





Outline

- introduction to chiral magnetic crystals
- crystallization process
- helimagnon band structure
- skyrmion topology & magnon-driven skyrmion motion
- spinwaves in skyrmion crystals
- orientation of skyrmion crystals
- topological defects and domain walls in helimagnets

Introduction to chiral magnetic crystals



Chiral magnets

magnets with a non-centrosymmetric chiral atomic crystal lattice

- hexagonal Cr_{1/3}NbS₂ with strong crystal anisotropies
 - monoaxial chiral magnet

Togawa et al PRL (2012)



• cubic B20 compounds: MnSi, FeGe, Fe_xCo_{1-x}Si, Cu₂OSeO₃, ...

Bravais lattice: simple cubic space group: P2₁3 (B20)



chiral atomic crystal lattice



cubic chiral magnets

 n_2

Ginzburg-Landau theory for cubic chiral magnets

Landau-Lifshitz Vol. 8 (2nd edition), §52 helicoidal magnetic structures:

in the limit of small spin-orbit coupling λ_{SOC} : clear separation of energy/length scales

 \Rightarrow universal theory at low energies

Bak & Jensen (1980), Nakanishi et al. (1980)



competition between exchange and DMI rightarrow chiral magnetic crystals





2d skyrmion crystal

MnSi in a magnetic field

Ishikawa *et al.* PRB (1977) Thessieu *et al.* J. Phys.: Condens. Matter (1997)



Cubic chiral magnets

"universal" magnetism shared by whole class of materials

metal MnSi



Mühlbauer et al. Science (2009)

15

7(K)

5 10 40



Grigoriev et al. PRB (2007)





Wilhelm et al. PRL (2011)

Phase diagram of MnSi con't.



under pressure

Chiral magnetic crystals

incommensurate periodic magnetic structures



share many aspect with crystalline order

lattice constant: $\lambda \approx 18 \text{ nm}$ in MnSi 70 nm in FeGe

Chiral magnetic transition: weak crystallization process



Weak crystallization process

crystal breaks translation symmetry and also isotropy of space!

 \Box

critical mode becomes soft on a sphere in momentum space

$$\omega(\vec{q}) = \Delta + (|\vec{q}| - Q)^2$$

Brazovskii, JETP (1975) Kats, Lebedev, Muratov, Phys. Rep, (1993)

> one-dimensional divergence in density of states

$$\rho(\varepsilon) = \int d^3k \delta(\omega(\mathbf{k}) - \varepsilon) \sim \frac{1}{\sqrt{|\varepsilon - \Delta|}}$$

Fluctuation-induced first-order transition to avoid the entropy of the critical state!

Paramagnons soften at finite momentum



on a sphere in momentum space

Strongly correlated critical paramagnet

strong correlations above T_c even at ambient pressure:



Strongly correlated critical paramagnet

strong correlations above T_c even at ambient pressure:



Fluctuation-induced first order transition



Fluctuation-induced first order transition



Brazovskii suppression of correlation length

temperature dependence of chiral correlation length $\kappa = 1/\xi$



Janoschek et al. PRB (2013)

Brazovskii renormalization in thermodynamics



Janoschek et al. PRB (2013)

Helimagnon band structure



Magnetization dynamics

Landau-Lifshitz equation



$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{B}_{\text{eff}} + \dots$$

in effective field

damping, driving currents etc.

effective field is determined by the magnetic texture: $\vec{B}_{\rm eff} = -\frac{\delta F}{\delta \vec{M}}$ with the Ginzburg-Landau functional F



Magnon excitations:

expansion around the static magnetization

> Bogoliubov wave equation

$$i\hbar\tau^z\partial_t\vec{\Psi}=H\vec{\Psi}$$

$$\hat{M} = \hat{M}_s \sqrt{1 - 2|\psi|^2} + \hat{e}^+ \psi + \hat{e}^- \psi^*$$

with magnon spinor wave function

$$\vec{\Psi}^T = (\psi, \psi^*)$$

U(1) charge = spin angular momentum of magnon not conserved!

due to spin-orbit coupling, texture and dipolar interactions

Magnon excitations of the magnetic helix



helix = 1d magnetic crystal

magnon excitations obey Bloch's theorem

rightarrow magnon band structure

magnon Hamiltonian:

$$\mathcal{H}_0 = \mathcal{D}\left[\mathbbm{1}(q_\perp^2 - \partial_z^2) - i2\tau^z Qq_\perp \cos(Qz) + \frac{Q^2}{2}(\mathbbm{1} - \tau^x)\right]$$

variant of the Mathieu equation

⇒ particle in a one-dimensional periodic cosine potential

Kugler *et al* PRL (2015) Weber, *et al* arXiv:1708.02098

Magnon excitations of the magnetic helix



helix = 1d magnetic crystal

magnon excitations obey Bloch's theorem

rightarrow magnon band structure

transversal momentum q_{\perp} tunes strength of periodic potential crossover from weak to tight-binding limit



Magnon excitations of the magnetic helix



helix = 1d magnetic crystal

magnon excitations obey Bloch's theorem

Inelastic neutron scattering on MnSi:





five magnon bands well-resolved

> Kugler *et al* PRL (2015) Weber, *et al* arXiv:1708.02098

Skyrmions



Skyrmions

Tony Skyrme (1961,1962)

solutions of a non-linear field theory, model for baryons



stereographic projection from sphere to plane:

(isospin doublet ³H/³He)



topologically stable object with quantized winding number

$$W = \frac{1}{4\pi} \int d^2 \mathbf{r} \, \hat{M} \left(\partial_x \hat{M} \times \partial_y \hat{M} \right)$$

counts skyrmions!

$$\Pi_2(S^2) = \mathbb{Z}$$
 baby-skyrmions

Observation of skyrmion crystals in B20 compounds



2d skyrmion crystal

First observation by neutron scatterin in MnSiS. Mühlbauer *et al.* Science (2009) T. Adams et al. PRL (2011) in Fe_{1-x}Co_xSi W. Münzer *et al.* PRB (2010) G. 04 0.040

transmission electron microscopy on films:

in $Fe_{0.5}Co_{0.5}Si$

X. Z. Yu et al. Nature (2010)

in FeGe

X. Z. Yu et al. Nature Materials (2011)

in Cu₂OSeO₃

S. Seki et al. Science (2012)



Fe_{0.5}Co_{0.5}Si

Magnetic force microscopy



in Fe_{0.5}Co_{0.5}Si P. Milde *et al.* Science (2013)

Consequences of non-trivial topology



Skyrmion hosting materials





Material	Crys. Class	SG	Skyrm. Type	T _c	λ (nm)	Trans.	References
MnSi	Т	P2 ₁ 3	Bloch	30 K	18	Metal	S. Mühlbauer <i>et al.</i> , Science 323 , 915 (2009)
Fe Ge	Т	P2 ₁ 3	Bloch	279 K	70	Metal	X.Z. Yu et al., Nat. Mater. 10, 106 (2010)
Fe _{1-x} Co _x Si	Т	P2 ₁ 3	Bloch	< 36 K	40-230	Semi- cond.	W. Münzer <i>et al.,</i> PRB 81 , 041203(R) (2010) X.Z. Yu <i>et al.</i> , Nature 465 , 901 (2010)
Mn _{1-x} Fe _x Si	Т	P2 ₁ 3	Bloch	< 17 K	10-12	Metal	S.V. Grigoriev et al., PRB 79, 144417 (2009)
Mn _{1-x} Fe _x Ge	Т	P2 ₁ 3	Bloch	< 220 K	5–220	Metal	K. Shibata <i>et al.</i> , Nat. Nanotech. 8 , 723– 728 (2013)
Cu ₂ OSeO ₃	Т	P2 ₁ 3	Bloch	58 K	60	Insulator	S. Seki <i>et al.</i> , Science 336 , 198 (2012) T. Adams <i>et al.</i> , PRL 108 , 237204 (2012)
Co _x Zn _y Mn _z	0	P4 ₁ 32 /P4 ₃ 32	Bloch	150 K – 500 K	120 — 200	Metal	Y. Tokunaga <i>et al.</i> , Nat. Commun. 6 , 7638 (2015)
(Fe,Co) ₂ Mo ₃ N	0	P4 ₁ 32 /P4 ₃ 32	Bloch	< 36 K	110	Metal	W. Li et al., Phys. Rev. B 93, 060409(R) (2016)
GaV₄S ₈	C _{3v}	<i>R</i> 3m	Néel	13 K	17	Semicond/ Insulator	I. Kézsmárki <i>et al.,</i> Nat. Mater. 14 , 1116 (2015)

Table from Jonathan White

Skyrmions in magnetic multilayers

lack of inversion at interfaces:

interfacial Dzyaloshinskii-Moriya interaction



Fert & Levy, PRL (1980)



small Neel skyrmions in Fe monolayer on Ir(111) S. Heinze et al, Nat Phys. (2011)



W. Jiang et al, Science (2015) HM/F/I trilayer controlled creation and annihilation in a PdFe bilayer N. Romming et al, Science (2013)



Ir/Co/Pt multilayer with additive interfacial chiral interactions \rightarrow strong DMI at RT

C. Moreau-Luchaire et al., Nat. Nanotech. 11, 444 (2016)





reviews:

A. Soumyanarayanan *et al.* Nature 539, 509 (2016).
 R. Wiesendanger, Nat. Rev. Mater. 1, 16044 (2016).
 W. Jiang *et al.* AIP Advances **6**, 055602 (2016)

Skyrmion in a field-polarised background

standard model for chiral magnets:

$$\mathcal{L} = A(\nabla \hat{M})^2 + D\hat{M}(\nabla \times \hat{M}) - M_s \hat{M} \mu_0 \vec{H}$$

Dzyaloshinskii-Moriya interaction

static soliton solution $\hat{M}_s(\mathbf{r})$



Equation of motion for the skyrmion

Thiele approach:

project Landau-Lifshitz equation onto translational zero mode



center-coordinate of a skyrmion

A. A. Thiele, PRL (1973)

$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{B}_{\text{eff}} + \dots$$

$$\vec{G} \times \partial_t \vec{R}(t) = \vec{F}$$

F: damping, disorder, driving currents etc.

effective equation of motion: massless particle in an effective magnetic field G

$$\vec{G} = S\hat{z} \int d^2 \mathbf{r} \hat{M} (\partial_x \hat{M} \times \partial_y \hat{M})$$



gyrocoupling vector S: spin density

Skyrmion topology \Rightarrow spin magnus force \vec{G}

$$\vec{G} \times \partial_t \vec{R}(t) = \vec{F}$$

massless particle in an effective magnetic field

(on tree-level)

magnetization dynamics of a skyrmion:



skyrmion moves perpendicular to the force!

similar to the guiding center of electrons in a strong magnetic field (lowest Landau level)

Magnons in the presence of a skyrmion

static skyrmion-soliton solution



spin-waves scatter off the skyrmion \square magnon scattering problem

Bogoliubov Hamiltonian

$$\begin{aligned} \mathcal{H} &= \frac{\hbar^2 (-i\tau^z \vec{\nabla} - \mathbf{1} \vec{a})^2}{2M_{\text{mag}}} + \mathbf{1} \mathcal{V}_0 + \tau^x \mathcal{V}_x \\ &\uparrow \qquad \uparrow \qquad \uparrow \qquad \\ &\text{scattering vector} \qquad \text{scattering potentials} \\ &\text{potential} \end{aligned}$$

Magnon-skyrmion bound states



Schütte & MG PRB (2014)

Emergent magnon Lorentz force

adiabatic adjustment of local frame

 \Rightarrow Berry phase vector scattering potential \vec{a}

with quantised total flux



$$\int d^2 \mathbf{r} (\nabla \times \vec{a}) = \int d^2 \mathbf{r} \hat{M} (\partial_x \hat{M} \times \partial_y \hat{M}) \qquad \begin{pmatrix} \Box & \text{topological winding} \\ & \text{number} \end{pmatrix}$$

magnon scatter off a localised emergent magnetic flux due to non-trivial topology of skyrmion



> emergent Lorentz force

Topological magnon skew scattering



emergent Lorentz force leads to skew scattering for high-energy magnons!

 \Box topological magnon Hall effect!

see also Iwasaki, Beekman & Nagaosa PRB (2014) Mochizuki *et al.* Nat. Mat. (2014)

Schütte & MG PRB (2014) Schroeter & MG LTP (2015)

Markus Garst

seminar Landau institute — March 2018

Skew & rainbow scattering



magnon differential cross section

asymmetric & oscillations

Different classical trajectories contribute and interfere!

rainbow scattering!





Schütte & MG PRB (2014) Schroeter & MG LTP (2015)

Skew & rainbow scattering



magnon differential cross section

asymmetric & oscillations

Different classical trajectories contribute and interfere!

rainbow scattering!





Schütte & MG PRB (2014) Schroeter & MG LTP (2015)

How to drive skyrmions with magnon currents?



magnon wave exerts a pressure on the skyrmion...

in which direction will it move?

momentum conservation?

Driving skyrmions with magnon currents

incoming magnon wave transfers momentum to the skyrmion

outgoing momentum



counting net flux of incoming & outgoing momentum using conservation of energy-momentum tensor

$$\partial_{\mu}T_{\mu\nu} = 0$$



force in the Thiele equation

$$\vec{G} \times \partial_t \vec{R}(t) = \vec{F}$$

Linear response approximation

Thiele equation with a magnon force

 $\vec{G} \times \partial_t \vec{R}(t) = \vec{F}$

Linear response: evaluate force F for skyrmion at rest $\dot{R} = 0$



momentum-transfer force

after some algebra using optical theorem:

$$\vec{F} = J_{\varepsilon} k \begin{pmatrix} \sigma_{\parallel}(\varepsilon) \\ \sigma_{\perp}(\varepsilon) \end{pmatrix}$$

magnon force determined by transport scattering cross sections:

with

$$\begin{pmatrix} \sigma_{\parallel}(\varepsilon) \\ \sigma_{\perp}(\varepsilon) \end{pmatrix} = \int_{0}^{2\pi} d\chi \begin{pmatrix} 1 - \cos \chi \\ -\sin \chi \end{pmatrix} \frac{d\sigma(\varepsilon)}{d\chi}.$$

skew scattering -> finite transversal force

Skyrmion caloritronics



skyrmion velocity: towards the magnon source $\partial_t \vec{R} = J_{\varepsilon} k \begin{pmatrix} \sigma_{\perp}(\varepsilon) \\ \sigma_{\parallel}(\varepsilon) \end{pmatrix}$

in the presence of a temperature gradient:



skyrmion moves towards hot region! Experiment: thermal ratchet



Mochizuki et al. Nat. Mat. (2014)

numerical Langevin dynamics:

Lin, Batista, Reichhardt, & Saxena, PRL (2014) Kong and Zang PRL (2013)

Magnon band structure of skyrmion lattices



Magnon-band structure of skyrmion lattice

magnon dispersion for in-plane momenta

skyrmion lattice **^**(1) 554 $rac{\omega}{\omega_{
m c2}}$ Κ $\omega/\omega_{\rm c2}^{\rm int}$ 3 М $\mathbf{2}$ 2**→**① 1 1 2d magnetic **Brillouin zone** $\frac{Q}{2}$ M K Γ Г $\frac{Q}{2}$ $\frac{Q}{2}$ 0 $\frac{Q}{2}$ q_y q_x

MG, J. Waizner, D. Grundler, J. Phys. D: Appl. Phys. 50, 293002 (2017)

Topological magnon-band structure

non-trivial topology of skyrmions
> topological magnon band structure

each skyrmion acts like a source of emergent magnetic flux

emergent magnon electrodynamics

- emergent magnon Landau levels
- bands with finite Chern numbers
- topologically protected magnon edge states



Chern numbers

Magnon normal modes with finite frequency



Magnetic resonances

ac magnetic field \Rightarrow exciting magnons at zero momentum

spectrum:



T. Schwarze, et al. Nature Materials (2015)

Two resonances of the helix:



Three resonances of the skyrmion crystal:







counter clockwise



breathing

Comparison experiment & theory

different field sweeps normalized with H_{c2}(T)



three different materials MnSi, $Fe_{0.8}Co_{0.2}Si$ and Cu_2OSeO_3

with three different shapes

(demagnization factors)

"universal" chiral magnetism

excellent parameter-free theoretical description

T. Schwarze, et al. Nature Materials (2015)

Orientation of the skyrmion crystal



Orientation of the skyrmion lattice



momentum space

six-fold symmetric Bragg peaks

orientation determined by weak cubic magnetic anisotropies!

How does the scattering pattern orient within the plane normal to H?

Bragg pattern defines six-fold symmetric tangent vector field on the H-unit sphere:



Effective theory for skyrmion lattice orientation





two anisotropy terms of sixth-order

single fitting parameter

tetrahedral point group

A. Bauer, et al. submitted

Experiment: skyrmion orientation

experimental setup

experiment excellently described by theory

[110][.]

in-plane alignment

Topology of wind patterns and skyrmion lattices

Hairy-Ball theorem (Poincare-Hopf):

continuous non-vanishing tangent vector cannot exist everywhere on the sphere

wind velocity has to vanish at least twice

eg. singularity on the north and south pole

also relevant for Abrikosov vortex lattice in type II SC

Larver & Forgan, Nat. Comm. (2010).

skyrmion lattice in MnSi

singularities of strengths 1/6 winding of the pattern by $\pi/3$

A. Bauer, et al. submitted

Topological domain walls of helimagnetic order

Domain walls in helimagnets

PRL 108, 107203 (2012)

PHYSICAL REVIEW LETTERS

week ending 9 MARCH 2012

S Vortex Domain Walls in Helical Magnets

 Fuxiang Li,¹ T. Nattermann,² and V. L. Pokrovsky^{1,3}
 ¹Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA
 ²Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany
 ³Landau Institute for Theoretical Physics, Chernogolovka, Moscow District, 142432, Russia (Received 17 November 2011; published 7 March 2012)

FIG. 3 (color online). DWs in noncentrosymmetric helical magnets. A detail of Fig. 1(g) of Ref. [29] (center) showing two types of DWs in the ferromagnet FeGe; the left one includes vortices, the right one is vortex-free. The panels are theoretically calculated DWs, right without vortices, left with vortices.

Domain walls in helimagnets

similar to grain boundaries in cholesteric liquid crystals

Y. Bouligand 1970ies

Topological defects of helimagnetic order

lacksquare

 \otimes

 (\cdot)

 \otimes

 (\cdot)

 \otimes

pitch \hat{Q} is a director $\Rightarrow \pm \pi$ vortices are possible = disclinations defects

disclinations combine to form a dislocation with Burgers vector B

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а

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Relaxation by climb motion of dislocations

MFM: surface of FeGe

climb motion of dislocations \Rightarrow 180° phase shift

A. Dussaux et al., Nat Comm 2016

Topological domain walls

depending on relative angle: three types of domain walls

MFM: surface of FeGe

curvature wall

dislocation wall

P. Schoenherr et al, Nat. Phys. (2018)

Domain wall angles

experiment:

P. Schoenherr et al, Nat. Phys. (2018)

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Domain wall angles

experiment:

micromagnetic simulations:

P. Schoenherr et al, Nat. Phys. (2018)

Domain wall angles

experiment:

micromagnetic simulations:

P. Schoenherr et al, Nat. Phys. (2018)

Skyrmion charge of dislocations

skyrmion number W = -1

skyrmion number W = -1/2

skyrmion embedded in a topologically trivial background

dislocation (with $B = \lambda$) = meron

general relation for skyrmion number of dislocation with Burger vector B:

$$|W| = \frac{1}{2} \operatorname{mod}_2\left(\frac{B}{\lambda}\right)$$

only dislocations with half-integer B contribute to

topological Hall effect & emergent electrodynamics

Topological skyrmion charge of domain walls

finite skyrmion charge if the distance D:

$$\frac{2D}{\lambda} = \frac{B}{\lambda} \quad \text{odd}$$

 λ : helix wavelength

topological Hall effect & emergent electrodynamics

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neutron scattering

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MFM

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magnetic resonance

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Spinwave spectroscopy

r Shinichiro Seki (Riken)

multilayers

C. Panagopolous (Singapore)

Summary: chiral magnetic crystals

crystallization process: fluctuation-driven 1st order

helimagnon band structure

skyrmion: internal modes, skew scattering

topological magnon band structure of skyrmion crystals

orientation of skyrmion crystals — hairy-ball

topological domain walls of helimagnets

Outlook: defect-mediated melting process? relation to NFL?