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Non-Abelian vortices: are they relevant in condensed matter physics?

Outline

☞ Vortices, strings (starting from Abrikosov)

☞ Moduli: (classically) gapless modes on vortices/strings

☞ Abelian and non-Abelian orientational moduli

☞ Example from dense quantum chromodynamics

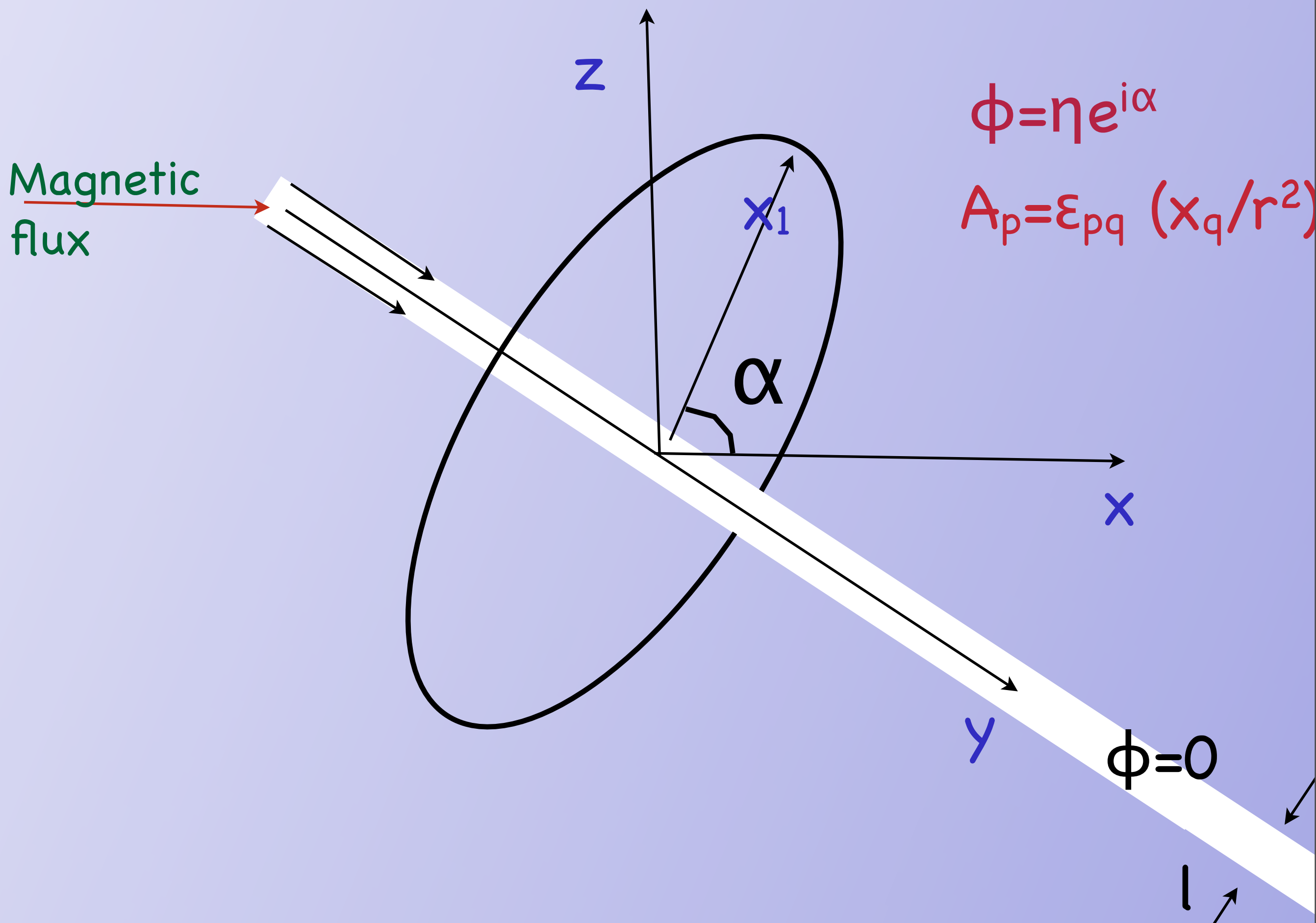
☞ Example from condensed matter (???) ^{3}He

☞ A few words in conclusion

Abrikosov vortices in type-2 superconductors

◇ Gauged U(1) symmetry spontaneously broken in the bulk

◇◇ $\Delta H_{GL} = D_k\phi^\dagger D_k\phi + \lambda(\phi^\dagger\phi - \eta^2)^2$ ➡ time derivatives can be rel. or non-relat.



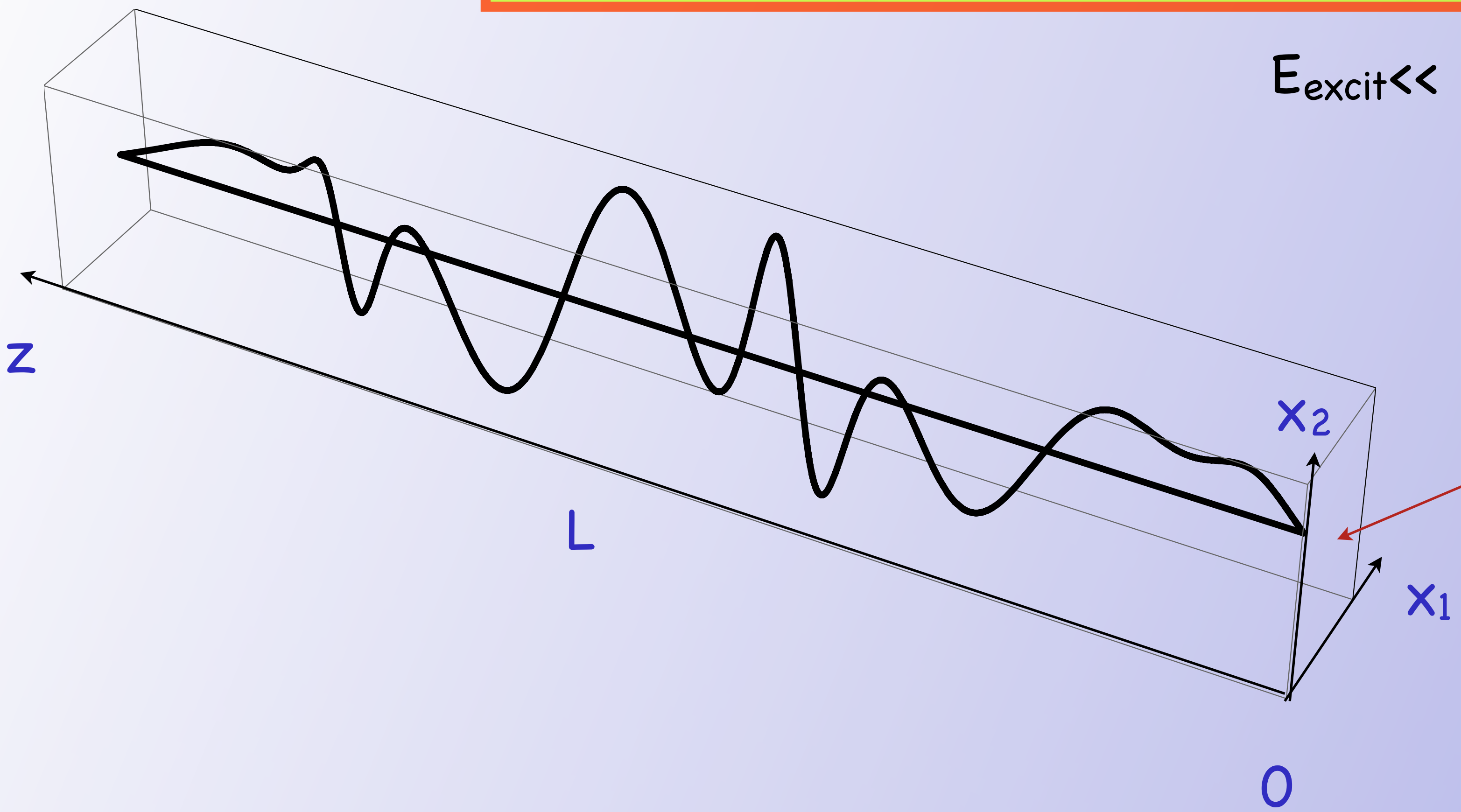
$m_\gamma \sim e\eta$

$T \sim \eta^2$

$l \sim 1/m_\gamma$

Low-energy excitations (gapless modes)

$E_{excit} \ll m_\gamma \sim e\eta$



Perpendicular plane

◇◇ $\Delta H_{GL} = (T/2)(\partial_z x_{perp}\, \partial_z x_{perp})$ + h.d. ➡ time derivatives can be rel. or non-relat.

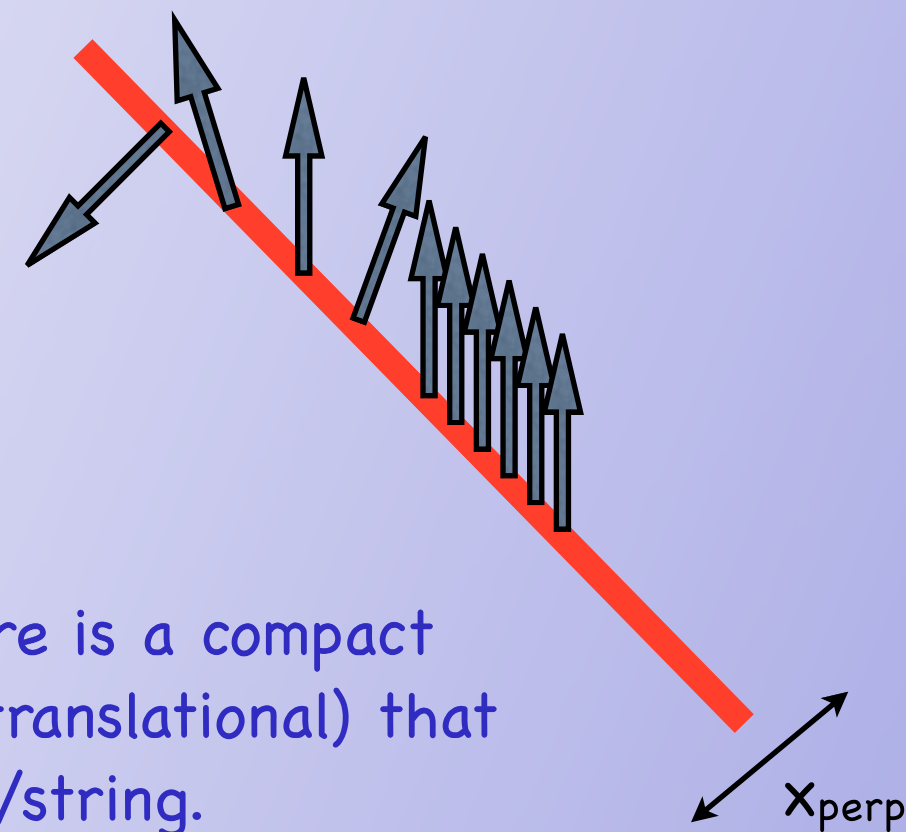
Nambu-Goto → String Theory

Kelvin modes or Kevlons
2 NG gapless modes in relat.
1 NG gapless mode in non-rel.

$E_{str} = TL + C/L$

Counts # of gapless modes !

Orientational moduli: classically extra gapless modes (gap may or may not develop quantum-mechanically



◇ Orientational modes develop IF there is a compact global symmetry in the bulk (i.e. not translational) that is spontaneously broken on the vortex/string.

$$\underset{\substack{\uparrow \\ \text{Bulk}}}{\text{E.g. } SU(2)}/\underset{\substack{\uparrow \\ \text{Vortex}}}{U(1)} = CP(1) \sim O(3) \text{ sigma model}$$

Abelian example: Belavin-Polyakov lump (Skyrmion in cond. matter)

$$\mathcal{H} = \frac{1}{2g^2}\partial_\mu \vec{n}\,\partial_\mu \vec{n}\ , \qquad \vec{n}^2 = 1$$

◇ O(3) sigma model in the bulk, poss. mass term: $\boxed{+\ m^2(1-n_3^2)}$

◇◇ In the bulk O(3) → O(2), i.e. $n_{gr.st.}$ = (0,0,1)

Along z axis

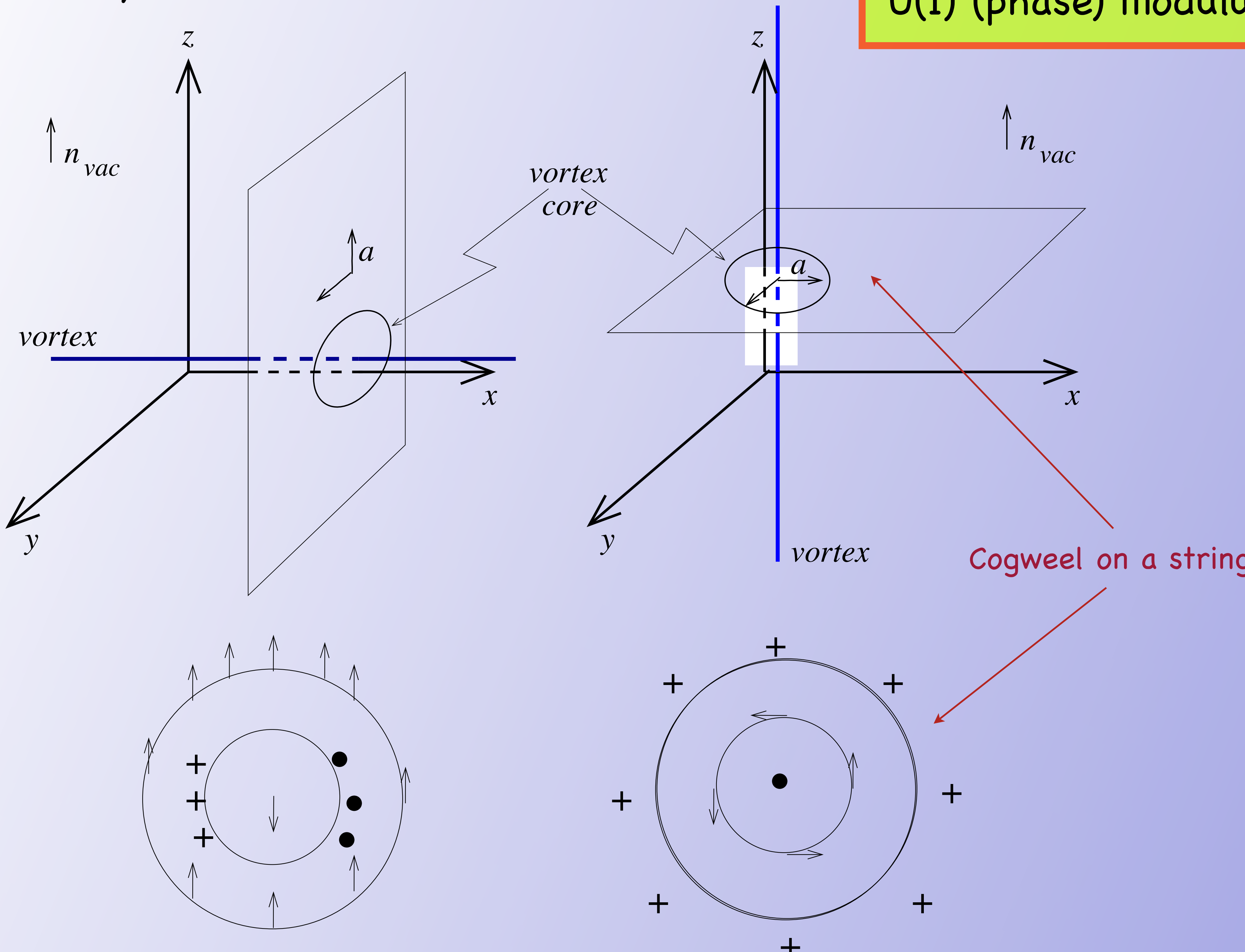
Arbitrary vector in the perp. plane

$$n_1 = \frac{2\vec{x}_\perp \cdot \vec{a}}{\vec{x}_\perp^2 + \vec{a}^2}\,, \qquad n_2 = \frac{2\vec{x}_\perp \times \vec{a}}{\vec{x}_\perp^2 + \vec{a}^2}\,, \,, \qquad n_3 = \frac{\vec{x}_\perp^2 - \vec{a}^2}{\vec{x}_\perp^2 + \vec{a}^2}$$

Belavin–Polyakov

$|a| \sim 1/m$

U(1) (phase) modulus



Amending Abrikosov to non-Abelian

◇ $\Delta H_{GL} = D_k\phi^\dagger D_k\phi + \lambda(\phi^\dagger\phi-\eta^2)^2$ ➠ time derivatives can be rel. or non-relat.

◇◇ $\Delta H_{NA} = \partial_k n^i\, \partial_k n^i + (-\mu^2+\phi^\dagger\phi)n^i n^i + \beta(n^i n^i)^2$ + time derivatives

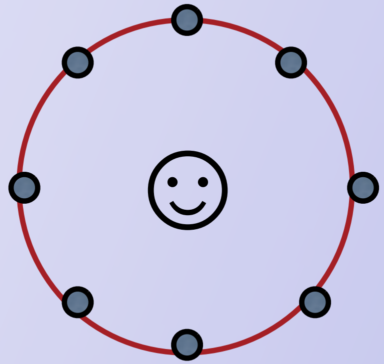
with $\eta^2>\mu^2$

✷ In ground state $\phi^\dagger\phi_{gr.st}= \eta^2$, hence the mass term of $n^i = \eta^2-\mu^2>0$ and O(3) is unbroken

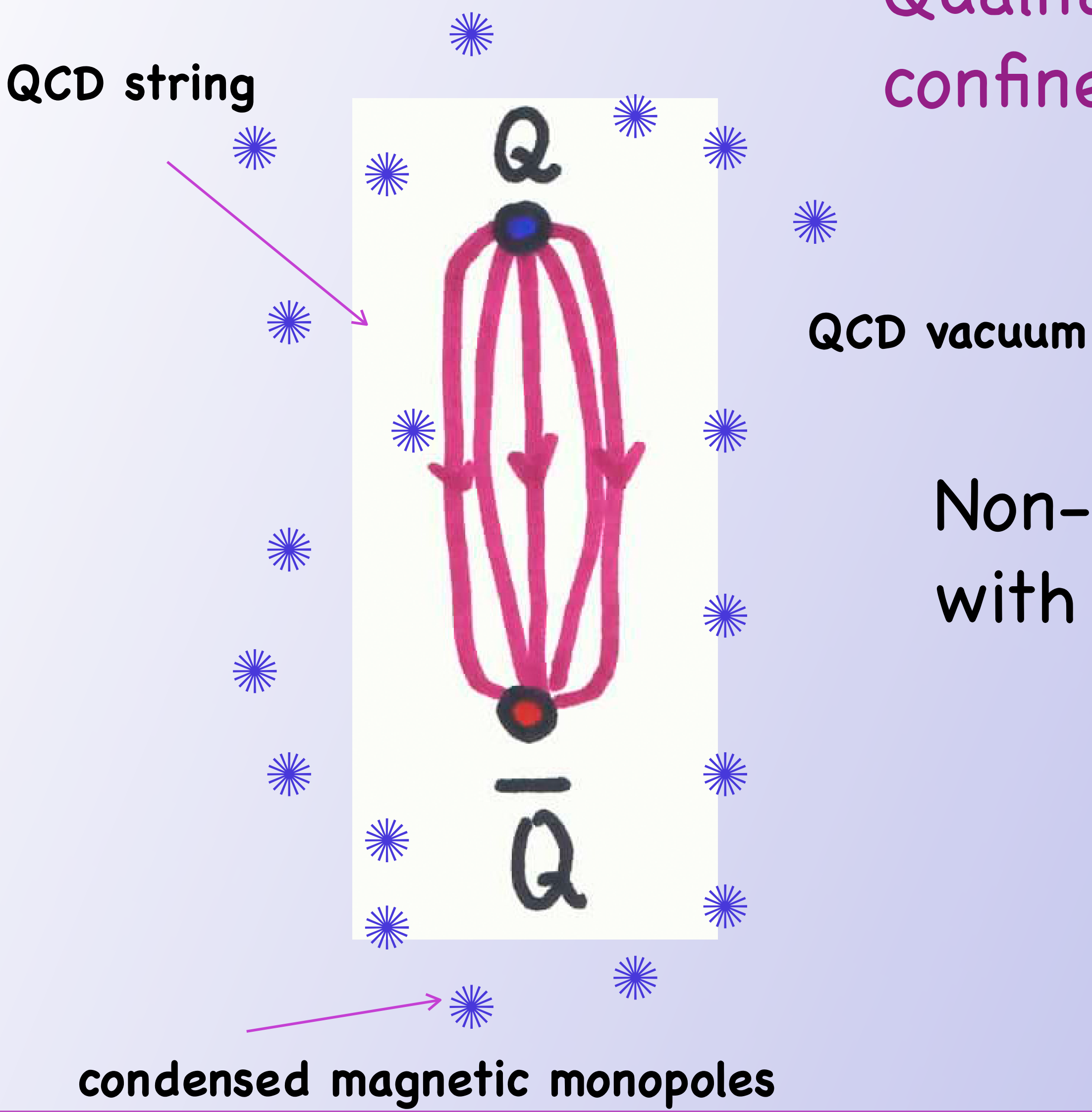
✷✷ Inside Abrikosov $\phi^\dagger\phi_{gr.st.}= 0$ hence the mass term of $n^i = -\mu^2<0$ and O(3) is broken down to O(2), while $n^i n^i = \mu^2/2\beta$

✷✷✷ Classically O(3) sigma model on vortex, 2 gapless interacting modes

Quantum-mechanically, because of IR interactions, both modes are lifted. However, if $\Lambda_{IR} \ll (T)^{1/2}$ they are quasi-NG!



History and HEP applications



Qualitative explanation of quark confinement: Dual Meissner effect:

Non-Abelian strings built in SUSY with CP(N-1) models on the w.-s.

Worldsheet theory

1) Confined monopoles in dense QCD

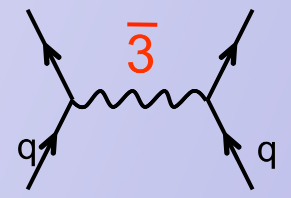
<u>*Color Superconductivity (CSC)*</u>

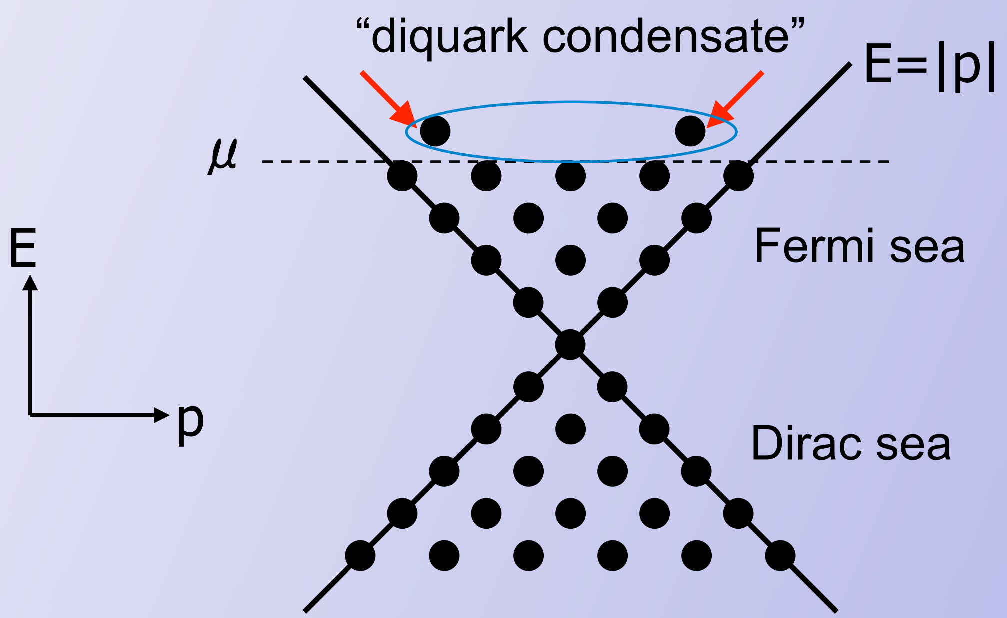
- QCD at high density → Fermi surface, weak-coupling

- Attractive channel → Cooper instability

  $[3]_C \times [3]_C = [6]_S + [\bar{3}]_A$

$$(\tau_a)_{ij}(\tau_a)_{kl} = \frac{2}{3}(\tau_S)_{ik}(\tau_S)_{lj} - \frac{4}{3}(\tau_A)_{ik}(\tau_A)_{lj}$$





Neutron stars?

2 Ginsburg–Landau effective description

At large μ QCD is in the CFL phase. Diquark condensate

3 colors and 3 flavors

$$\Phi^{kC} \sim \varepsilon_{ijk}\,\varepsilon_{ABC}\left(\psi_\alpha^{iA}\,\psi^{jB\,\alpha} + \tilde{\bar\psi}^{iA\,\dot\alpha}\,\tilde{\bar\psi}_{\dot\alpha}^{jB}\right)$$

At $T \to T_c$ gap fluctuations become important.

Chiral fluctuations (π-mesons) are considered less important

$$S = \int d^4x \left\{ \frac{1}{4g^2}\left(F_{\mu\nu}^a\right)^2 + 3\,\mathrm{Tr}\,(\mathcal{D}_0\Phi)^\dagger(\mathcal{D}_0\Phi) \right.$$

$$\left. + \quad \mathrm{Tr}\,(\mathcal{D}_i\Phi)^\dagger(\mathcal{D}_i\Phi) + V(\Phi) \right\}$$

with the potential

$$V(\Phi) = -m_0^2\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right) + \lambda\left(\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 + \mathrm{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right]\right)$$

$$\Phi_{\mathrm{vac}} = v\,\mathrm{diag}\,\{1,1,1\}$$

where

$$v^2 = \frac{m_0^2}{8\lambda} = \frac{4\pi^2}{3}\,\frac{T_c - T}{T_c}\,\mu^2$$

The symmetry breaking pattern

$$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_F \times \mathrm{U}(1)_B \to \mathrm{SU}(3)_{C+F}$$

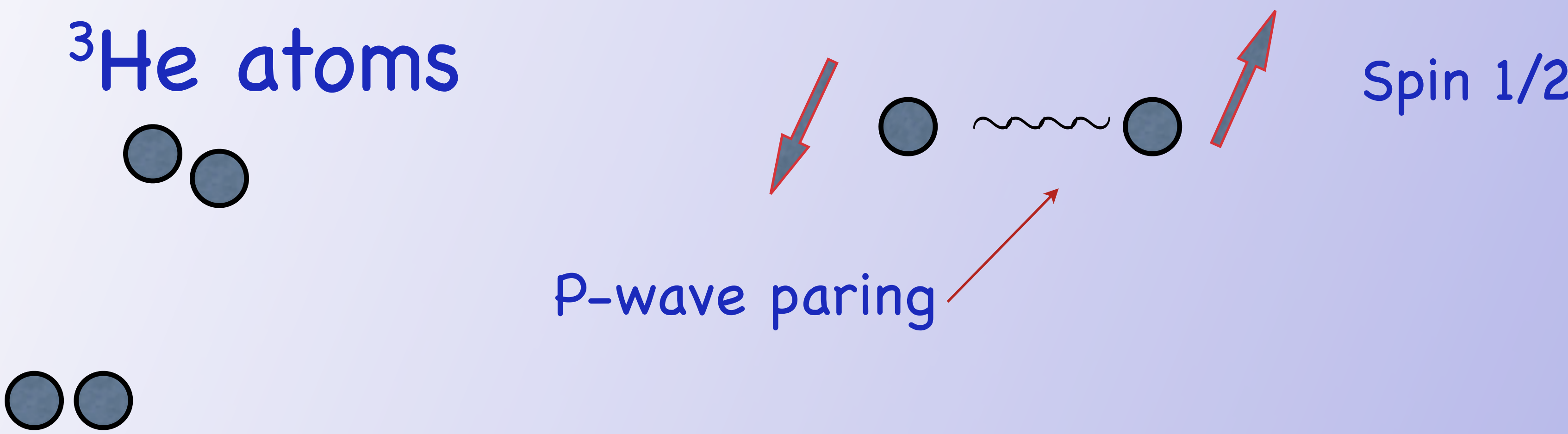
9 symmetries are broken.

8 are eaten by Higgs mechanism.

One Goldstone boson associated with broken $\mathrm{U}(1)_B$.

Broken → SU(2)×U(1) ➟ CP(2) model on the string w.-s. !

^{3}He-B example?



L=1, S=1 ➟ Cooper pair order parameter $e_{\mu i}$ $\leftarrow$ 3×3 matrix

Spin-orbit small, symmetry of H is $G = U(1)_p \times SO_S(3) \times SO_L(3)$

In the ground state $U_p(1) \times SO_S(3) \times SO_L(3) \to H_{\rm B} = SO(3)_{S+L}$

Hence, contrived NG modes in the bulk!

While:

In the conventional superfluid, the breaking of an Abelian phase symmetry

$$U_p(1) \to 1$$

leads (only) to phonons in the bulk.

^{3}He-B theory supports vortices ← Non-trivial topology:

$$\pi_1\left(U_p(1) \times SO_S(3) \times SO_L(3)/SO(3)_{S+L}\right) = \pi_1\left(U_p(1) \times SO(3)_{S-L}\right) = \mathbb{Z} \times \mathbb{Z}_2$$

The first $\mathbb{Z}$ factor corresponds to the breaking of the Abelian $U_p(1)$ symmetry and support the so called mass vortices. These vortices are created and stabilized in a lattice once the sample is rotated, and are characterized by a non-vanishing superfluid current and angular momentum, as vortices in conventional superfluid. The second $\mathbb{Z}_2$ factor also stabilize a second more exotic type of vortices called spin vortex. Spin vortices are not directly created by rotation of the superfluid.

✱The vortex solution breaks a part of symmetry preserved in gr.st.

✱✱ Known: it can break U(1) if the core is axially asymmetric. Hence U(1) NG mode.

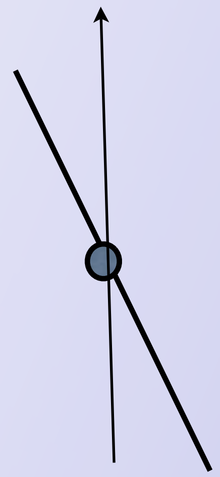
We say:

(M. Nitta+M.S.+W.Vinci)

On the vortex $SO(3)_{S+L} \to SO(2)=U(1)$. Hence, there are (classically) gapless modes corresponding to the $SU(2)/U(1) = O(3)$ sigma model. Quantum mechanically a gap is generated, but they remain quasi-NG if

$$\Lambda_{IR} \ll (T)^{1/2}$$

Objections: Gregory Volovik ➟ "spatial rotation is not a good symmetry since it rotates the vortex"!



Objection to objections: local rotation ≡ z dependent translation.

OK, to rotate the order parameter without rotating the string.

Instead of conclusions

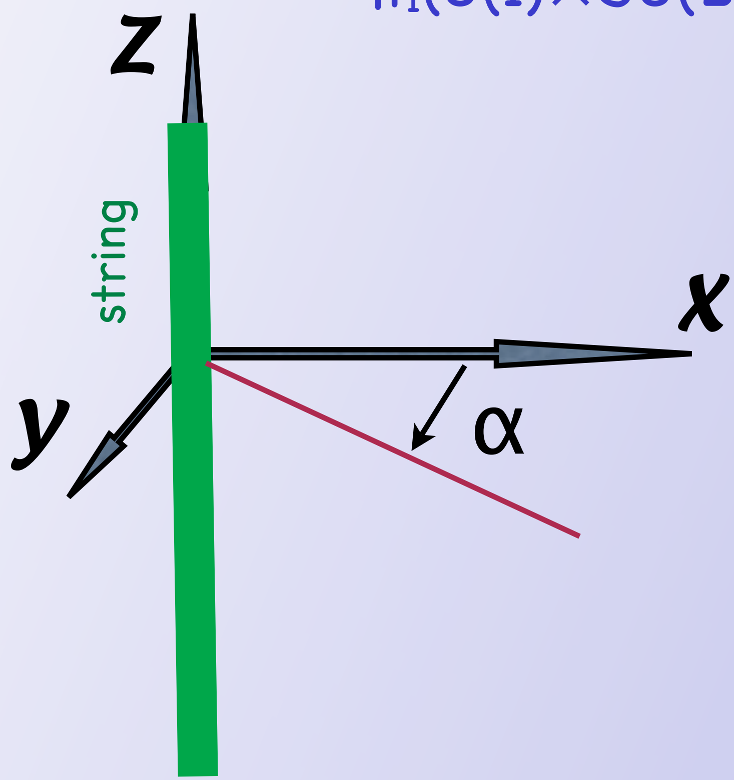
A few words about Tolya. Our offices were next door. Unfortunately, I did not work with him, but whenever I asked him something, he always tried to understand the question immersing himself in deep – and sometimes long – thinking, and then would always come up with an illuminationg answer. And he was very kind.

I am editing a book of recollections about Landau's students/ colleagues "Under the Spell of Landau: when theoretical physics was shaping destinies." My dream is to have a Chapter in this book devoted to Tolya. Insofar I have short articles from Sasha Larkin, Andrei Varlamov and Valentin Vaks. If you want to pay a tribute to Tolya, please, consider writing an article too. Even 1–2 pages about Tolya as a physicist and human being, as a teacher and collaborator, would be great!

☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



x_0 ← string center in perp. plane

ANO $\quad \sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$T = 4\pi\xi$

Non-Abelian $\quad \sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$

$T_{U(1)} \pm T^3{}_{SU(2)}$

$T = 2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

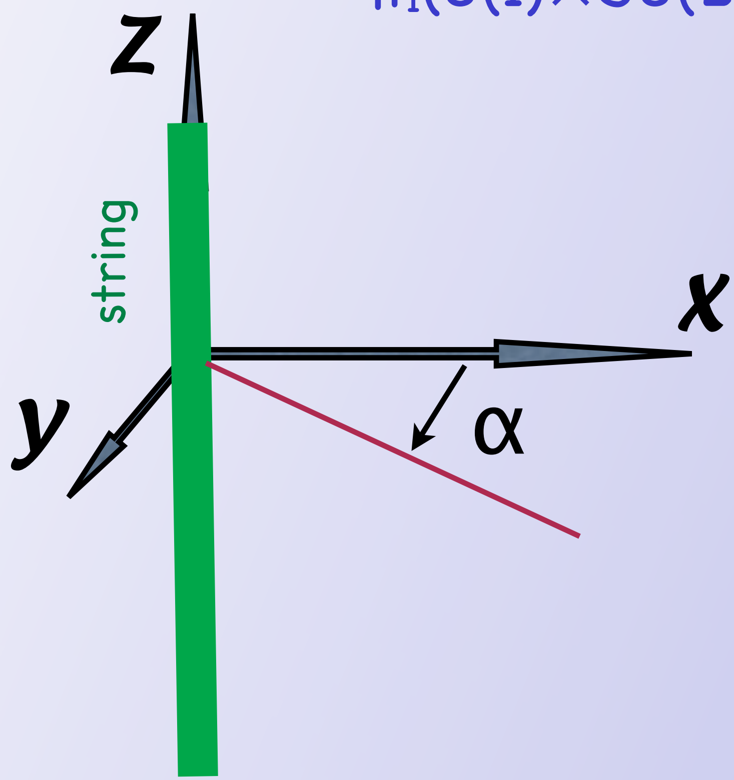
☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ -1; another -1 rotate by π in U(1)

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



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