A Josephson relation for e/3 and e/5 fractionally charged with anyons

Nanoelectronics Group

 OPEN POSITION for 18-24 months Post-doct. (urgent)

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OUTLINE

• Quantum Hall edge states and Fractional Quantum Hall Effect

• PHOTON-ASSISTED TRANSPORT
  • Photon-assisted processes
  • A JOSEPHSON Relation for Photon Assisted Shot Noise (PASN)

• Experimental Results
  • $e^* = e/3$
  • $e^* = e/5$

• CONCLUSION and PERSPECTIVES

\[ f_J = \frac{e^* V}{h} \]

X. G. Wen (1991)
Quantum Hall Effect (QHE)

III-V semi-conductor heterojunction GaAs/GaAlAs

$R_{Hall} = \frac{B}{e n_s} = \frac{h}{e^2} \frac{1}{(\nu = k)}$

$H = \frac{1}{2m} \left( \hat{p} + e \hat{A} \right)^2 = \frac{\tilde{\pi}^2}{2m}$

$x = x - \frac{\pi_y}{eB}$
$y = y + \frac{\pi_x}{eB}$

$(x, y)$

$\pi / eB$

$(X, Y)$

cyclotron motion

energy

$\frac{5}{2} h \omega_c$
$\frac{3}{2} h \omega_c$
$\frac{1}{2} h \omega_c$

density of state

$n_\omega = \frac{eB}{h}$

$\omega_c = \frac{eB}{m}$

$[\pi_x, \pi_y] = -i \hbar eB$

$E_n = \hbar \omega_c \left( n + \frac{1}{2} \right)$

$[X,Y] = -i \frac{\hbar}{eB}$

$B \Delta X \Delta Y = \frac{\hbar}{e}$

cyclotron motion is frozen $\rightarrow$ 1D dynamics
Integer Quantum Hall Effect (IQHE)

$$R_{\text{Hall}} = \frac{h}{e^2} \frac{1}{\nu} \quad \nu = 1, 2, 3, \ldots$$

$$\rho_{xx}$$

$$\rho_{xy}$$

Energy levels:

$$\omega_c = \frac{eB}{m}$$

Density of state:

$$n_\phi = \frac{eB}{h}$$

$$R_{\text{Hall}} = \frac{B}{en_s} = \frac{h}{e^2} \frac{1}{\nu} (\nu = k)$$
QHE and EDGE STATES

III-V semi-conductor heterojunction GaAs/GaAlAs

$\frac{5}{2} \hbar \omega_c$

$\frac{3}{2} \hbar \omega_c$

$\frac{1}{2} \hbar \omega_c$

$\vec{E}_{\text{conf.}}$

$\vec{V}_{\text{drift}}$

( no current in the bulk )

( edge current )

cyclotron motion drift $\rightarrow$ chiral 1D EDGE CHANNELS

$\vec{B} \hat{z}$

$V_g < 0$

2D electrons

III-V semi-conductor heterojunction GaAs/GaAlAs

$\vec{E}_{\text{conf.}} \times \hat{z}$

$\vec{V}_{\text{drift}} = \frac{\vec{E}_{\text{conf.}}}{B} \times \hat{z}$
Integer Quantum Hall Effect (IQHE)

\[ R_{\text{hall}} = \left( \frac{h}{e^2} \right)^{1/\nu} \quad \nu = 1, 2, 3, \ldots \]

Fractional Quantum Hall Effect (FQHE)

\[ R_{\text{hall}} = \left( \frac{h}{e^2} \right)^{1/\nu} \quad \nu = 1/3, 2/5, 3/7, \ldots 2/3, 3/5, 4/7, \ldots \]

Anyons \( \Psi(a,b) = e^{i\theta} \Psi(b,a) \quad \theta = 2\pi/3 \)
DC SHOT NOISE: Integer QHE

\[ S_I = 2eI_0 D(1 - D) \]

\[ I_0 = e^2 V / h \]

\[ I_0 = I + I_B \]

transmitted \((D)\)

reflected \((1-D)\)

\[ S_I = 2elD \ll 1 \]

Schottky (1918)

Poisson’s statistics

\[ S_I = 2el_B D \approx 1 \]


**strong barrier:**

\[ I_0 = e^2 V / h \]

\( I(t) \)

\( v = 1 \)

transmitted \((D)\)

reflected \((1-D)\)

(rarely transmitted electrons)

(rarely transmitted holes)

**weak barrier:**

\[ I_0 = e^2 V / h \]

\( I \approx I_0 \)

\( I_B(t) \)

(rarely transmitted electrons)

(rarely transmitted holes)
**DC SHOT NOISE: FQHE**

**strong barrier:**

\[ I_0 = \frac{e^2 V}{3h} \]

\[ I(t) \]

\[ \nu = \frac{1}{3} \]

\[ \nu = \frac{1}{3} \] Laughlin state

\[ h/3eV \]

transmitted \((D)\) reflected \((1-D)\)

\[ S_I = 2eI \quad D << 1 \]

(rarely transmitted electrons)

\[ e/3 \quad e/3 \]

(rarely transmitted holes)

**weak barrier:**

\[ I_0 = \frac{e^2 V}{3h} \]

\[ I \approx I_0 \]

\[ I_B(t) \]

\[ I_0 = I + I_B \]

First observation:

CEA Saclay 1997

Weizmann 1997

\[ S_I = \frac{2e}{3} I_B \quad D \approx 1 \]

derived from chiral-Luttinger liquid approach

Tunneling through a $\nu=2/5$ Jain FQHE state

**FQHE $\rightarrow$ C-F. IQHE**

- $\nu = 1/3 \rightarrow \nu = 1$
- $\nu = 2/5 \rightarrow \nu = 2$
- $\nu = 3/7 \rightarrow \nu = 3$
- $\ldots \rightarrow \ldots$

**Plateau 1/3**

- $\nu_B = 2/5$

**Plateau 1/3**

- $\nu_B = 2/5$

**B: WB in 2/5**

- $e^* = e/5$

**A: WB in 1/3 while 2/5 reflected**

- $e^* = e/3$

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\[ f_J = \frac{e^*V}{h} \]

X. G. Wen (1991)
\[ \mu_R - \mu_L = e^* V_{dc} \]

\[ I_B(V_{dc}) \]

\[ S_{I_{DC}} = 2e^* |I_B| \]
### Photon-Assisted transport (weak coupling)

\[ V(t) = V_{dc} + V_{ac}\cos(2\pi ft) \]

\[ H_L \rightarrow H_L + e^* V_{ac}\cos(2\pi ft) \]

→ all carriers get extra time dependent phase:

\[ \phi(t) = \frac{1}{\hbar} \int_{-\infty}^{t} e^* V_{ac}(t')dt' \]

with: \( \exp(-i\phi(t)) = \sum p_i e^{-i2\pi ft} \)

\( p_i \): photo-absorption (Floquet) probability amplitude
\[ V(t) = V_{dc} + V_{ac}\cos(2\pi ft) \]

\[ H_L \rightarrow H_L + e^*V_{ac}\cos(2\pi ft) \]

→ all carriers get extra time dependent phase:

\[ \phi(t) = \frac{1}{\hbar}\int_{-\infty}^{t} e^*V_{ac}(t')dt' \]

with:

\[ \exp(-i\phi(t)) = \sum_{l} p_{l}e^{-i2\pi l ft} \]

\( p_{l} \): photo-absorption (Floquet) probability amplitude

global energy scattering for all left carrier energies \( \epsilon \) shifted by \( \epsilon \rightarrow \epsilon + l \ hf \)

\[ S_{I}^{PASN} = |p_0|^2 S_{I}^{DC}(V_{dc}) + |p_1|^2 S_{I}^{DC}(V_{dc} + hf/e^*) + |p_{-1}|^2 S_{I}^{DC}(V_{dc} - hf/e^*) + \ldots \]

\((e^* = e)\) Lesovik and Levitov (1994)

\((e^* = e/m)\) Chamon and Wen (1995)
Photon-Assisted Shot Noise (PASN)

\[ V(t) = V_{dc} + V_{ac}\cos(2\pi ft) \]

\[ H_L \rightarrow H_L + e^* V_{ac}\cos(2\pi ft) \]

→ all carriers get extra time dependent phase: \[ \phi(t) = \frac{1}{\hbar} \int_{-\infty}^{t} e^* V_{ac}(t') dt' \]

with: \[ \exp(-i\phi(t)) = \sum_{l} p_l e^{-i2\pi ft} \] \( p_l \): photo-absorption probability amplitude

global energy scattering for all left carrier energies \( \epsilon \) shifted by \( \epsilon \rightarrow \epsilon + L hf \)

\[ S_{I_{PASN}} = |p_0|^2 S_{I_{DC}}(V_{dc}) + |p_1|^2 S_{I_{DC}}(V_{dc} + hf/e^*) + |p_{-1}|^2 S_{I_{DC}}(V_{dc} - hf/e^*) + \ldots \]

\( (e^* = e) \) Lesovik and Levitov (1994)
\( (e^* = e/m) \) Chamon and Wen (1995)
Photon-Assisted Shot Noise (PASN)

\[ V(t) = V_{dc} + V_{ac}\cos(2\pi ft) \]

\( p_i \): photo-absorption probability amplitude

\( \mu_L \) shifted by \( \mu_L \to \mu_L + hf \) with probability \( |p_i|^2 \)

\[ |p_0|^2 + |p_1|^2 + |p_{-1}|^2 + \ldots = 1 \]

\[ S_i^{\text{PASN}} = |p_0|^2S_i^{\text{DC}}(V_{dc}) + |p_1|^2S_i^{\text{DC}}(V_{dc} + hf/e^*) + |p_{-1}|^2S_i^{\text{DC}}(V_{dc} - hf/e^*) + \ldots \]
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\[ f_J = \frac{e^*V}{h} \]

X. G. Wen (1991)
Experimental Set-up and samples

Samples: $n_s = 1.07 \times 10^{11} \text{ cm}^{-2}$ $\mu = 3 \times 10^6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ (from I. Farrer, D. Ritchie, Cambridge UK)
Experimental Set-up and samples

CROSS-SPECTRUM

- 300 K
  - $V_A$
  - $V_{dc}$
  - $V_{RF}$ 0-26GHz
  - $V_B$

- 4 K
  - -60 dB

- 25 mK
  - $V_2$
  - $I_B$
  - $V_{0}(t)$

2.2 MHz
$\Delta f \approx 150$ kHz

Home-made Cryo-amp.
(0.22 nV)^2/Hz

Helium-free Cryoconcept® cryostat

14 Tesla Dry Magnet
13 mK base temperature
case A: $\nu = \frac{2}{5}$

$e/3$

case B: $\nu = \frac{2}{5}$

$e/5$
DC Shot noise for the 1/3-FQHE state

\( V_{dc} \)

\( I_i + \Delta I_i(t) \)

\( v_B = 2/5 \)

\( v = e/3 \)

\( v_B = 2/5 \)

\( I_B + \Delta I_B(t) \)

\( e^* = e/3 \)

\( S^D (I_i) = 2 e^* I_B \left[ \coth \left( \frac{e^* V_{dc}}{2 k_B T} \right) - \frac{2 k_B T}{e^* V_{dc}} \right] \)

\( e^* = e/3 \)

Con confirms ’97-’98 experiments

(Saclay PRL 97, Weizmann Nat. 97 and 99, NTT 2015)
Photon-Assisted Shot Noise for the 1/3-FQHE state

\[ V(t) = V_{dc} + V_{ac} \cos(2\pi f t) \]

\[ V_{ac} \approx 100 \mu V \text{ for } -67\text{dBm} \]
Photon-Assisted Shot Noise for the 1/3-FQHE state

\[ V_{dc} + V_{ac} \cos(2\pi ft) \]

\[ v_B = \frac{2}{5} \]

\[ v = \frac{e}{3} \]

\[ v_B = \frac{2}{5} \]

\[ f = 22 \text{GHz} \]

\[ V(t) = V_{dc} + V_{ac} \cos(2\pi ft) \]

\[ V_{ac} \approx 200 \mu V \text{ for } -61 \text{dBm} \]
Photon-Assisted Shot Noise for the 1/3-FQHE state

\[ V(t) = V_{dc} + V_{ac} \cos(2\pi ft) \]

\[ p_0 = J_0 \left( \frac{e^* V_{ac}}{hf} \right) \]

\[ p_1 = -p_{1-1} = J_1 \left( \frac{e^* V_{ac}}{hf} \right) \]

\[ S_{l,\text{PASN}}(V_{dc}) = \left| p_0 \right|^2 S_{l,\text{DC}}^{\text{DC}}(V_{dc}) + \left| p_1 \right|^2 \left[ S_{l,\text{DC}}^{\text{DC}}(V_{dc} - hf / e^*) + S_{l,\text{DC}}^{\text{DC}}(V_{dc} + hf / e^*) \right] \]

\[ f = 22 \text{GHz} \]

\[ V_{ac} \approx 200 \mu \text{V} \text{ for } -61 \text{dBm} \]
Excess PASN for the 1/3-FQHE state

Killing the non photon-assisted part!

Excess PASN:

\[ \Delta S_i = S_i^{PASN}(V_{dc}) - |p_0|^2 S_i^{DC}(V_{dc}) \]

\[ = |p_1|^2 \left[ S_i^{DC}(V_{dc} - hf/e^*) + S_i^{DC}(V_{dc} + hf/e^*) \right] \]

Finding a flat variation for the low |V_{dc}| range provides a determination of |p_0|^2
Excess PASN for the 1/3-FQHE state

WHY a FLAT VARIATION?

$\begin{align*}
\text{Excess PASN} & \quad V_{dc} \\
-hf/e^* & \quad hf/e^* \\
\end{align*}$

$\begin{align*}
S_i & \\
\text{DC Shot Noise} & \\
|p_0|^{2S_i^{DC}}(V_{dc}+hf/e^*) & \quad \text{for } V_{dc} > hf/e^* \\
|p_{-1}|^{2S_i^{DC}}(V_{dc}-hf/e^*) & \quad \text{for } V_{dc} < hf/e^* \\
\end{align*}$
Finding a flat variation for the low $|V_{dc}|$ range provides a determination of $|p_0|^2$

as $|p_0|^2 + 2|p_1|^2 \approx 1$ this gives $|p_1|^2$

Excess PASN:

$$\Delta S_I = S_I^{PASN}(V_{dc}) - |p_0|^2 S_I^{DC}(V_{dc})$$

$$= |p_1|^2 \left[ S_I^{DC}(V_{dc} - hf / e^*) + S_I^{DC}(V_{dc} + hf / e^*) \right]$$

Killing the non photon-assisted part!
Finding a flat variation for the low $|V_{dc}|$ range provides a determination of $|p_0|^2$ as $|p_0|^2 + 2|p_1|^2 \approx 1$ this gives $|p_1|^2$

Excess PASN:

$$\Delta S_i = S_i^{PASN}(V_{dc}) - |p_0|^2 S_i^{DC}(V_{dc})$$

$$= |p_1|^2 \left[ S_i^{DC}(V_{dc} - hf/e^*) + S_i^{DC}(V_{dc} + hf/e^*) \right]$$

Killing the non photon-assisted part!

Excess PASN: $22\text{GHz} \quad -61\text{dBm}$

- $e^* = e/3$
- $\alpha = 0.85$
- $T_e = 60\text{mK}$
Josephson relation for the 1/3-FQHE state

CHECKING the FREQUENCY DEPENDENCE of Excess PASN:

\[ \Delta S_I = S_{I_{PASN}}(V_{dc}) - |p_0|^2 S_{I_{DC}}(V_{dc}) \]

\[ = |p_1|^2 \left[ S_{I_{DC}}(V_{dc} - hf/e^*) + S_{I_{DC}}(V_{dc} + hf/e^*) \right] \]

threshold voltage: \( V_J = hf/e^* \) scales with frequency!
New Measurement of $e^*$ for the 1/3-FQHE State

MEASURING $e^*$ from Excess PASN:

$$\Delta S_I = S_I^{PASN} (V_{dc}) - |p_0|^2 S_I^{DC} (V_{dc})$$

$$= |p_1|^2 \left[ S_I^{DC} (V_{dc} - hf/e^*) + S_I^{DC} (V_{dc} + hf/e^*) \right]$$

threshold voltage: $V_J = hf/e^*$ scales with frequency!

Best fit of data with $e^*$ free parameter

$e^* = 1/(3.07 \pm 0.05)$
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  • \( e^* = e/5 \)

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\[ f_j = \frac{e^*V}{h} \]

X. G. Wen (1991)
DC Shot noise for the 2/5-FQHE state

\[ S_I^{DC} = 2e^* I_B \left[ \coth \left( \frac{e^* V_{dc}}{2k_B T} \right) - \frac{2k_B T}{e^* V_{dc}} \right] \propto -<\Delta I_B \Delta I_t> \]

\( e^* = e/5 \)

confirms Weizmann results (Reznikov 1999) on 2/5
Photon-Assisted Shot Noise for the 2/5-FQHE state

\[ V(t) = V_{dc} + V_{ac} \cos(2\pi ft) \]

\[ V_{ac} \approx 300 \, \mu V \text{ for } -58 \text{dBm} \]

\[ V_{ac} \approx 400 \, \mu V \text{ for } -55 \text{dBm} \]
Excess PASN for the 2/5-FQHE state

Killing again the non photon-assisted part!

Excess PASN:

\[ \Delta S_i = S_i^{\text{PASN}}(V_{dc}) - |p_0|^2 S_i^{\text{DC}}(V_{dc}) = |p_i|^2 \left[ S_i^{\text{DC}}(V_{dc} - hf / e^*) + S_i^{\text{DC}}(V_{dc} + hf / e^*) \right] \]

Finding a flat variation for the low \(|V_{dc}|\) range provides a determination of \(|p_0|^2\)
Excess PASN for the 2/5-FQHE state

Killing again the non photon-assisted part!

Excess PASN:

$$\Delta S_l = S_l^{PASN} (V_{dc}) - |p_0|^2 S_l^{DC} (V_{dc})$$

$$= |p_1|^2 \left[ S_l^{DC} (V_{dc} - \hbar f / e^*) + S_l^{DC} (V_{dc} + \hbar f / e^*) \right]$$

Finding a flat variation for the low $|V_{dc}|$ range provides a determination of $|p_0|^2$

as $|p_0|^2 + 2|p_1|^2 \approx 1$ this gives $|p_1|^2$

comparison using $f_{Josephson} = e^*V_{dc}/\hbar$ with $e^* = e/5$
Josephson relation for the 2/5-FQHE state

CHECKING the FREQUENCY DEPENDENCE of Excess PASN:

\[ \Delta S_I = S_{I_{PASN}}^{PASN} (V_{dc}) - |p_0|^2 S_{I_{DC}}^{DC} (V_{dc}) \]
\[ = |p_1|^2 \left[ S_{I_{DC}}^{DC} (V_{dc} - hf / e^*) + S_{I_{DC}}^{DC} (V_{dc} + hf / e^*) \right] \]

threshold voltage: \( V_J = hf / e^* \) scales with frequency!
New Measurement of $e^*$ for the 2/5-FQHE State

MEASURING $e^*$ from Excess PASN:

$$
\Delta S_l = S_l^{PASN} (V_{dc}) - |p_0|^2 S_l^{DC} (V_{dc})
= |p_1|^2 \left[ S_l^{DC} (V_{dc} - hf / e^*) + S_l^{DC} (V_{dc} + hf / e^*) \right]
$$

threshold voltage: $V_J = hf / e^*$ scales with frequency!

Best fit of data with $e^*$ free parameter
• FQHE $e*=e/3$ and $e/5$ abelian anyons can be manipulated with microwave by well-defined photon-assisted processes. What about $e/4$ in non-abelian 5/2 FQHE state?

• Validates the possibility to realize on-demand single anyon sources for time domain anyon braiding.

• Based on Photon-Assisted Shot Noise (PASN)

• Shows evidence of the Josephson relation $e^*V/h = f$ predicted in 1991 by X.G. Wen*

(Old 1997 exp.)

\[
S_1 = 2 \ e^* \ I_B
\]

weak signal but accurate

good signal but lack of accuracy, model dependent

(PASN Josephson Relation (photon quantum))

\[
hf = e^* \ V
\]

very accurate

good accuracy

*predicted for the current, see also I. Safi + Sukhorukov (2010).
The Josephson Frequency of fractionally charge anyons
M. Kapfer, P. Roulleau, I. Farrer, D. A. Ritchie, and D. C. Glattli,
arXiv:1806.03117,
Published 24 January 2019 on Science
DOI: 10.1126/science.aau3539

Levitons:
J. Dubois et al., Nature 502, 659 (2013)
T. Jullien et al., Nature 514, 603 (2014)
IDEA: Weak backscattering beaks the leviton into e/3, 2e/3 quasiparticles.

- Anyons inherit from the time properties of Levitons
- Non-deterministic: Poissonian source
\(- \langle \Delta I_u \Delta I_d \rangle \propto 1 + g_2(\tau) \cos(\theta_{\text{stat}})\)
Braiding Anyons

1) Unveiling the anyon statistical angle with Hong Ou Mandel braiding interference

Photon 1
Photon 2
Anyon 1
Anyon 2

\[ P(1,2) = \frac{1}{2} |b(1,2) + b(2,1)|^2 \]

\[ P(1,2) = \frac{1}{2} (1 - \cos \theta) \]

0 : boson bunching (\(\theta=0\))
1 : fermion antibunching (\(\theta=\pi\))
\(\frac{1}{4} \) : for \(\nu=1/3\) FQHE abelian anyons (\(\theta=\pi/3\))