Unconventional Magnons and their Impact on Spin Pumping Transport

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Outline/Results

- Introduction and motivation
- Quasiparticles in Ferromagnets with spin $\hbar^* > \hbar$
- Spin current shot noise and quantum of transport
- Quasiparticles in Ferrimagnets and Antiferromagnets $\rightarrow$ squeezing ($\hbar^* > \hbar$) and hybridization ($0 \leq \hbar^* < \hbar$)
- Spin pumping in Ferrimagnets and Antiferromagnets $\rightarrow$ interface asymmetries, cross sublattice coupling
- Phenomenology of non-standard Gilbert damping in Ferri- and Antiferromagnets
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Quasiparticles in a ferromagnet

Quasiparticles in a ferromagnet

Magnon

Magnon

Considering only exchange interaction and Zeeman energy!

Quasiparticles in a ferromagnet

With magnon operators \( \tilde{b}_q \) and \( \tilde{b}_q^+ \) ("bosons")

\[
\hat{\mathcal{H}}_F = \sum_q A_q \tilde{b}_q^+ \tilde{b}_q
\]

Effect of dipolar interactions!

Quasiparticles: squeezed magnons

Bogoliubov transformation to new quasi-particles

\[
\tilde{\beta}_q = u_q \tilde{b}_q - v^*_q \tilde{b}_q^+
\]

\[
\hat{\mathcal{H}}_F = \sum_q \hbar \omega_q \tilde{\beta}_q^+ \tilde{\beta}_q
\]

Ground state: squeezed vacuum

\[
\tilde{\beta}_q |\psi_G\rangle_q = 0 \rightarrow |\psi_G\rangle_q \neq |0\rangle
\]
Classical Hamiltonian

\[ \mathcal{H}_F = \int_{V_F} d^3r \left( H_Z + H_{aniso} + H_{ex} + H_{dip} \right) \]

Linearization around equilibrium magnetization \((M_s)\):
Magnetization saturated along z direction

\[ H_Z + H_{aniso} = \frac{E_{za}}{M_s^2} \left( M_x^2 + M_y^2 \right) \]

\[ H_{ex} = \frac{A}{M_s^2} \left( (\vec{\nabla}M_x)^2 + (\vec{\nabla}M_y)^2 \right) \]

Classical Hamiltonian

Simplification for dipolar interaction:

- Homogeneous demagnetization field

- Demagnetization tensor:

\[ \hat{N} = \text{diag}(N_x, N_y, N_z) = \text{diag}(1, 0, 0) \]

\[ H_{dip} = -\frac{1}{2} \mu_0 \vec{H}_m \vec{M} = \frac{1}{2} \mu_0 M_x^2 \]

\[ \vec{H}_m = -\hat{N} \vec{M} = -M_x \hat{e}_x \]

Quantization: HP transformations

Magnon field operators in real space: $\tilde{b}(\vec{r})$, $\tilde{b}^+(\vec{r})$

Ladder operators: $\tilde{M}_\pm = \tilde{M}_x \pm i\tilde{M}_y$  ($[\tilde{M}_x, \tilde{M}_y] = i\hbar\tilde{M}_z$)

$$
\tilde{M}_+ = M_s \sqrt{1 - \tilde{b}^+\tilde{b}} / S \tilde{b}
$$

$$
\tilde{M}_- = M_s \tilde{b}^+ \sqrt{1 - \tilde{b}^+\tilde{b}} / S
$$

$$
\tilde{M}_z = M_s - \hbar \tilde{b}^+\tilde{b} / S
$$

$$
S = 2M_s / \gamma \hbar
$$

$$
\tilde{M}_x^2 \sim (\tilde{b}_0^+ + \tilde{b}_0)^2 = 2\tilde{b}_0^+\tilde{b}_0 + \tilde{b}_0^+\tilde{b}_0^+ + \tilde{b}_0\tilde{b}_0
$$


Quantum Hamiltonian

Magnon annihilation operators in momentum space: $\tilde{b}_{\vec{q}}$

$$\tilde{H}_F = \tilde{H}_0 + \sum_{\vec{q} \neq 0} \tilde{h}_{\vec{q}}$$

with

$$\tilde{H}_0 = A_0 \tilde{b}_0^+ \tilde{b}_0 + B_0 \tilde{b}_0^+ \tilde{b}_0^+ + B_0 \tilde{b}_0 \tilde{b}_0$$

$$A_0 = \hbar \omega_{za} + \hbar \omega_s / 2$$

$$B_0 = \hbar \omega_s / 4$$

$\hbar \omega_{za} = \text{Anisotropy and external fields}$

$\hbar \omega_s = \mu_0 \mu_B M_s \rightarrow \text{Energy of a moment in the dipole field!}$

- Finite momentum Hamiltonian has similar form with $A_{\vec{q}}, B_{\vec{q}}$
- Hamiltonian can be diagonalized by a Bogolubov transformation (for each $(\vec{q}, -\vec{q})$ separately)
- Non-trivial ground state and excitations
  $\rightarrow$ squeezed vacuum and squeezed magnons
Squeezed-magnons

Bogoliubov transformation to new quasi-particles

\[ \tilde{\beta}_q = u_q \tilde{b}_q - v_q^* \tilde{b}_{-q}^{\dagger} \]

\[ \tilde{\mathcal{H}}_F = \sum_q \hbar \omega_q \tilde{\beta}_q^{\dagger} \tilde{\beta}_q \]

E.g. for \( \tilde{q} = 0 \)

- \( \omega_0 = \sqrt{\omega_{za}^2 + \omega_{za} \omega_s} \)
- \( u_0^2 - v_0^2 = 1 \)
- \( v_0^2 = \frac{\omega_s^2 / 4 \omega_0}{\omega_0 + \omega_{za} + \omega_s / 2} \)

Squeezed Vacuum

Ground state defined by

\[ \tilde{\beta}_q |\psi_G\rangle_q = 0 \quad \leftrightarrow \quad |\psi_G\rangle_q \sim |0\rangle_q + \frac{v_q}{u_q} |2\rangle_q + \cdots \]
Ground state: Squeezed Vacuum

Heisenberg uncertainty, viz. \(\langle \Delta \tilde{M}_x^2 \rangle \langle \Delta \tilde{M}_y^2 \rangle \geq \Delta M_0^4\)

\[
\langle \Delta \tilde{M}_x^2 \rangle = \Delta M_0^2 e^{-\xi} \quad \langle \Delta \tilde{M}_y^2 \rangle = \Delta M_0^2 e^{+\xi} \quad \text{tanh} \; \xi = \frac{v_0}{u_0}
\]

Squeezed magnons

\[ |n_\beta = 1\rangle = \tilde{\beta}_0^+ |\psi_G\rangle \]
\[ = (u_0 \tilde{b}^+ - v_0 \tilde{b}_0) |\psi_G\rangle \]
\[ \sim |1\rangle + \cdots |3\rangle + \cdots |5\rangle + \cdots \]

→ Complex superposition of elementary magnons

Spin of squeezed magnon?

\[ \langle n_\beta = 1 | \tilde{S}_z | n_\beta = 1 \rangle - \langle \psi_G | \tilde{S}_z | \psi_G \rangle = \hbar (1 + 2v_0^2) \equiv \hbar^* \geq \hbar \]

Note: Bogolubov trafo depends in general on \( \tilde{q} \) : \( \hbar^* \rightarrow \hbar^*_q \)

Spin of squeezed magnon

\[ \hbar^* = \hbar (1 + 2\nu_0^2) \]

\[ 2\nu_0^2 = \frac{\omega_s^2/4\omega_0}{\omega_0 + \omega_{za} + \omega_s/2} \]

\[ \omega_0 = \sqrt{\omega_{za}^2 + \omega_{za}\omega_s} \]

\[ \omega_{za} = \mu_0\gamma H_{za} \]

→ Large \( \hbar^* \) for realistic material parameters
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Quantum noise spectral density of a quantum point contact

Conductance $G = G_Q \sum T_n$

Fano factor $F = \frac{\sum T_n (1 - T_n)}{\sum T_n}$

Conductance quantum $G_Q = \frac{2e^2}{h}$

Transmission probabilities $T_n$

$S(\omega) = \int dt e^{i \omega t} \langle \{ \hat{I}(0), \hat{I}(t) \} \rangle = 2eG \left[ F \left(eV - |\omega|\right) \theta(eV - |\omega|) + |\omega| \right]$
Spin detection via spin pumping

\[ \tilde{H}_{\text{int}} = \int d^2 \rho J_I \tilde{M}(\tilde{\rho}) \tilde{s}_N(\tilde{\rho}) \]
\[ = \sum_{\tilde{k} \tilde{k}', \tilde{q}} J_{\tilde{k} \tilde{k}', \tilde{q}} \tilde{b}_{\tilde{q}} \tilde{c}_{\tilde{k} \uparrow}^+ \tilde{c}_{\tilde{k}' \downarrow} + h.c. \]

2\text{nd–order perturbation in } J \text{ for the z-component of the spin current } I_{dc}:

\[ I_{dc} = G_S \hbar^* \omega |\chi|^2 \]

Spin detection via spin current shot noise

Spin noise: uncorrelated transfer of bunches $\hbar^*$ (Poissonian statistics) in time period $t_0$

Statistics of number of events $N$:
$\langle N \rangle = \bar{N} = \Gamma t_0$
$\langle \Delta N^2 \rangle = \bar{N} = \Gamma t_0$

Statistics of Spin $S = \hbar^* N$:
$\langle S \rangle = \hbar^* \bar{N} = \hbar^* \Gamma t_0$
$\langle \Delta S^2 \rangle = \hbar^*^2 \langle \Delta N^2 \rangle = \hbar^* \langle S \rangle$

Full noise spectral density

$$S(\Omega) = 2 \int dt e^{i\Omega t} \langle \Delta I_z(t) \Delta I_z \rangle$$
$$= \hbar^*^2 G_s |\chi|^2 [|\omega - \Omega| + |\omega + \Omega|]$$

- Zero-frequency noise $S(0) = 2 \hbar^* I_{dc}$
- Analog electric current noise with $\omega = eV$!

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Two interpenetrating sublattices

Similar model as before, but generalized to two sublattices:

\[ J \rightarrow J_{AA}, J_{BB}, J_{AB} \quad E_{za} \rightarrow E_{za}^{A/B} \quad M_s \rightarrow M_{A0/B0} \quad \tilde{b}_q \rightarrow \tilde{a}_q, \tilde{b}_q \]
Two-sublattice magnon Hamiltonian

\[ \tilde{\mathcal{H}} = \sum_k \left[ \frac{A_k}{2} \tilde{a}_k \tilde{\alpha}_k + \frac{B_k}{2} \tilde{b}_k \tilde{\beta}_k + C_k \tilde{a}_k \tilde{b}_{-k} + D_k \tilde{a}_k \tilde{\alpha}_{-k} \\
+ E_k \tilde{b}_k \tilde{\beta}_{-k} + F_k \tilde{a}_k \tilde{\beta}_k^\dagger \right] + \text{H.c.} \]

\[ A_k, B_k : \text{intra-sublattice exchange and external field} \]
\[ C_k : \text{inter-sublattice exchange} \]
\[ D_k, E_k : \text{dipolar interaction-induced squeezing} \]
\[ F_k : \text{dipolar interaction-induced hybridization} \]
4-D Bogoliubov Transform

Ferromagnet: \( b_q \to \beta_q \)
\[
\tilde{\beta}_q = u_q \tilde{b}_q - v^*_q \tilde{b}^+_q
\]

Ferrimagnet: \( \tilde{a}_q, \tilde{b}_q \to \tilde{a}_q, \tilde{\beta}_q \)
\[
\tilde{\alpha}_q = u_q \tilde{a}_q + v_q \tilde{b}^+_q + w_q \tilde{a}^+_q + x_q \tilde{b}_q
\]

Magnon spin: \( \hbar^* = \hbar \left( 1 + 2|w_q|^2 - 2|x_{\bar{q}}|^2 \right) \)

Squeezing \( \rightarrow \hbar^* > 1 \)

Hybridization \( \rightarrow \hbar^* < 1 \)

Magnons in Ferrimagnets

- Two branches crossing
- Effect of dipolar interaction
  \( \sim \text{GHz} \) (solid lines)
  \( \rightarrow \) anticrossing

Ferrimagnet

\[ M_{A0} = 5M_{B0} \]

„squeezing“
(analog quasi-ferromagnet)

Ferrimagnets

Ferrimagnet

\[ M_{A0} = 2M_{B0} \]

Hybridization

Antiferromagnets

\[ M_{A0} = 2M_{B0} \]

Magnon spin:

\[ \hbar^* = \hbar \left( 1 + 2|w_{\vec{q}}|^2 - 2|x_{\vec{q}}|^2 \right) = 0 \]

Zero-spin quasiparticles!

Antiferromagnets

Zero-spin quasiparticles!

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Spin pumping in ferrimagnets

Bulk and Interfacial asymmetry!


Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets
Interface (matters!)

A/B sublattice

$$\tilde{H}_{int} = -\frac{1}{\hbar^2} \int d^2 \rho [J_{iA} \tilde{M}_A(\hat{\rho}) + J_{iB} \tilde{M}_B(\hat{\rho})] \tilde{s}_N(\hat{\rho})$$


Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets
Spin pumping in antiferromagnets (previous work)

Different interface model and assumptions:

\[
\frac{e}{\hbar} I_s = G_r (\mathbf{n} \times \dot{n} + \mathbf{m} \times \dot{m})
\]

\(G_r\): real part of the mixing conductance

\[
\mathbf{m} = \frac{1}{2} (\mathbf{m}_A + \mathbf{m}_B)
\]

\[
\mathbf{n} = \frac{1}{2} (\mathbf{m}_A - \mathbf{m}_B)
\]

Spin pumping in ferrimagnets

\[ I_{sz} = 2\hbar|\chi|^2 \left[ \Gamma_{AA} \left( |u|^2 - |w|^2 \right) + \Gamma_{BB} \left( |v|^2 - |x|^2 \right) \right. \]

\[ \left. -2\Gamma_{AB} \mathcal{R} \left( u^* v - wx^* \right) \right], \]

With \( \Gamma_{AA} \sim J_{iA}^2 \), \( \Gamma_{BB} \sim J_{iB}^2 \), \( \Gamma_{AB} \sim J_{iA} J_{iB} \)

In terms of sublattice magnetizations:

\[ I_{sz} = G_{AA} \left( \vec{m}_A \times \dot{\vec{m}}_A \right)_z + G_{BB} \left( \vec{m}_B \times \dot{\vec{m}}_B \right)_z \]

\[ + G_{AB} \left( \vec{m}_A \times \dot{\vec{m}}_B + \vec{m}_B \times \dot{\vec{m}}_A \right)_z \]

\[ G_{AA} \sim J_A^2 M_A^2 v_N^2, \ldots \]

- Under generic assumptions: \( G_{AA} G_{BB} = G_{AB}^2 \)
- In general NO direct connection to \( \hbar^* \)
- Shot noise → detailed information on quasiparticles (provided \( G_{AA/BB} \) are known)

Spin pumping in ferrimagnets

\[
\frac{e}{\hbar} I_{sz} = G_{AA}(\vec{m}_A \times \vec{m}_A)_z + G_{BB}(\vec{m}_B \times \vec{m}_B)_z \\
+ G_{AB}(\vec{m}_A \times \vec{m}_B + \vec{m}_B \times \vec{m}_A)_z \\
= G_{mm}(\vec{m} \times \vec{m})_z + G_{nn}(\vec{n} \times \vec{n})_z \\
+ G_{mn}(\vec{m} \times \vec{n} + \vec{n} \times \vec{m})_z
\]

\[
\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B) \\
\vec{n} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)
\]

\[
G_{mm} = G_{AA} + G_{BB} + 2G_{AB} \\
G_{nn} = G_{AA} + G_{BB} - 2G_{AB} \\
G_{mn} = G_{AA} - G_{BB} \\
G_{AB} = \sqrt{G_{AA}G_{BB}}
\]

Spin pumping in ferrimagnets

\[ G_{mm} = G_{AA} + G_{BB} + 2G_{AB} \]
\[ G_{nn} = G_{AA} + G_{BB} - 2G_{AB} \quad G_{AB} = \sqrt{G_{AA}G_{BB}} \]
\[ G_{mn} = G_{AA} - G_{BB} \]

• Compensated interface:
  \[ G_{AA} = G_{BB} \rightarrow G_{mn} = 0 = G_{nn} \]

• Fully uncompensated interface
  \[ G_{BB} = 0 \rightarrow G_{mm} = G_{nn} = G_{mn} \]

• Chen et al.: \( G_{AA} = G_{BB}, G_{AB} = 0 \)
  \[ \rightarrow G_{mm} = G_{nn}, G_{mn} = 0 \]


Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets
Role of asymmetry for spin pumping

Vanishes for compensated AF-N interface

Strongly enhanced for AF with asymmetries

\[ \frac{1}{\hbar} I_{sz} = G_{AA}(\vec{m}_A \times \dot{\vec{m}}_A)_z + G_{BB}(\vec{m}_B \times \dot{\vec{m}}_B)_z + G_{AB} (\vec{m}_A \times \dot{\vec{m}}_B + \vec{m}_B \times \dot{\vec{m}}_A)_z \]


Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets
Role of cross-sublattice terms

\[ G_{AB} = \sqrt{G_{AA}G_{BB}} \]

By hand: \[ G_{AB} = 0 \]

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Simple dissipation model

\[ \mathcal{L} = \mathcal{T} - \mathcal{U} = \frac{1}{2}m\ddot{x}^2 - \frac{1}{2}kx^2. \]

Dissipation:
\[ \mathcal{R} = \frac{1}{2}\Gamma\dot{x}^2 \]

\[
\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} + \frac{\partial \mathcal{R}}{\partial \dot{x}} = 0, \]
\[ m\dddot{x} + kx + \Gamma \ddot{x} = 0. \]

Rayleigh dissipation function captures viscous damping
Dynamical equation for a Ferromagnet

\[
\frac{d}{dt} \frac{\delta L[M, \dot{M}]}{\delta \dot{M}} - \frac{\delta L[M, \dot{M}]}{\delta M} + \frac{\delta R[\dot{M}]}{\delta \dot{M}} = 0
\]

Euler-Lagrange

\[
R[\dot{M}(r, t)] = \frac{\eta}{2} \int \dot{M}(r, t) \cdot \dot{M}(r, t) \, dr
\]

Rayleigh dissipation

\[
\frac{\partial M(r, t)}{\partial t} = \gamma M(r, t) \times \left[ H(r, t) - \eta \frac{\partial M(r, t)}{\partial t} \right]
\]

Landau-Lifshitz-Gilbert equation

• Rigorous treatment of Kinetic Energy term is tricky, and require the equations to reduce to Landau-Lifshitz equation in the absence of damping
• Note: Landau-Lifshitz equation $\dot{M} = \gamma M \times H - \lambda M \times (M \times H)$ gives $\dot{M} \to \infty$ for $\lambda \to \infty$! Cured by reduced gyromagnetic ratio $\gamma \to \gamma^*/(1 + \lambda^2)$ reducing the precession.

Rayleigh dissipation functional captures viscous damping

Two-sublattice magnet

\[
\frac{d}{dt} \frac{\delta \mathcal{L}[\cdot]}{\delta \dot{M}_{A,B}} - \frac{\delta \mathcal{L}[\cdot]}{\delta M_{A,B}} = - \frac{\delta R[\dot{M}_A, \dot{M}_B]}{\delta \dot{M}_{A,B}}
\]

\[
R[\dot{M}_A, \dot{M}_B] = \int_V d^3 r \left( \frac{\eta_{AA}}{2} \dot{M}_A \cdot \dot{M}_A + \frac{\eta_{BB}}{2} \dot{M}_B \cdot \dot{M}_B + \eta_{AB} \dot{M}_A \cdot \dot{M}_B \right)
\]

\[
\dot{M}_A = - |\gamma_A| (M_A \times \mu_0 H_A) + |\gamma_A| \eta_{AA} (M_A \times \dot{M}_A) + |\gamma_A| \eta_{AB} (M_A \times \dot{M}_B)
\]

\[
\dot{M}_B = - |\gamma_B| (M_B \times \mu_0 H_B) + |\gamma_B| \eta_{AB} (M_B \times \dot{M}_A) + |\gamma_B| \eta_{BB} (M_B \times \dot{M}_B)
\]

\[
\mu_0 H_{A,B} = - \frac{\delta F[M_A, M_B]}{\delta M_{A,B}}
\]

Two-sublattice magnet

\[
\dot{\mathbf{m}}_A = -|\gamma_A| (\mathbf{m}_A \times \mu_0 \mathbf{H}_A) + \alpha_{AA} (\mathbf{m}_A \times \dot{\mathbf{m}}_A) + \alpha_{AB} (\mathbf{m}_A \times \dot{\mathbf{m}}_B)
\]

\[
\dot{\mathbf{m}}_B = -|\gamma_B| (\mathbf{m}_B \times \mu_0 \mathbf{H}_B) + \alpha_{BA} (\mathbf{m}_B \times \dot{\mathbf{m}}_A) + \alpha_{BB} (\mathbf{m}_B \times \dot{\mathbf{m}}_B)
\]

\[
\tilde{\alpha} = \begin{pmatrix}
\alpha_{AA} & \alpha_{AB} \\
\alpha_{BA} & \alpha_{BB}
\end{pmatrix}
= \begin{pmatrix}
|\gamma_A| \eta_{AA} M_{A0} & |\gamma_A| \eta_{AB} M_{B0} \\
|\gamma_B| \eta_{AB} M_{A0} & |\gamma_B| \eta_{BB} M_{B0}
\end{pmatrix}
\]

\[
\frac{\alpha_{AB}}{\alpha_{BA}} = \frac{|\gamma_A| M_{B0}}{|\gamma_B| M_{A0}}.
\]

Gilbert damping matrix

Collinear Ground State

\[ F[M_A, M_B] = \int_V d^3r \left[ -\mu_0 H_0 (M_{Az} + M_{Bz}) - K_A M_{Az}^2 - K_B M_{Bz}^2 + JM_A \cdot M_B \right] \]

Resonance frequencies:

\[ \omega_{r\pm} = \frac{\pm (\Omega_A - \Omega_B) + \sqrt{(\Omega_A + \Omega_B)^2 - 4J^2 |\gamma_A||\gamma_B|M_{A0}M_{B0}}}{2} \]

Line widths:

\[ \frac{\omega_{i\pm}}{\omega_{r\pm}} = \bar{\alpha} \frac{(\Omega_A + \Omega_B) - 2J |\gamma_B|M_{A0}\alpha_{AB}}{\omega_{r+} + \omega_{r-}} \pm \Delta \bar{\alpha} \]

\[ \bar{\alpha} \equiv (\alpha_{AA} + \alpha_{BB})/2 \quad \Delta \bar{\alpha} \equiv (\alpha_{AA} - \alpha_{BB})/2 \]

- Renormalization of resonance width by off-diagonal damping \( \alpha_{AB} > 0 \)
- Difference resonance widths due to different sublattice dampings \( \alpha_{AA} \neq \alpha_{BB} \)

Compensated Ferrimagnets


Spin pumping mediated damping

\[ I_s = \frac{\hbar}{e} \sum_{i,j=\{A,B\}} G_{ij} (\hat{m}_i \times \hat{m}_j) \]

\[ \alpha'_{ij} = \frac{\hbar G_{ij} |\gamma_i|}{e M_i V} \]

\[ \frac{\omega_{i\pm}}{\omega_{r\pm}} = \tilde{\alpha} (\Omega_A + \Omega_B) - 2J |\gamma_B| M_{A0} |\alpha_{AB} + \Delta\tilde{\alpha} \]

Summary:

- A two-sublattice ferromagnet is not described by a simple ferromagnet
- Non-diagonal damping has observable but difficult in collinear magnets
- No need for artificially enhanced damping around the compensation point
Summary: aspects of quantum magnonics

- Dipolar interaction-mediated squeezing and hybridization of magnons with spin greater (lesser) than $\hbar$
- Spin current-shot noise as probe of non-integer spins
- Spin-zero excitations in Ferri- and Antiferromagnets
- Importance of interface/damping asymmetry and cross terms for spin pumping and spin shot noise in Fi/AF-N

- **A. Kamra** and W. Belzig, 
  Super-Poissonian shot noise of squeezed-magnon mediated spin transport, 

- **A. Kamra** and W. Belzig, 
  Magnon-mediated spin current noise in ferromagnet|nonmagnetic conductor hybrids, 

- **A. Kamra**, U. Agrawal, and W. Belzig 
  Noninteger-spin magnonic excitations in untextured magnets 

- **A. Kamra** and W. Belzig 
  Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets, 

- **A. Kamra**, R. E. Troncoso, W. Belzig, and A. Brataas 
  Gilbert damping phenomenology for two-sublattice magnets. 