Cherenkov emission of magnons by a slow monopole

P. V. Vorob'ev[a)] and I. V. Kolokolov[b)]

G. I. Budker Institute of Nuclear Physics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk State University, 630090 Novosibirsk, Russia

(Submitted 25 March 1998; resubmitted 6 May 1998)
Pis'ma Zh. Éksp. Teor. Fiz. **67**, No. 11, 866–868 (10 June 1998)

The energy losses due to Cherenkov emission of magnons during the interaction of a slow heavy monopole with magnetically ordered media are discussed. [S0021-3640(98)00611-2]

PACS numbers: 14.80.Hv

The observation of slow, heavy monopoles[1,2] would be of fundamental significance for elementary particle physics and cosmology. However, most modern detectors possess a low efficiency when detecting slow monopoles with velocities $v/c<10^{-4}$ (Ref. 3). For this reason, for both detector physics and astrophysics it is of interest to consider different mechanisms of the interaction of monopoles with matter.

In the present letter we study the passage of a slow monopole through a magnetically ordered medium. In this case the main mechanism of kinetic energy loss is Cherenkov emission of magnons. This is because the phase velocities of magnons reach zero and the monopole–magnon coupling is linear and large.

For definiteness, we shall study a ferromagnet but the estimates obtained below are more general.

The magnon Hamiltonian in the presence of the magnetic field of a moving monopole can be written in the form

$$H=\sum_{\mathbf{k}} \hbar\,\omega_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \sum_{\mathbf{k}} \left(f_{\mathbf{k}} e^{-i\Omega_{\mathbf{k}} t} a_{\mathbf{k}}^{\dagger} + \text{c.c.} \right), \tag{1}$$

where $a_{\mathbf{k}}^{\dagger}$ is the creation operator of a magnon with wave vector $\mathbf{k}$, $\omega_{\mathbf{k}}$ is the magnon dispersion law, $\Omega_{\mathbf{k}}=\mathbf{k}\cdot\mathbf{v}$, $\mathbf{v}$ is the monopole velocity vector, and $f_{\mathbf{k}}$ is the coupling coefficient between the monopole magnetic field $\mathbf{B}=g\nabla(1/r)$ and the magnon.

The energy of magnons emitted per unit time equals

$$\epsilon=\frac{2\pi}{\hbar}\sum_{\mathbf{k}} \omega_{\mathbf{k}} |f_{\mathbf{k}}|^2 \delta(\Omega_{\mathbf{k}}-\omega_{\mathbf{k}}). \tag{2}$$

Let the velocity $\mathbf{v}$ of the monopole be directed along the spontaneous magnetization, fixing the direction of the Z axis.[c)] Then

$$f_{\mathbf{k}}=\frac{4\pi g\mu_B}{a^{3/2}\sqrt{V}}\sqrt{\frac{S}{2}}\frac{k_x-ik_y}{k^2}, \tag{3}$$

where a is the lattice constant, V is the volume of the sample, S is the spin at a lattice site, and μ_B is the Bohr magneton. Substituting expression (3) Eq. (2) for ϵ becomes

$$\epsilon=\frac{2g^2\mu_B^2 S}{a^3\hbar}\int d^3\mathbf{k}\omega_{\mathbf{k}}\frac{k_x^2+k_y^2}{k^4}\delta(k_z v-\omega_{\mathbf{k}}). \tag{4}$$

The integration in Eq. (4) extends over the first Brillouin zone.

If $v\gg u$, where u is the velocity of magnons near the boundary of the Brillouin zone, then the magnons with large $\mathbf{k}$ are important. Then

$$\epsilon\simeq\frac{\overline{\omega}g^2\omega_M}{v}, \tag{5}$$

where the frequency $\omega_M=4\pi\mu_B^2 S/\hbar a^3$ characterizes the magnetization of the medium,[4]

$$\overline{\omega}=\frac{1}{2\pi}\int\frac{d^2\mathbf{k}_\perp}{k_\perp^2}\omega_{k_\perp} \tag{6}$$

$\mathbf{k}_\perp=(\mathbf{k}_x,\mathbf{k}_y)$, and $\overline{\omega}$ is of the order of the maximum magnon frequency.

For $g^2\simeq 4700\cdot e^2$ we obtain

$$\epsilon\simeq 10^3\cdot \mathrm{Ry}\cdot\omega_M(\overline{\omega}\tau), \tag{7}$$

where $\tau=a/v$ is the characteristic interaction time.

Typical values for magnetically ordered dielectrics are $\overline{\omega}\simeq 10^{-13}$ s^{-1}, $\omega_M\simeq 10^{-11}$ s^{-1} and for $v/c\simeq 10^{-4}$ we have $\epsilon\simeq 10^{14}$ eV/s, which corresponds to losses per unit length $dE/dl\simeq 10^8$ eV/cm.

It is evident from Eq. (5) that the losses ϵ and dE/dl increase with decreasing velocity v of the monopole. When the velocity v becomes $v<u$, magnons from the bottom of the spectrum make the main contribution to the losses. For them $\omega_{\mathbf{k}}=\omega_{\mathrm{ex}}(ak)^2$, where ω_{ex} is the frequency characterizing the exchange interaction,[4,5] the expressions for the losses now become

$$\epsilon=g^2\frac{\omega_M v}{4\omega_{\mathrm{ex}}a^2}, \tag{8}$$

$$\frac{dE}{dl}=\frac{\epsilon}{v}=g^2\frac{\omega_M}{4\omega_{\mathrm{ex}}a^2}. \tag{9}$$

As one can see, as the velocity of the monopole decreases, the energy losses per unit length become constant. The characteristic values are $\omega_M/\omega_{\mathrm{ex}}\simeq 10^{-2}$, and for $v/c\simeq 10^{-4}$ and $a\simeq 10^{-8}$ cm we have $dE/dl\simeq 10^8$ eV/cm.

It is evident from these estimates that the level of the energy losses of a slow magnetic monopole in a magnetically ordered medium is comparable to that of the ionization losses of a fast monopole. This opens up new possibilities for constructing detectors in the range $v/c < 10^{-4}$. Conversion of spin waves into electromagnetic waves[5] makes it possible to register the passage of a monopole through a slab of magnetic material by standard radio electronic means.

A detailed analysis of other mechanisms of the interaction of a slow monopole with matter is given in Ref. 6.

We thank L. M. Barkov, I. B. Khriplovich, and V. V. Yanovskiĭ for their interest in this work and for helpful remarks.

[a] e-mail: vorobyov@inp.nsk.su
[b] e-mail: kolokolov@inp.nsk.su
[c] The general case can be investigated completely analogously and the answer differs only by a numerical factor of the order of 1.

1. A. P. Polyakov, JETP Lett. **20**, 194 (1974).
2. G. 't Hooft, Nucl. Phys. B **79**, 276 (1974).
3. H. V. Klapdor-Kleingrofthaus and A. Staudt, *Non-Accelerator Particle Physics*, IOP Publishers, 1994 [Russian translation, Nauka, Moscow, 1997].
4. A. G. Gurevich, *Magnetic Resonance in Ferrites and Antiferromagnets* [in Russian], Nauka, Moscow, 1973.
5. A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, *Spin Waves*, North-Holland, Amsterdam, 1968 [Russian original, Nauka, Moscow, 1967].
6. P. V. Vorob'ev, I. V. Kolokolov, and V. V. Yanovskiĭ, ''Interaction of slow magnetic monopoles with matter'' [in Russian], in *Proceedings of the 32nd Winter School of St. Petersburg Institute of Nuclear Physics,* Gatchina, February 21–28, 1998 (to be published).

Translated by M. E. Alferieff