Dephasing by a Zero-Temperature Detector and the Friedel Sum Rule

Workshop "The Science of Nanostructures: New Frontiers in the Physics of Quantum Dots", Chernogolovka, September 13th, 2012

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based on work done with Yehuda Dinaii and Yuval Gefen, important discussions with Moty Heiblum and Emil Weisz

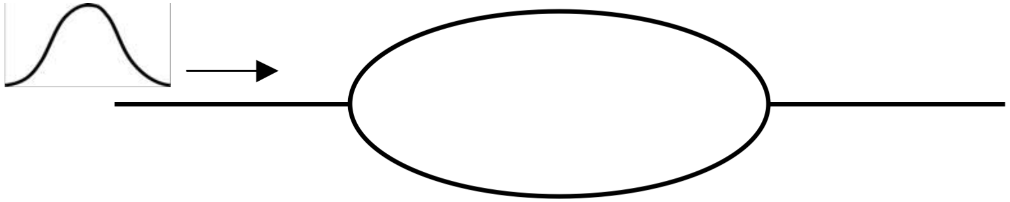
B. Rosenow and Y. Gefen, PRL 108, 256805 (2012)

Outline

- motivation
- model: MZ interferometer with detector at equilibrium
- thermal regime
- quantum regime
- Friedel sum rule

“Which Path” experiment I

$$|\Psi\rangle = |\Psi_u\rangle + |\Psi_d\rangle$$



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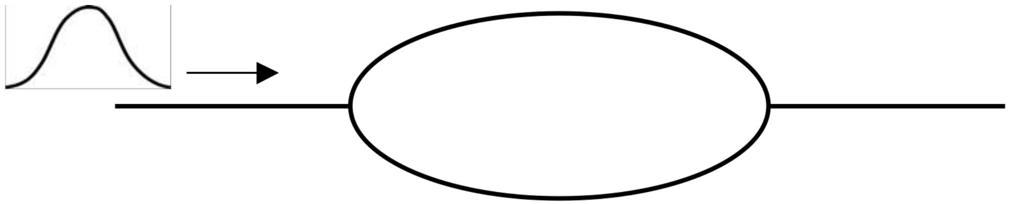

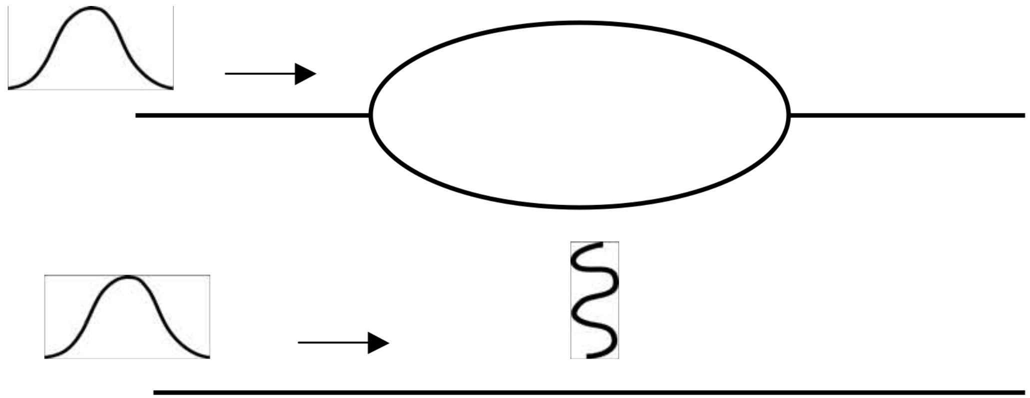

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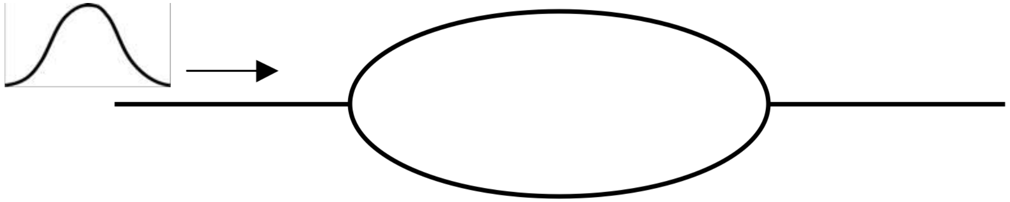

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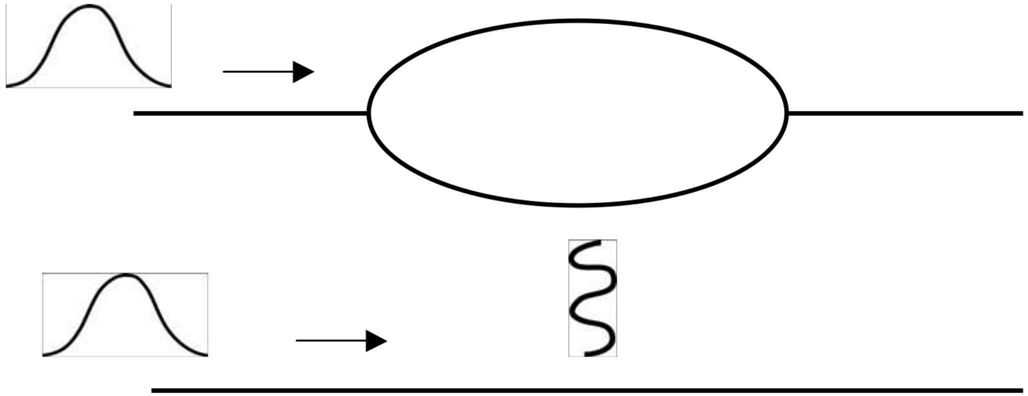

$$P_{\mathrm{det}} \;=\; |\langle \mathrm{det}|\Psi_u\rangle|^2 \;\cdot \langle \chi_u|\chi_u\rangle \;+\; |\langle \mathrm{det}|\Psi_d\rangle|^2 \;\cdot \langle \chi_d|\chi_d\rangle \;+\; 2\mathrm{Re}\left[\langle \Psi_d|\mathrm{det}\rangle\langle \mathrm{det}|\Psi_u\rangle \cdot \langle \chi_u|\chi_d\rangle\right]$$

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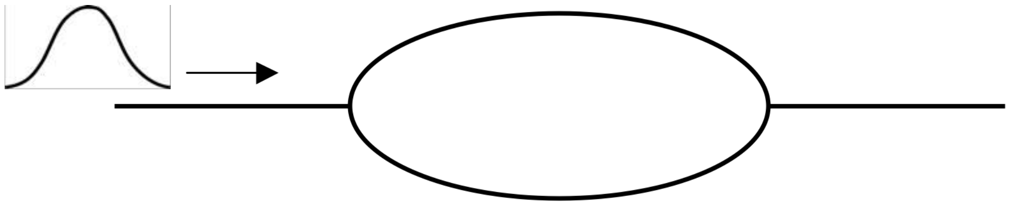

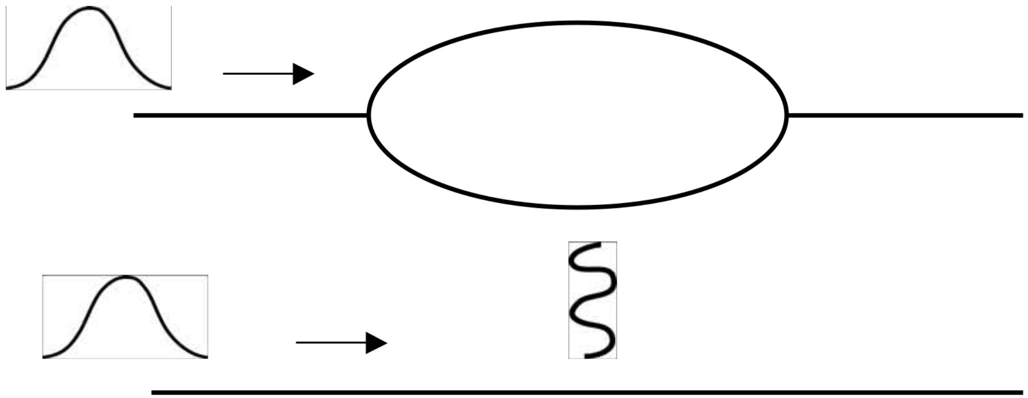

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entanglement between system and detector reduces interference visibility

"Which Path" Experiment II

- electronic interferometry for studying QM phenomena like AB effect, fractional statistics, non-abelian anyons

- visibility of interference signal suppressed when trajectory of interfering particle is measured $\Rightarrow$ dephasing of system Stern, Aharonov & Imry, PRA 1990

- "which path" measurement observed experimentally and studied theoretically Buks, Schuster, Heiblum, Mahalu & Umansky, Nature 1998; Gurvitz PRB 1997; Levinson, EPL 1997; Aleiner, Wingreen & Meir, PRL 1997

- standard detection schemes involve out of equilibrium detector, e.g. biased QPC with electrostatic coupling to interferometer

Dephasing

- directly related to entanglement between system and detector

Neder, Heiblum, Mahalu & Umansky, PRL 2007

- observed and discussed in the context of electronic interferometers

van der Wiel et al., PRB 2003; McClure et al., PRL 2009; Yamauchi et al., PRB 2009; Ji, Chung, Sprinzak, Heiblum, Mahalu & Shtrikman, Nature 2003; Chalker, Gefen & Veillette, PRB 2007; Levkivskyi & Sukhorukov, PRB 2008; Neder & Marquardt, New J. Phys. 2007; Neder & Ginossar, PRL 2008; Youn, Lee & Sim, PRL 2008; Heyl, Kehrein, Marquardt & Neuenhahn, PRB 2010; Schneider, Bagrets & Mirlin, PRB 2011; Ngo Dinh & Bagrets, PRB 2012; Marquardt & Bruder, PRB 2002; Seelig & Buttiker, PRB 2001; Treiber, Yevtushenko, Marquardt, von Delft & Lerner, PRB 2009; Horovitz & Le Doussal, PRB 2010

- special features when system-detector coupling is strong

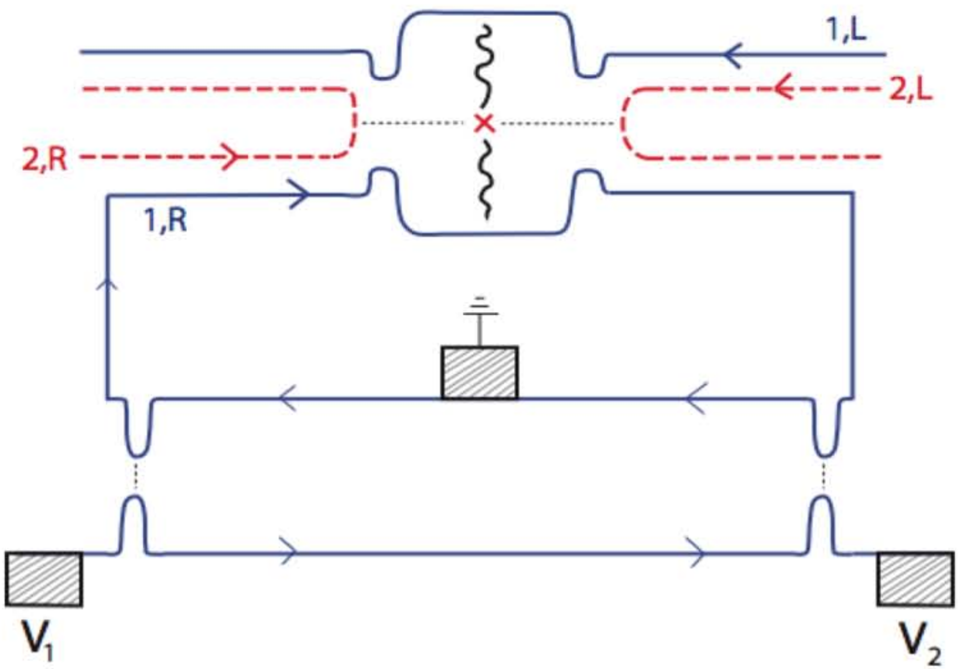
Averin & Sukhorukov, PRL 2005; Grishin, Yurkevich & Lerner, PRB 2005; Abel & Marquardt, PRB 2008

Equilibrium Detector?

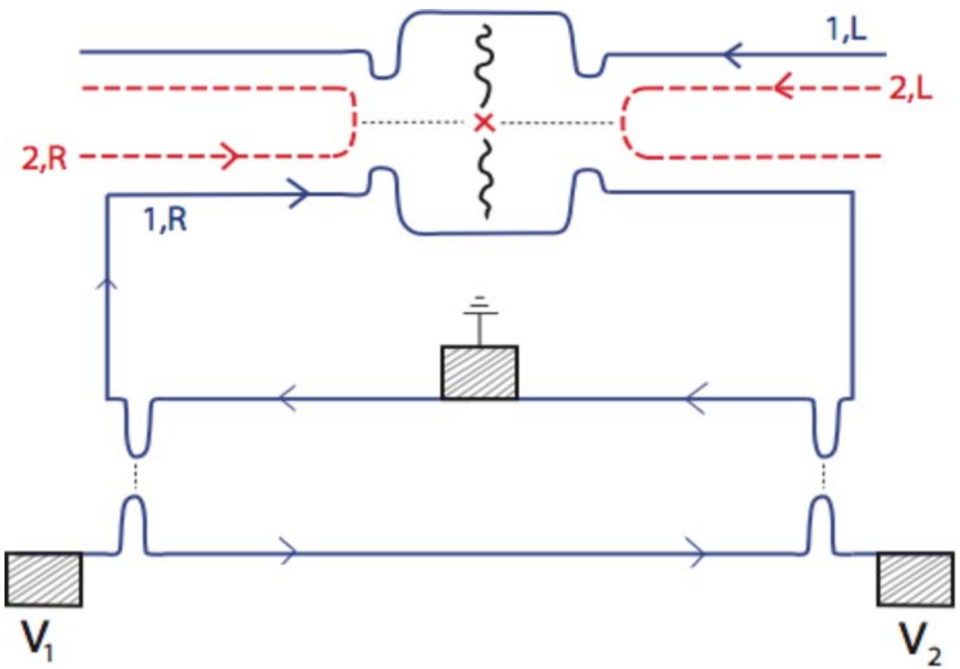
- what happens when detector set to equilibrium?
- coupling to dissipative environment may give rise to dephasing through thermal fllucuations Marquardt & Bruder, PRB 2002; Seelig & Buttiker, PRB 2001; Treiber, Yevtushenko, Marquardt, von Delft & Lerner, PRB 2009; Horovitz & Le Doussal, PRB 2010; Altshuler, Aronov & Khmelnitskii, J. Phys. C 1982; von Delft, in Fundamental Problems of Mesoscopic Physics, Lerner et al. (eds.) 2004; change of the ground state through Anderson orthogonality catastrophy mechanism Chakravarty & Leggett, PRL 1984; Aleiner, Wingreen &Meir, PRL 1997; Anderson, PRL 1967; Neuenhahn & Marquardt, PRL 2009
- dephasing due to thermal fluctuations in a quantum dot detector coupled to AB ring Meier, Fuhrer, Ihn, Ensslin, Wegscheider & Bichler, PRB 2004

Model

- consider electronic Mach-Zehnder interferometer with one arm coupled electrostatically to detector
- interferometer defined by outer edge channel of a ν=2 quantum Hall setup
- detector consists of localized state tunnel-coupled to inner edge



Model - Hamiltonian

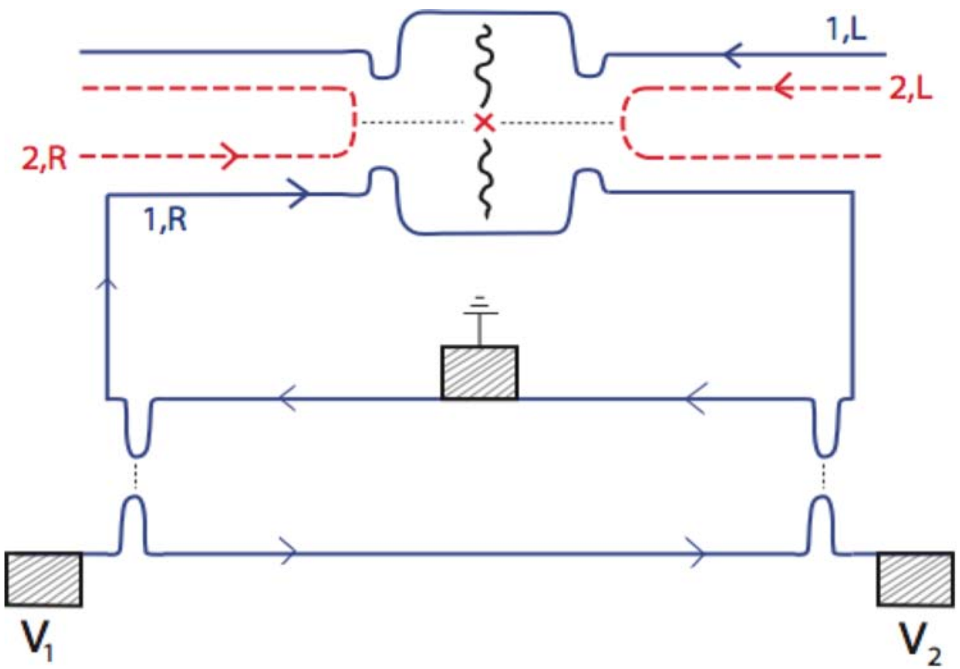


Model - Hamiltonian

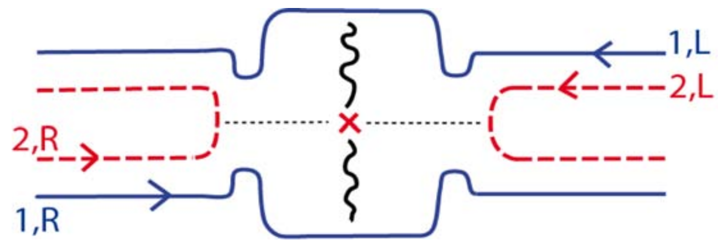
$$H_0 = v \sum_{\alpha=1,2;k} c^\dagger_{\alpha R,k} c_{\alpha R,k} (k - k_F) - c^\dagger_{\alpha L,k} c_{\alpha L,k} (k + k_F) + \ \epsilon_0 \, d^\dagger d$$

$$H_{\mathrm{tunnel}} = \sum_k \left(\gamma_{2L,k} c^\dagger_{2L,k} + \gamma_{2,R,k} c^\dagger_{2R,k} \right) d \ + \ \mathrm{h.c.}$$

$$H_{\mathrm{imp-edge}} = \int_{-D/2}^{D/2} dr \left(\rho_{1R}(r) V_R(r) + \rho_{1L}(r) V_L(r) \right) d^\dagger d$$



Coulomb phase



localized state is occupied with $\langle\, d^\dagger\, d\,\rangle = 1$

$\Rightarrow$ density change $$\langle \rho_{R/L}(r) \rangle = -\frac{1}{2\pi v\hbar} V_{R/L}(r)$$

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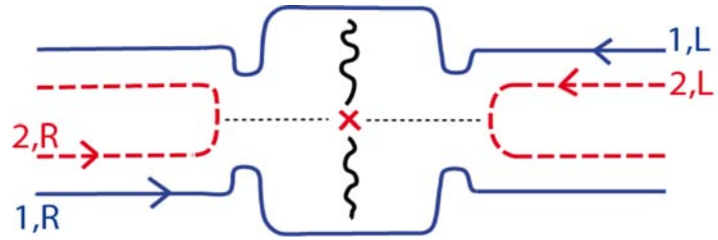

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bosonize $\quad \psi_R(r) = \exp[i\varphi_R(r)] \quad \text{with} \quad \rho_R(r) = \frac{1}{2\pi} \partial_r \varphi_R(r)$

$\Rightarrow$ electron Green function can be expressed as

$$g_R(x,t) \;=\; \langle e^{i\varphi_R(x,t) - i\varphi_R(0,0)} \rangle = e^{2\pi i \int_{-D/2}^{x} dy \langle \rho_R(y) \rangle} \langle e^{i\delta\varphi(x,t) - i\delta\varphi(0,0)} \rangle$$

where $\quad \delta\varphi(x) = \varphi(x) - 2\pi \int_{-D/2}^{x} dy \langle \hat{\rho}(y) \rangle$

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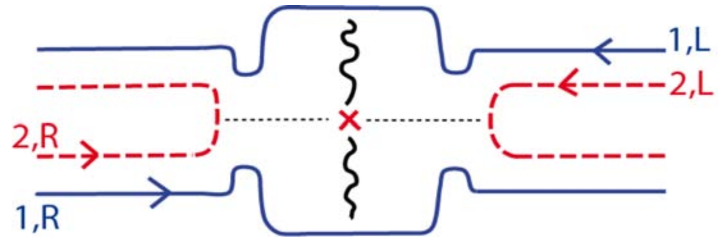

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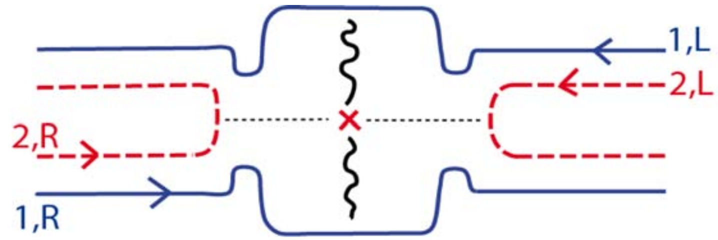

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full and symmetric screening of impurity charge yields $\int_{-D/2}^{D/2} dy \langle \rho_R(y) \rangle = -1/2$

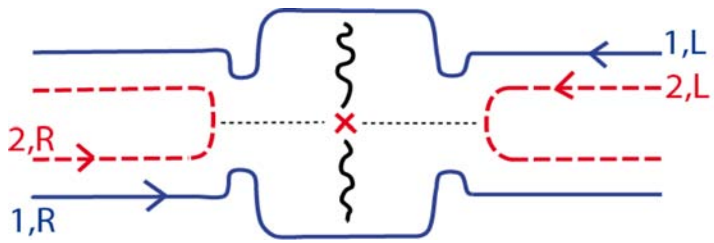
$\Rightarrow$ $\delta = \pi$ for symmetric screening

Dephasing by statistical averaging

$e\,V_{sd} \ll \Gamma \ll k_B\,T \;\Rightarrow$ occupancy of localized level fluctuates thermally

transmission phase of (1,R) channel depends on dot occupancy $\Rightarrow$ average yields

$$\langle e^{i\delta d^\dagger d}\rangle = \frac{e^{i\delta}}{e^{(\epsilon_0-\mu)/T}+1} + \frac{1}{e^{-(\epsilon_0-\mu)/T}+1}$$



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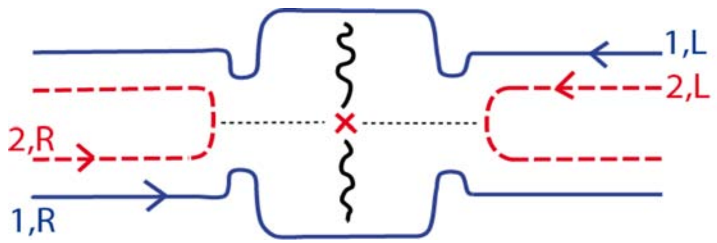


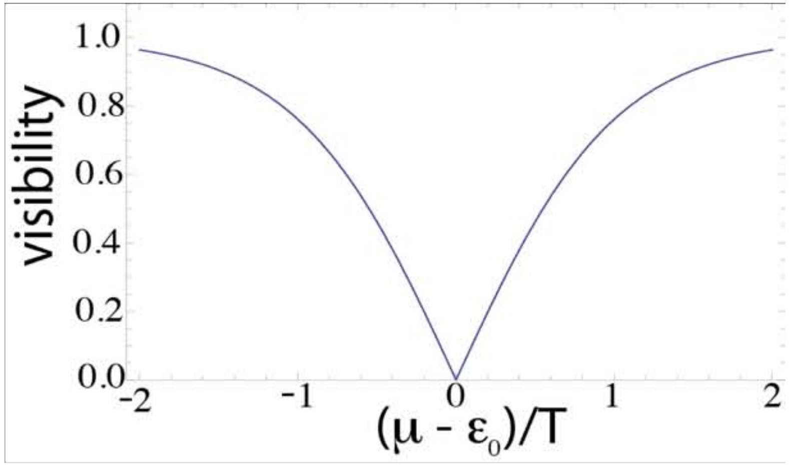
fully symmetric dot with $e^{i\delta} = -1$ yields

$$\langle e^{i\delta d^\dagger d}\rangle = \tanh((\epsilon_0-\mu)/T)$$

$\Rightarrow$ phase lapse of π at the degeneracy point,

and a visibility $\nu \propto |\tanh((\epsilon_0 - \mu)/T)|$ with

FWHM $\approx 2.197\ k_B\ T$



Dephasing by entanglement

- consider now the limit $k_B\,T \ll \Gamma \ll eV_{sd} \ll \mu$ -B, where B denotes the bottom of the band

- discuss the change an interfering electron on outer edge makes to its environment, which consists of the localized state coupled to the inner edge

- describe electron wave function after passage through the interferometer by

$$|u\rangle \otimes |\chi_u\rangle \; + \; e^{i\phi}|d\rangle \otimes |\chi_d\rangle$$

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$\|u\rangle$, $\|d\rangle$	partial waves through upper or lower arm of the MZ
$\|\chi_u\rangle$, $\|\chi_d\rangle$	respective states of the environment
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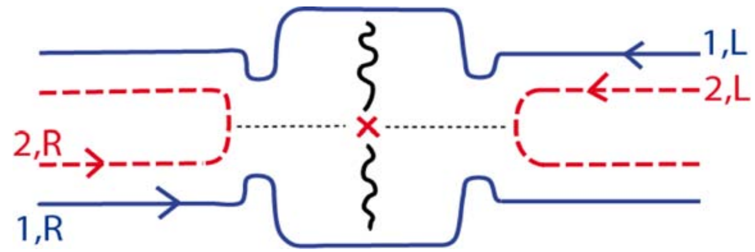
- as interference term is superposition of partial waves travelling along upper and lower arm, it is reduced by the factor $|\langle\chi_u|\chi_d\rangle|$

Impurity coupled to chiral edge

- couple localized state to only one chiral edge
- eigenstate of the inner edge plus localized state



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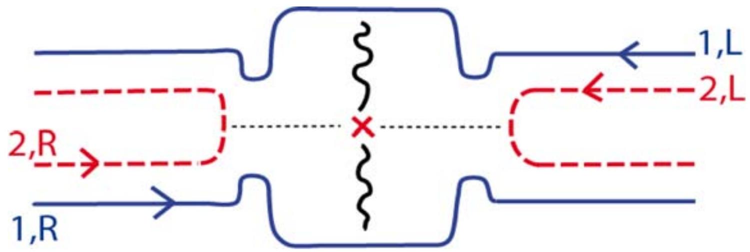

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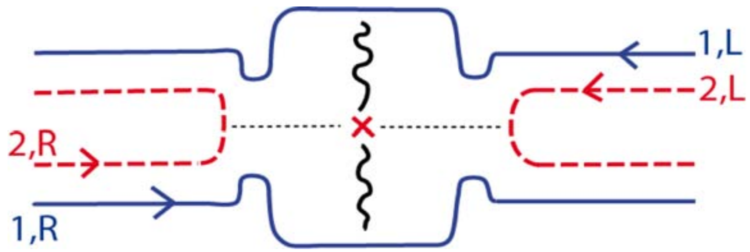

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previously: assign Coulomb phase $e^{i\delta}$ to electronic wave function $|u\rangle$

now: assign Coulomb phase $e^{i\delta}$ to environmental state $|\chi_u\rangle$

only component with weight on the localized state will be affected $\Rightarrow$

$$|\epsilon_\delta\rangle = \mathcal{N}_\epsilon \left(\int_{-D/2}^{D/2} dx \varphi(x)|x\rangle \;+\; e^{i\delta} A(\epsilon)|d\rangle \right)$$

"Dephasing by quantum fluctuations"

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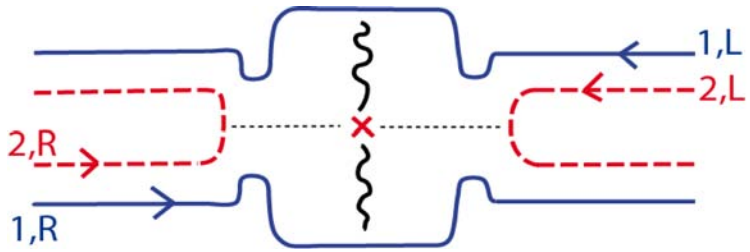

- more formally: apply operator $e^{i\delta d^\dagger d}$ state $|\epsilon\rangle$
- denote expection value w.r.t. ground state Slater determinant $|\epsilon\rangle$ by $\langle\ \ldots\ \rangle$

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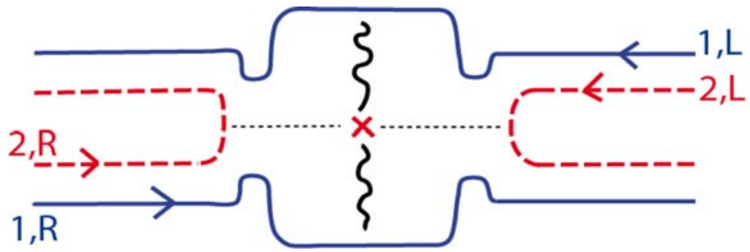

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- for $\mu \gg \epsilon_0$ or $\mu \ll \epsilon_0$, dot occupancy is 1 or 0, and $d^\dagger$ d can be replace by eigenvalue
- for $|\mu - \epsilon| \approx \Gamma$, fluctuations in $d^\dagger d$ are large and reduce the expectation value

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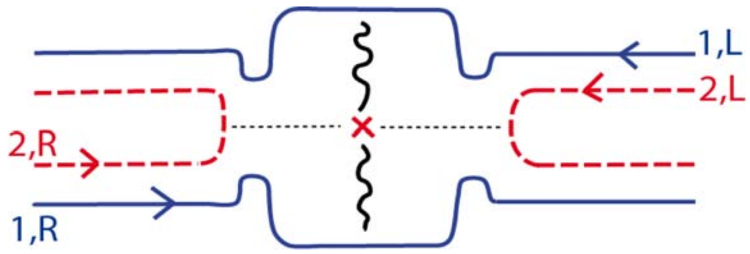

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$\Rightarrow$ dephasing by the dot intimately related to strength of quantum fluctuations

however: change in environment only possible when energy $\approx \Gamma$ is transferred

Reduction of interference visibility I

- to calculate $\langle \chi_u | \chi_d \rangle$, need to know the matrix of wave-function overlaps

$$M_{n,n'} = \delta_{n,n'} + \mathcal{N}_{\epsilon_n} A^*(\epsilon_n) \mathcal{N}_{\epsilon_{n'}} A(\epsilon_{n'}) \left(e^{i\delta} - 1 \right)$$

- where

$$A(\epsilon) = \frac{\gamma}{\epsilon - \epsilon_0 + i\Gamma} \quad \text{with} \quad \Gamma = \frac{|\gamma|^2}{2\hbar v}$$

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- calculate determinant by diagonalizing $M_{n,n'} = \delta_{n,n'} + \alpha\, u_n\, u^*_{n'}$ $\Rightarrow$ one eigenvector with components u_n and eigenvalue $1 + \alpha \sum_\nu u^*_n u_n$, as well as degenerate eigenspace orthogonal to vector $\{ u_n \}$ with eigenvalue 1

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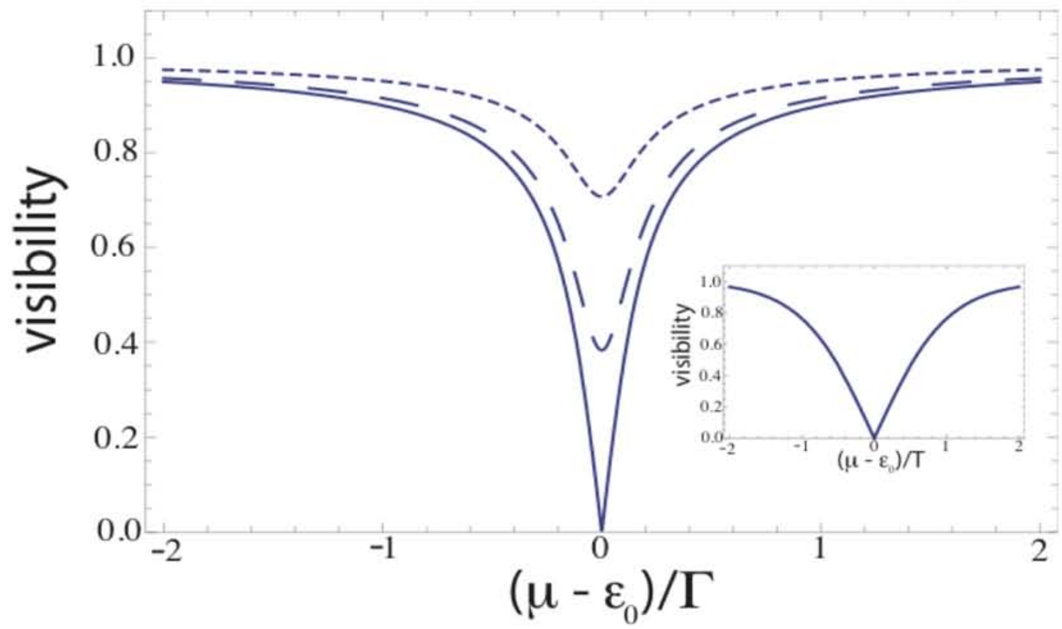
$$\Rightarrow \quad |\langle \chi_u | \chi_d \rangle| = \left| 1 + \left(e^{i\delta} - 1\right) \frac{\arctan \frac{(\mu - \epsilon_0) 2\pi}{\Gamma} + \frac{\pi}{2}}{\pi} \right|$$

Reduction of interference visibility II

$$|\langle \chi_u | \chi_d \rangle| = \left| 1 + \left(e^{i\delta} - 1 \right) \frac{\arctan \frac{(\mu - \epsilon_0) 2\pi}{\Gamma} + \frac{\pi}{2}}{\pi} \right|$$



full line $\delta = \pi$

dashed line $\delta = 3\pi/4$

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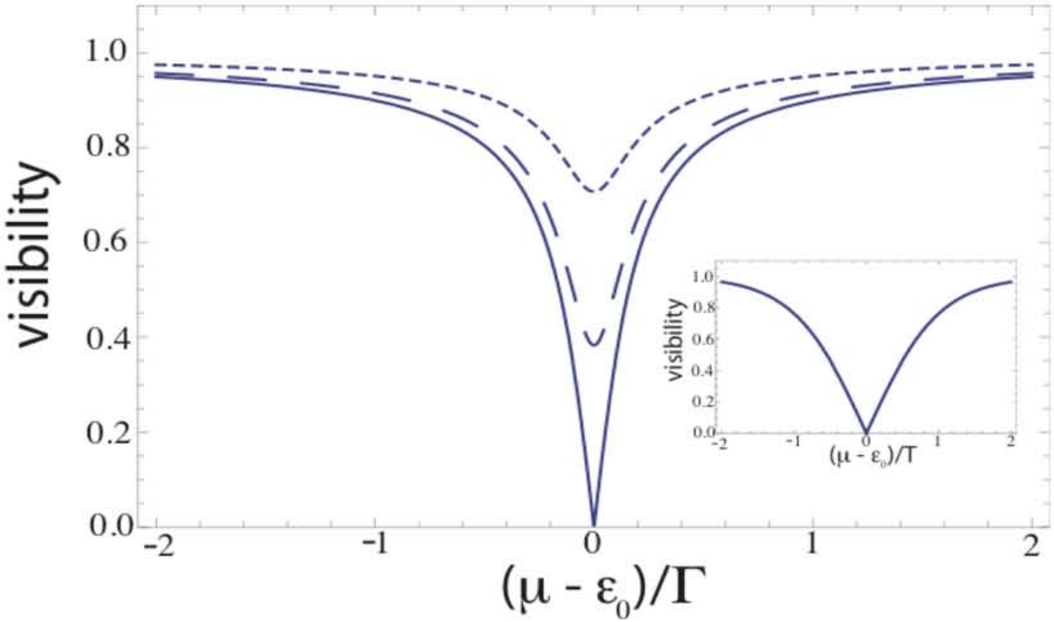
- interestingly, $\langle\ \chi_u\ |\ \chi_d\ \rangle$ does not scale with system size, in contradistinction from determinants in the Anderson orthogonality case

- symmetric coupling with $\delta = \pi \Rightarrow$ complete suppresssion of interference for $\mu = \epsilon_0$

- interference visibility fully recovered in case of empty or fully occupied localized state



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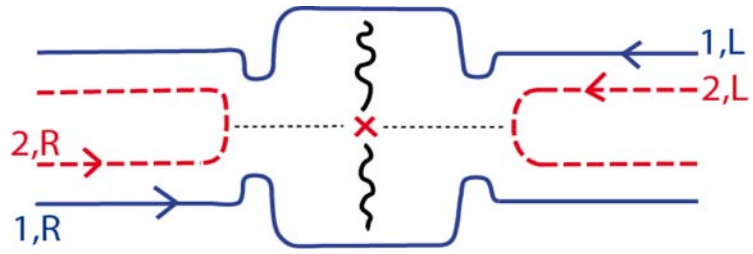
Condition for dephasing to occur

• energy difference between Slater determinants of $|\epsilon\ \rangle$ and of $|\epsilon_\delta\rangle$ from expectation values of Hamiltonian

$$\Delta E \;=\; \frac{\Gamma}{\pi}(1-\cos\delta)\ln\left(\frac{B^2+\Gamma^2}{\mu^2+\Gamma^2}\right)$$



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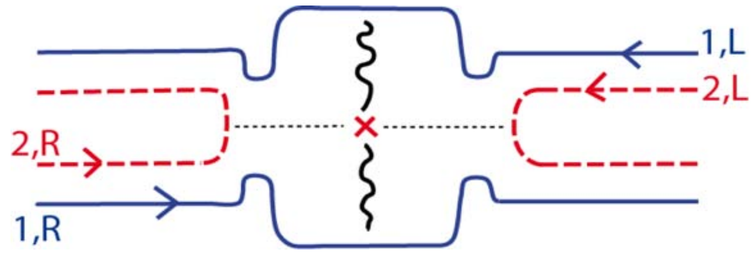

• environmental state $|\chi_u\rangle$ is by amount ΔE higher in energy than state $|\chi_d\ \rangle$ $\Rightarrow$ energy difference must be provided by interfering electron

• as interfering electron needs to have energy ΔE above Fermi level, we arrive at requirement e$V_{sd} \geqslant \Gamma$

• as an energy transfer is involved in the reduction of MZ interference visibility, dephasing can be described as backaction of the environment onto the interfering electron

Friedel sum rule - electrostatic coupling

- change of charge in a spatially confined region in a Fermi sea by one $\Rightarrow$ sum of the scattering phases changes by 2 π

$$\Delta N_{\text{confined}} = \frac{1}{2\pi i} \Delta \mathrm{Tr} \ln \mathcal{S}(\mu)$$

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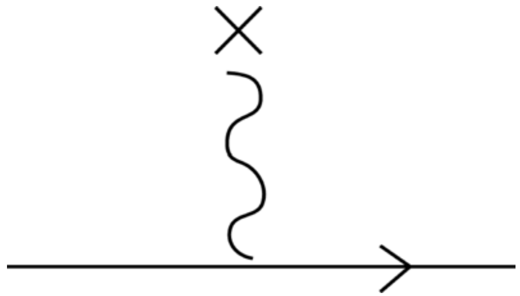

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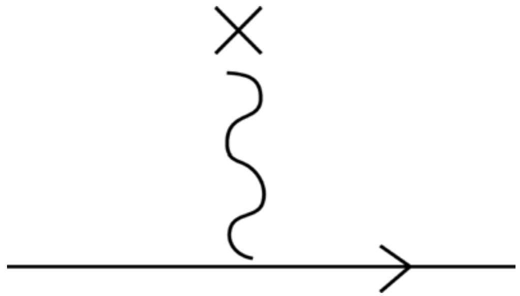
• in order to make contact with the sign convention of the Friedel sum rule, we refer here to a chiral edge embedded into a Fabry-Perot interferometer



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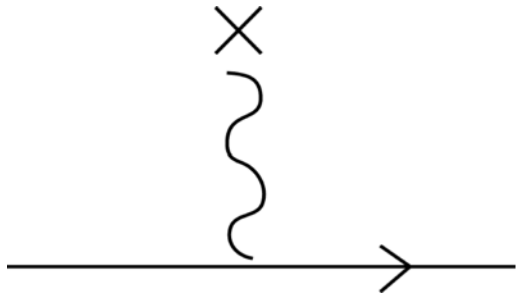
• coupling between chiral lead and localized state is electrostatic

$\Rightarrow$ localized state acts like external potential $\Rightarrow$ occupation by an electron gives rise to screening cloud of charge |e|

$\Rightarrow$ change of transmission phase through chiral by - 2 π



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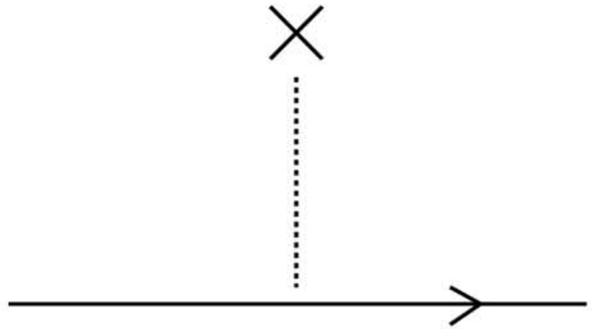

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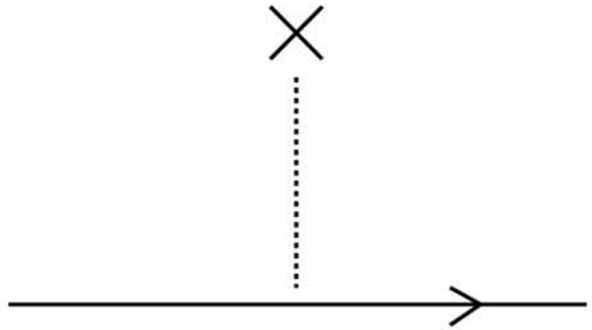

- consider tunnel-coupling between chiral lead and localized impurity $\Rightarrow$ now localized state is part of the sytem
- direct calculation of scattering phase yields

$$\tan \frac{\delta_t}{2} = -\frac{\Gamma}{\epsilon}$$

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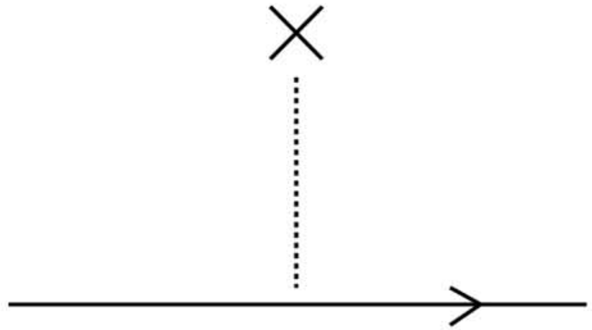

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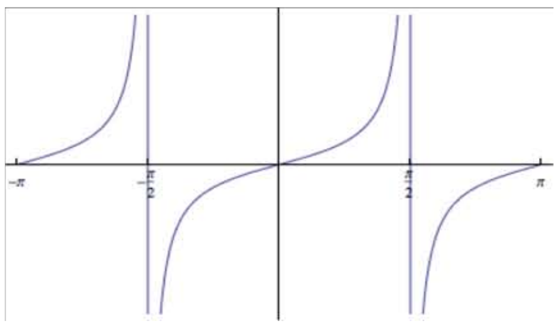
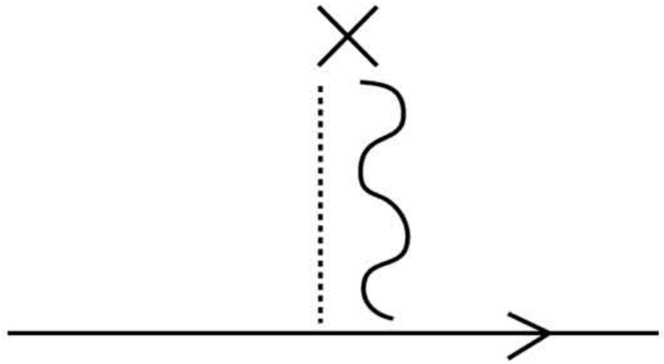
- filling up the state (i.e. varying ϵ from - ∞ to + ∞) results in phase change of 2 π, in agreement with Friedel sum rule



Friedel sum rule - tunnel and Coulomb coupling

$$\Delta N_{\mathrm{confined}} = \frac{1}{2\pi i} \Delta \mathrm{Tr} \ln \mathcal{S}(\mu)$$



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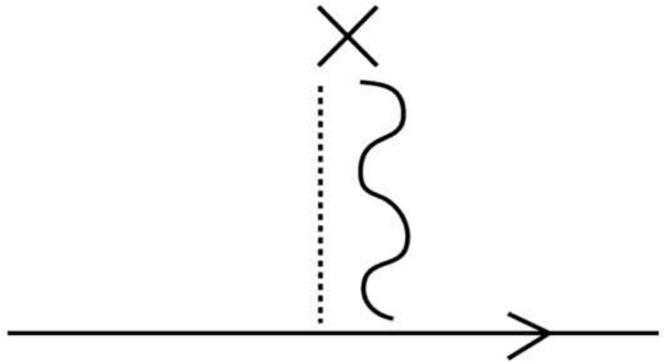
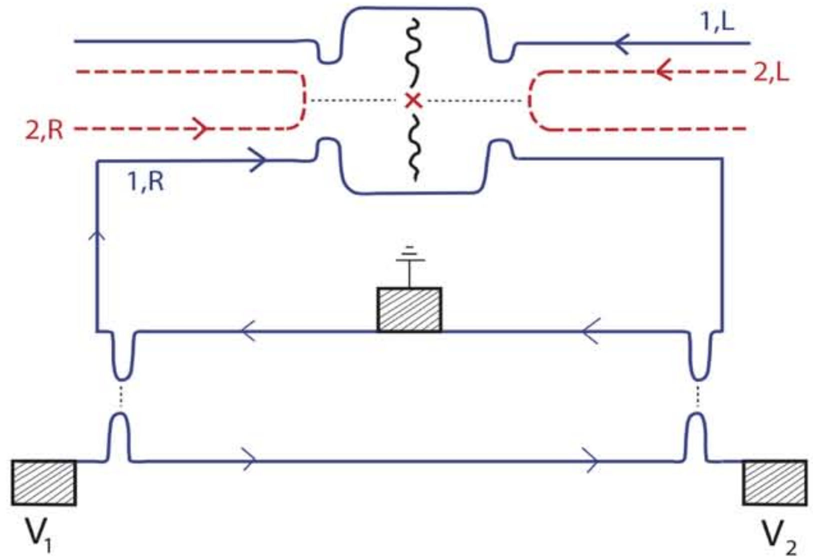


• now consider a case with both tunnel and Coulomb coupling ⇒ the two contributions cancel each other

• in agreement with Friedel sum rule, as filling up the localized state will induce an opposite sign screening cloud ⇒ amounts to redistribution of charge (with the net total charge unchanged)

Friedel sum rule - full model

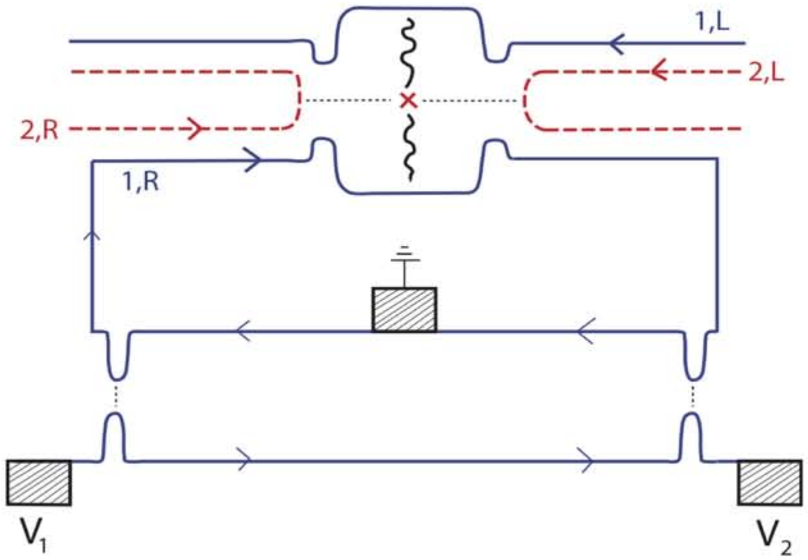


Friedel sum rule – full model

• now discuss realistic situation with a second chiral

• "blue" chirals have Coulomb coupling, "red" chirals have tunnel coupling ⇒ need 4x4 scattering matrix for full description

• if Coulomb coupling between localized state and the blue chirals is symmetric, Coulomb phase shift is – π



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- now discuss realistic situation with a second chiral
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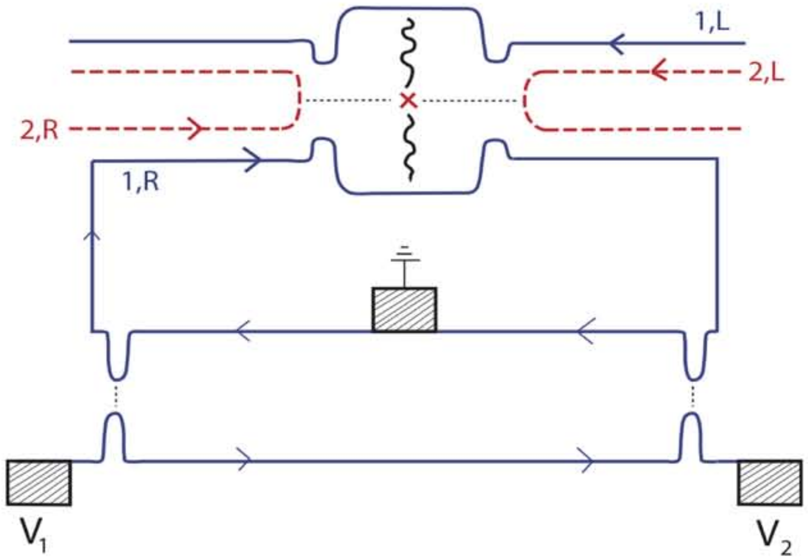
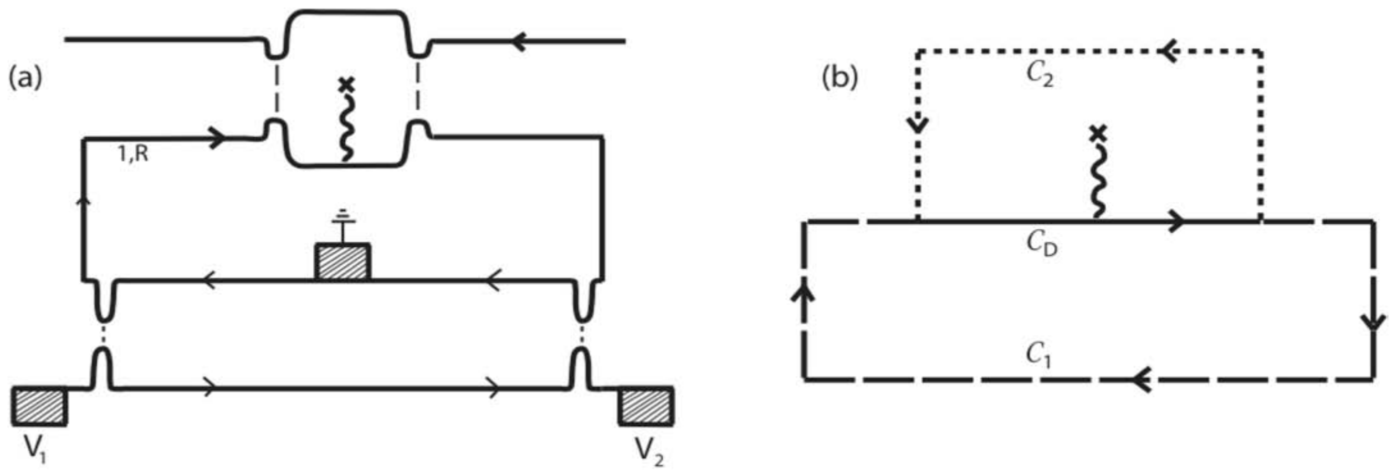


- if coupling to the two blue chirals is not symmetric, there is no full dephasing
- with 4x4 scattering matrix, sum of all scattering phases is again zero
- for weakly pinched inner edge mode, screening charge is distributed over blue and red chirals $\Rightarrow$ Friedel phase is spread over more channels and reduces the the magnitude of Coulomb phase shift δ

Fabry-Perot interference phase vs. Mach-Zehnder interference phase

MZ interferometer has path $\mathcal{C}_{\mathrm{MZI}} = \mathcal{C}_1 + \mathcal{C}_D$

FP interferometer has path $\mathcal{C}_{\mathrm{FPI}} = \mathcal{C}_2 + \mathcal{C}_D$



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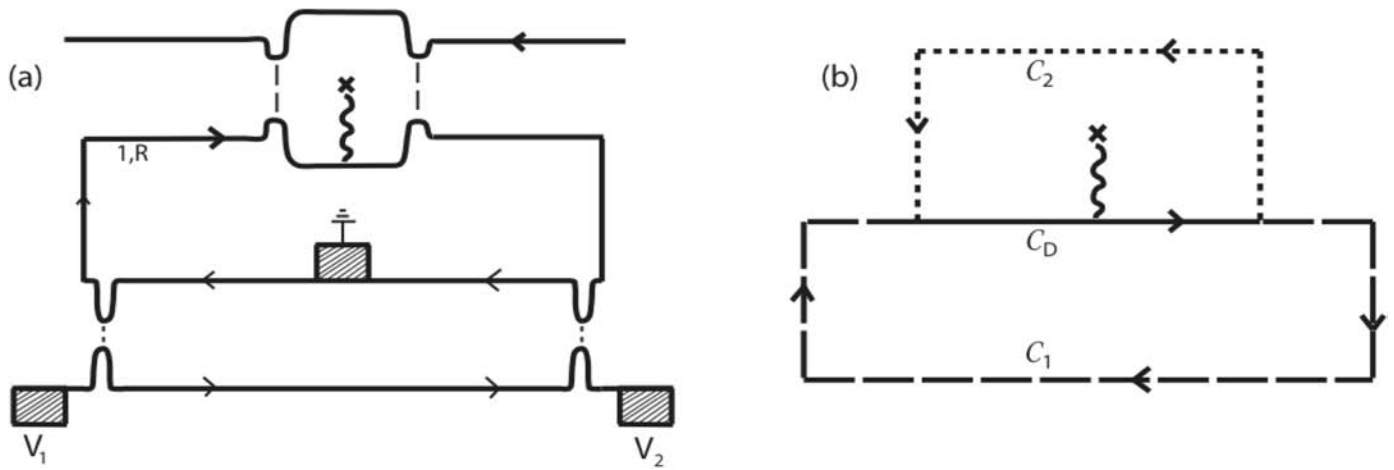
as edge states are projected onto single Landau level, inerference phase is pure Aharonov-Bohm, and in the absence of phase shift δ

$$\varphi_{\rm MZI} = \frac{2\pi}{\Phi_0}\int_{\mathcal{C}_1+\mathcal{C}_D} d\underline{s}\cdot\underline{A} \ < \ 0 \quad , \quad \varphi_{\rm FPI} = \frac{2\pi}{\Phi_0}\int_{\mathcal{C}_2+\mathcal{C}_D} d\underline{s}\cdot\underline{A} \ > \ 0$$



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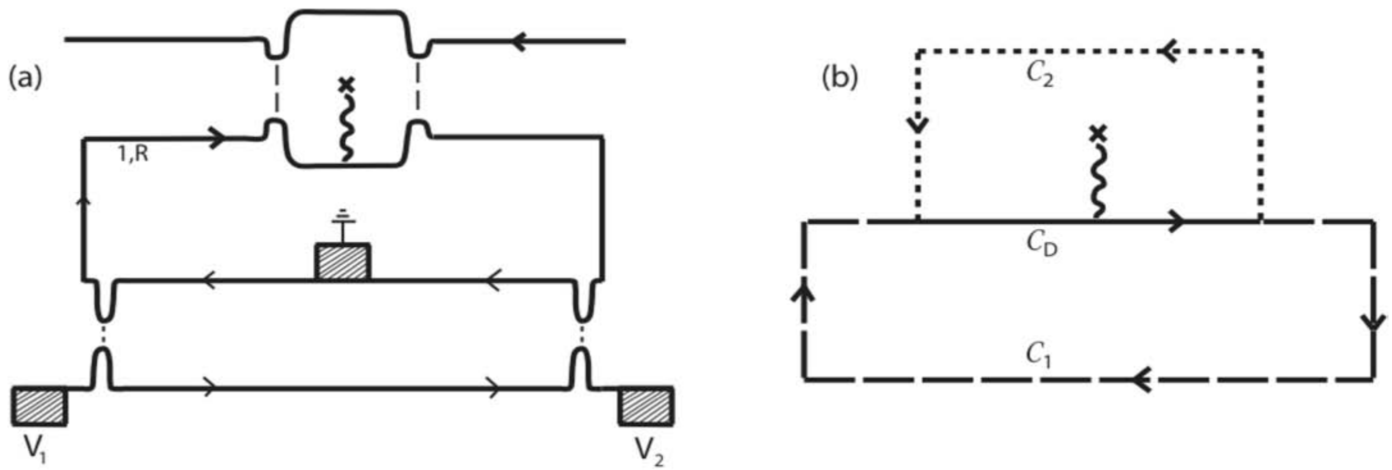
phase change δ is purely local and changes only what happens along C_D $\Rightarrow$ appears as additive correction



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$$\tilde{\varphi}_{\mathrm{MZI}} = \frac{2\pi}{\Phi_0} \int_{\mathcal{C}_1 + \mathcal{C}_D} d\underline{s} \cdot \underline{A} + \delta \quad , \quad \tilde{\varphi}_{\mathrm{FPI}} = \frac{2\pi}{\Phi_0} \int_{\mathcal{C}_2 + \mathcal{C}_D} d\underline{s} \cdot \underline{A} + \delta$$

since φ_{MZI} and $\varphi_{\Phi\Pi}$ differ in sign, the magnitude of the two phases is affected in opposite ways by the change δ

Conclusions

- dephasing of a MZI interference signal by a detector at equilibrium
- at finite temperature, dephasing is due to a thermal average over different states of the detector
- in the zero temperature limit, dephasing is due to quantum fluctuations of the detector in a nonequilibrium interferometer
- strength of dephasing is determined by phase shift due to Coulmb coupling between detector and interferometer
- Friedel sum rule relates this phase shift to the screening charge that an occupied detector state induces on the interferometer arm