$ density of the water and η is its viscosity. Besides the linear attenuation μq^4 figuring in (13) the nonlinear fluctuation contribution to the attenuation of the shear sound βq^3 exists [2, 3]. Here $\beta \sim T/(\rho_2 c_s)$ and ρ_2 is the surface mass density of the film (for the soap film $\rho_2 \sim \rho h$). Comparing these contributions ($\propto q^4$ and $\propto q^3$) we conclude that the fluctuation attenuation become essential at wave vectors

$$qh \sim (T/\gamma h^2)(\eta^2/\gamma\rho h)^{1/2} \quad , \tag{14}$$

both combinations in brackets being small parameters.

Bending fluctuations (as well as squeezing ones) give anomalous contributions to the viscosity coefficients. If qh exceeds the value (14) the contributions to η_1 and η_2 are of the order of $T/(\mu c_s q)^{1/2}$. For qh smaller than the scale (14) they are of the order of $T/\beta^{2/3}c_s q^{1/3}$. Note that the above contributions to η_1 and η_2 can be neglected in the region (3) since really Θ does not exceed T [5]. The schematic dependence of η_1 and ζ on scales is depicted in Fig.2.

This work was supported in part by the Russian Fund for the Fundamental Investigations (93-02-2687). E.K. and V.L. thank prof. Y.Stavans and dr. O.Krichevskii for the stimulating discussions of the experimental situation. A.M. thanks the Soros fundation for partial support.

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