Coulomb drag in graphene

Igor Gornyi
Karlsruhe Institute of Technology

Collaboration:
Mikhail Titov (Nijmegen)
Boris Narozhny (Karlsruhe)
Pavel Ostrovsky (Stuttgart)
Michael Schütt (Karlsruhe)
Alexander Mirlin (Karlsruhe)

Phys. Rev. B **85**, 195421 (2012)
+ arXiv:1205.5018

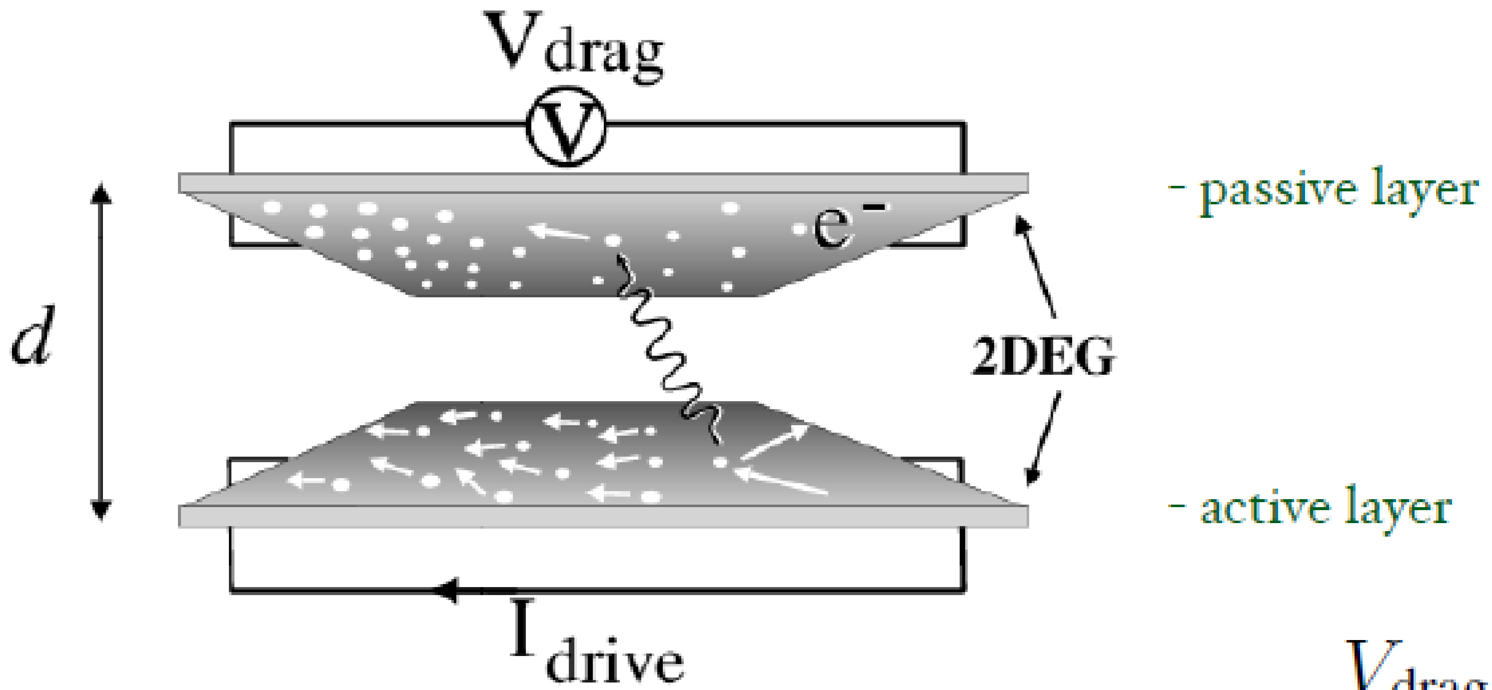
Chernogolovka, 14 September 2012

What is DRAG?

- **Merriam-Webster Online Dictionary,** http://www.webster.com
  - 3: a draw on a pipe, cigarette, or cigar
  - 4: one that is boring or gets in the way of enjoyment
  - 7: clothing typical of one sex worn by a person of the opposite sex
- **American Heritage Dictionary of the English Language,** http://www.dictionary.com
  - 7: one that impedes or slows progress; a drawback, clog, or burden
  - 12: one that is obnoxiously tiresome

    My lectures were only a pleasure to me, and no drag.

What is Coulomb drag?



$$R_D = \frac{V_{\rm drag}}{I_{\rm drive}} = \frac{L}{W}\rho_D$$

Coulomb drag = response of the passive layer
to a current in the active layer
mediated by Coulomb interaction

Coulomb drag measurements

Gramila, Eisenstein, MacDonald *et.al* ., Phys. Rev. Lett. 66, 1216 (1991)

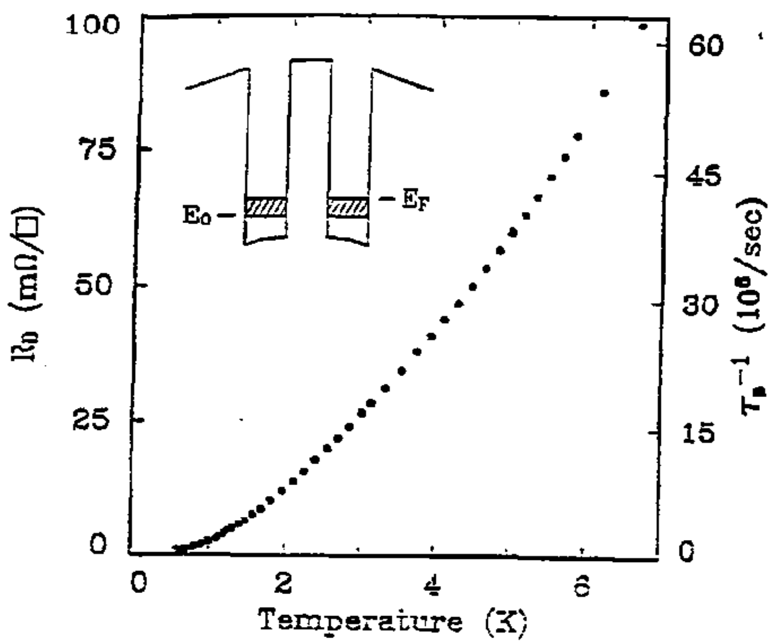


FIG. 2. Temperature dependence of observed frictional drag between two 2D electron systems separated by 175-Å barrier. Data are plotted as an equivalent resistance and a momentum-transfer rate. Inset: An idealized conduction-band diagram for a DQW structure indicating the ground subband energy E_0 and the Fermi energy E_F.

Theory: 2D

Pogrebinskii (1977) – introduced Coulomb drag

Zheng, MacDonald (1993) – memory function

Jauho, Smith (1993) – kinetic equation

Kamenev, Oreg (1995) – diagrammatics

Flensberg et al. (1995) – diagrammatics

...

Coulomb drag: why interesting?

- no drag without interaction:

  probe of inter-electron correlations

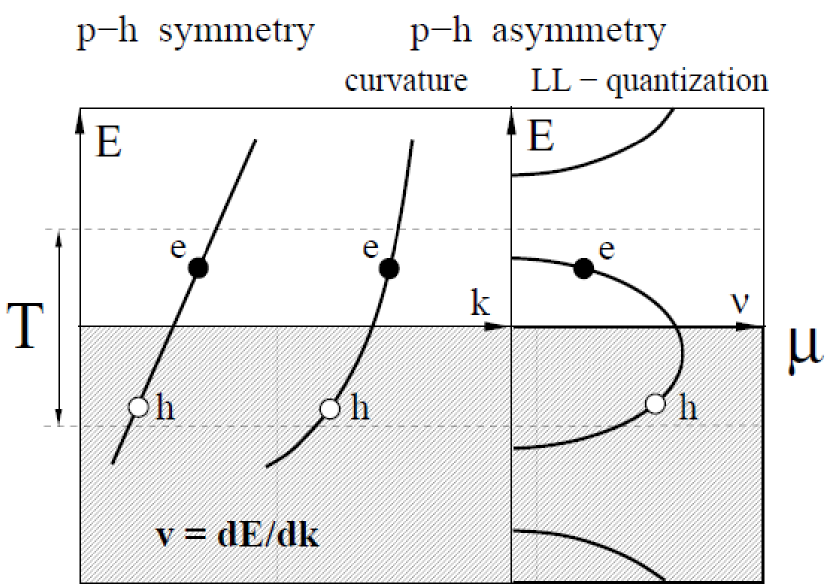
- provides information about inelastic processes,

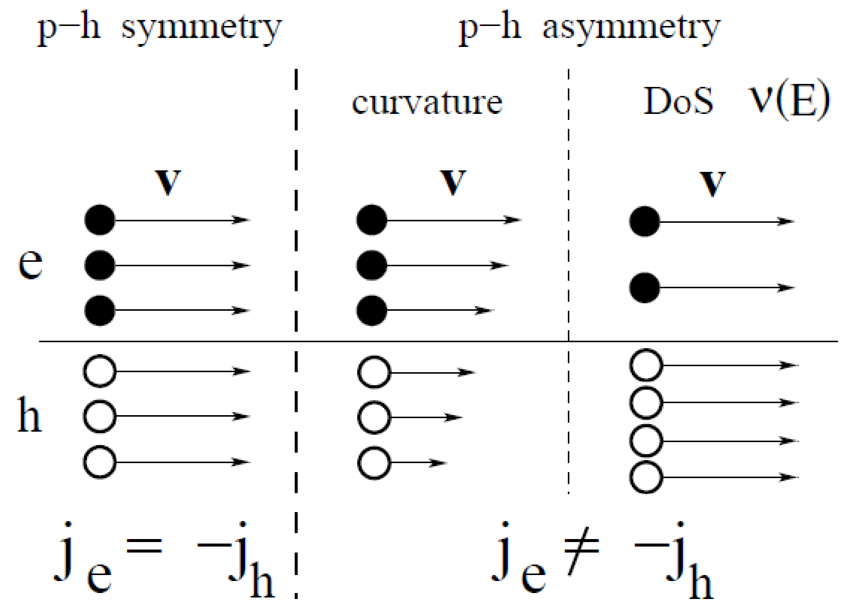
  phase-coherent phenomena

- drag is related to particle-hole asymmetry

Drag in graphene near the Dirac point ?

Particle-hole asymmetry





Example: strong magnetic field

(i) Curvature ➔ normal positive drag
(ii) Landau levels DoS ➔ anomalous oscillatory drag

IG, Mirlin, von Oppen (2004)

Drag in 2D: standard theory

transconductivity:

more convenient for diagrammatics

$$\sigma_D \ll \sigma_{1(2)}$$

$$\rho_D = \sigma_D / (\sigma_1 \sigma_2)$$

$$\boldsymbol{\sigma}^D_{\alpha\beta} = -j_{2\alpha}/\boldsymbol{E}_{1\beta}$$

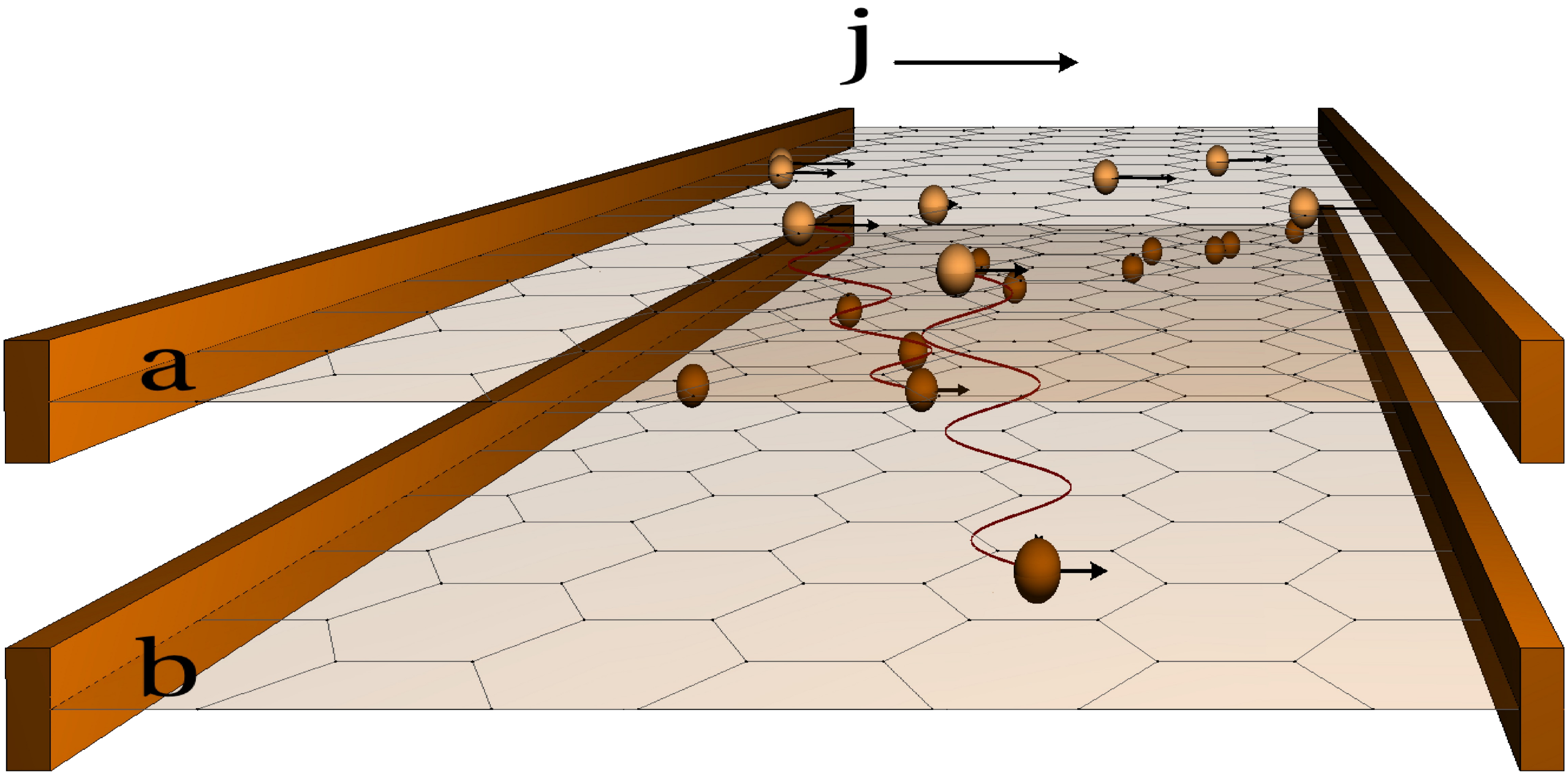
$$\boldsymbol{\sigma}^D_{ij} = \frac{e^2}{16\pi TS} \sum_{\mathrm{q}} \int_{-\infty}^{\infty} \frac{d\omega}{\sinh^2(\omega/2T)} \Gamma_i^{(1)}(\mathrm{q},\omega,B)\Gamma_j^{(2)}(\mathrm{q},\omega,-B)|U(\mathrm{q},\omega)|^2$$

$\boldsymbol{\Gamma(\mathrm{q},\omega)}$ $\longleftrightarrow$ quasiclassical rectification coefficient

$\longleftrightarrow$ *dc* response to *ac* scalar potential $\phi(\mathrm{q},\omega)$:

$$\mathrm{J}^{\mathrm{dc}} = \frac{e}{2} \sum_{\mathrm{q},\omega} \Gamma(\mathrm{q},\omega)|e\phi(\mathrm{q},\omega)|^2$$

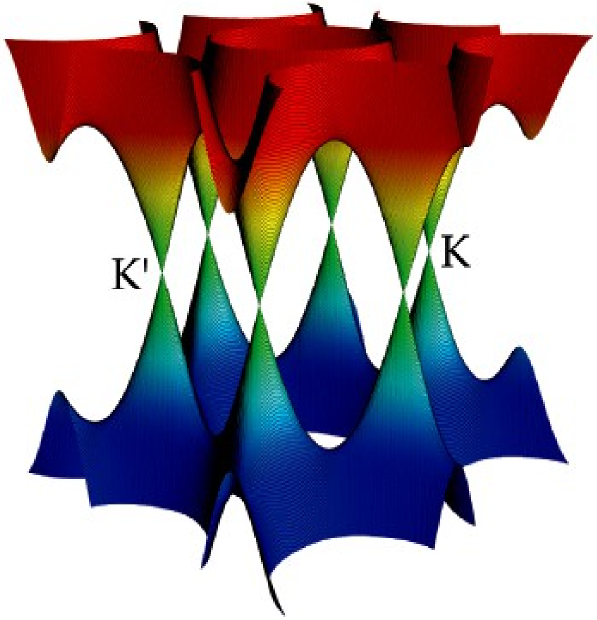
Drag in graphene



News in graphene

Dirac spectrum at low energies

$$E_\nu(\boldsymbol{k}) = \nu v k, \quad \nu = \pm$$



electron-hole symmetry at the Dirac point

linear spectrum – no Galilean invariance

- non-trivial single-layer conductivity

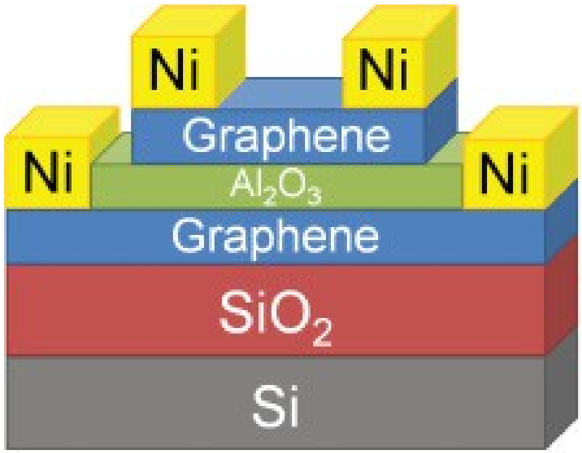
small interlayer distance *d*

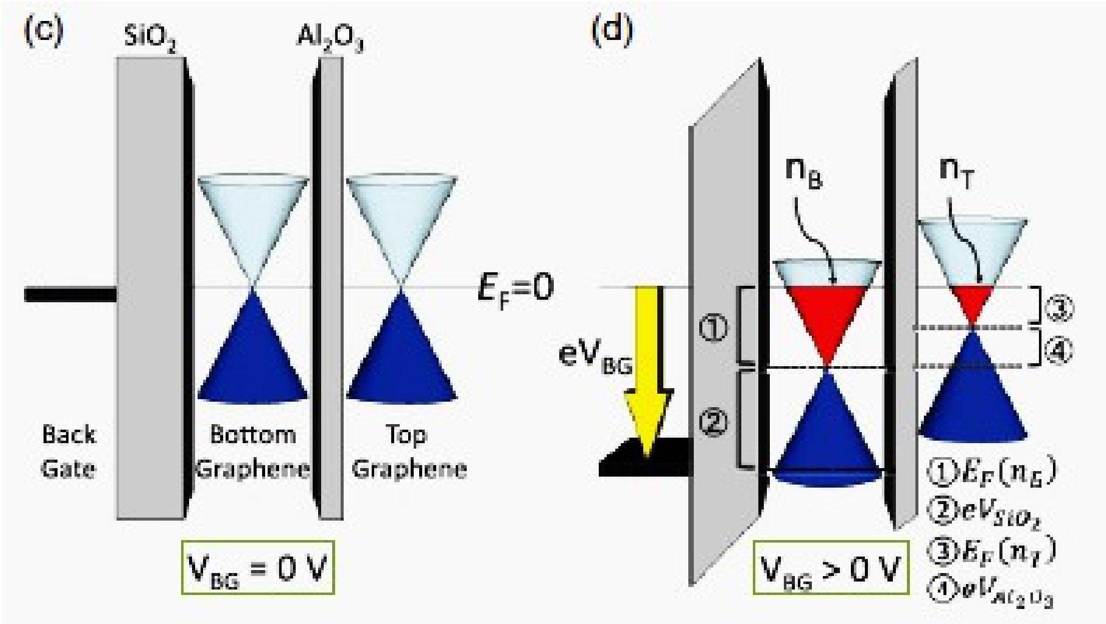
- electron density can be positive, negative, large and low
- Fermi wave length at low density is much larger than *d*
- screening length at low density is much larger than *d*

Drag in graphene: experiment

single-gate device

Kim, Jo, Nah, Yao, Banerjee, and Tutuc, Phys. Rev. B **83**, 161401 (2011)





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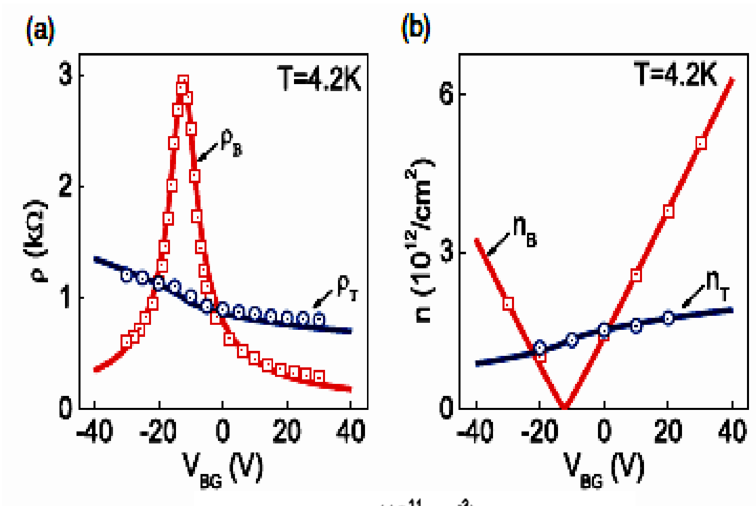
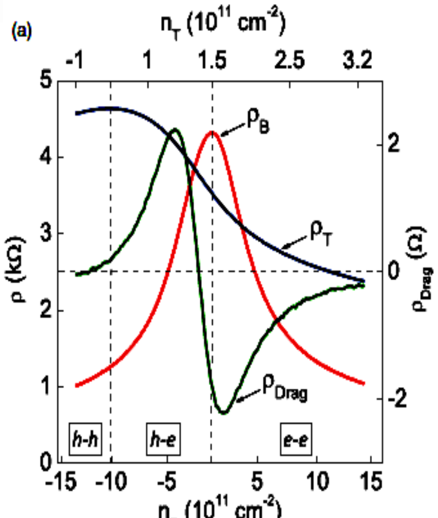
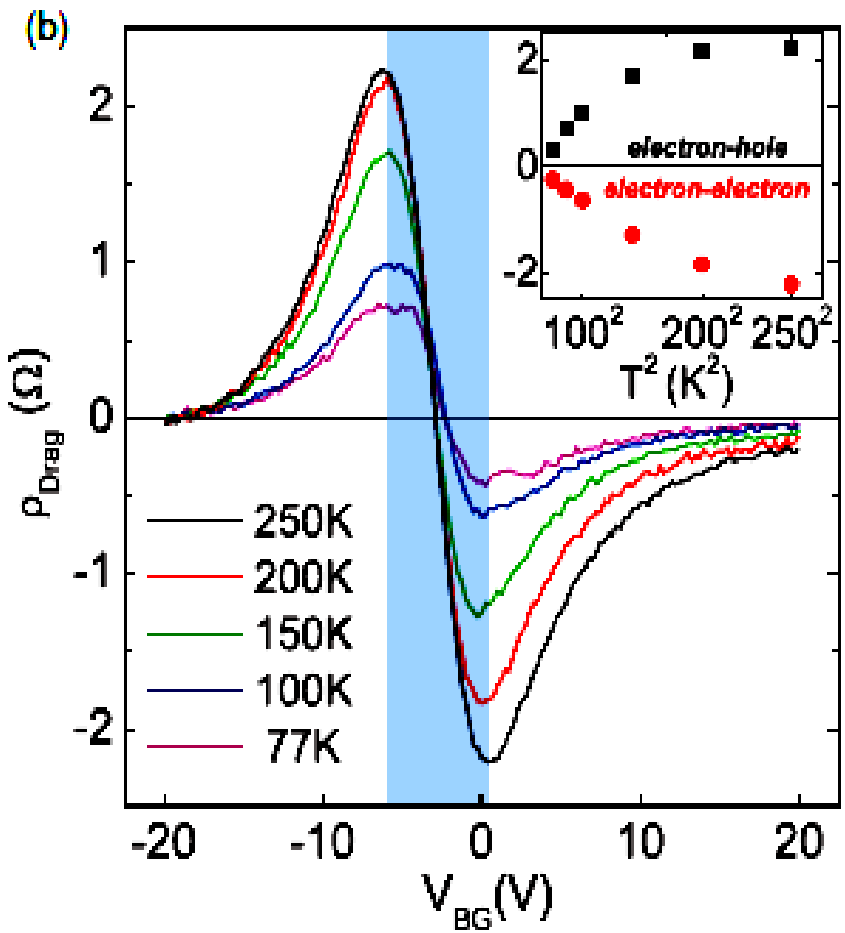


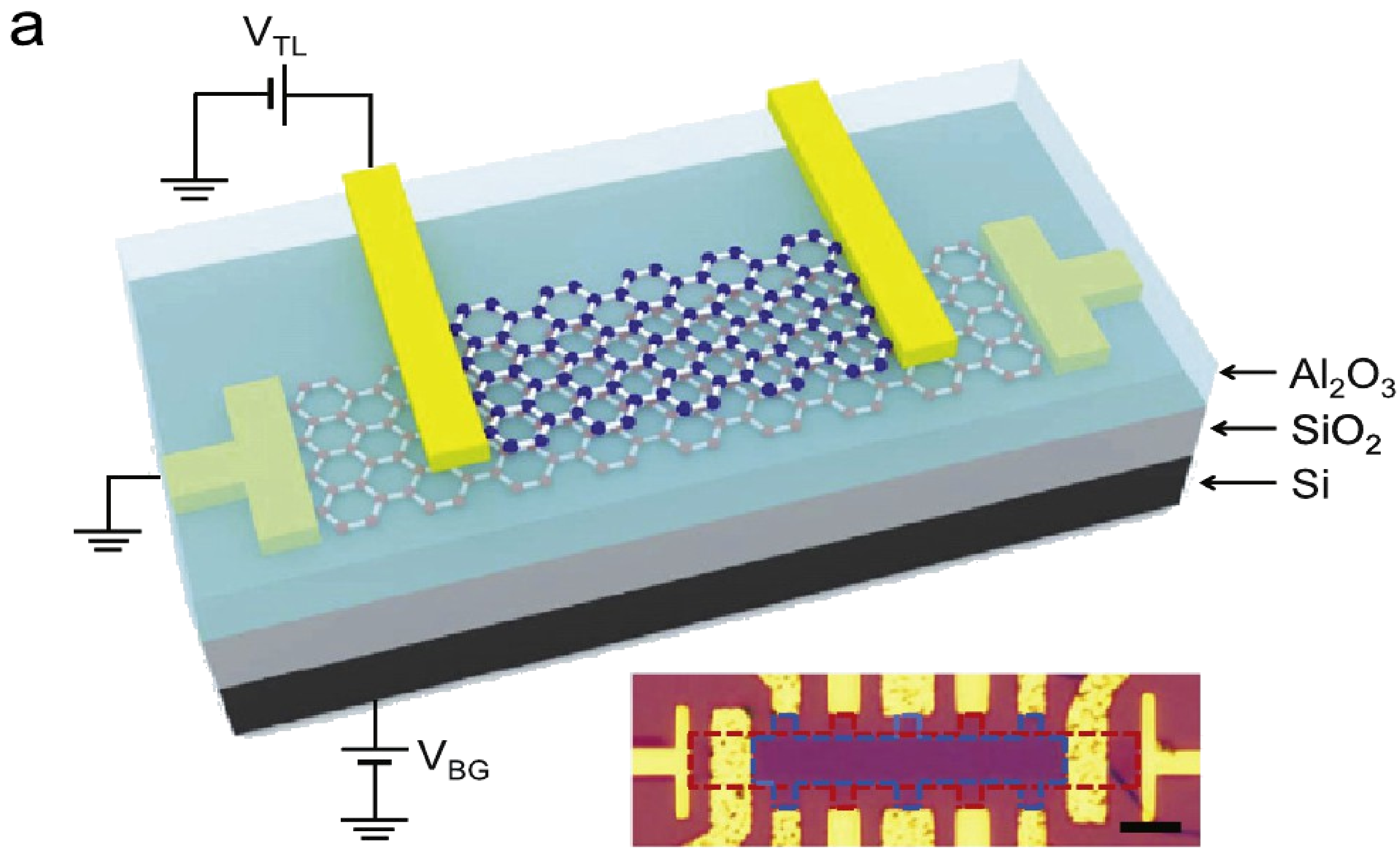




Drag in graphene: experiment

double-gate device

Tutuc and Kim, Solid State Comm. (2012)



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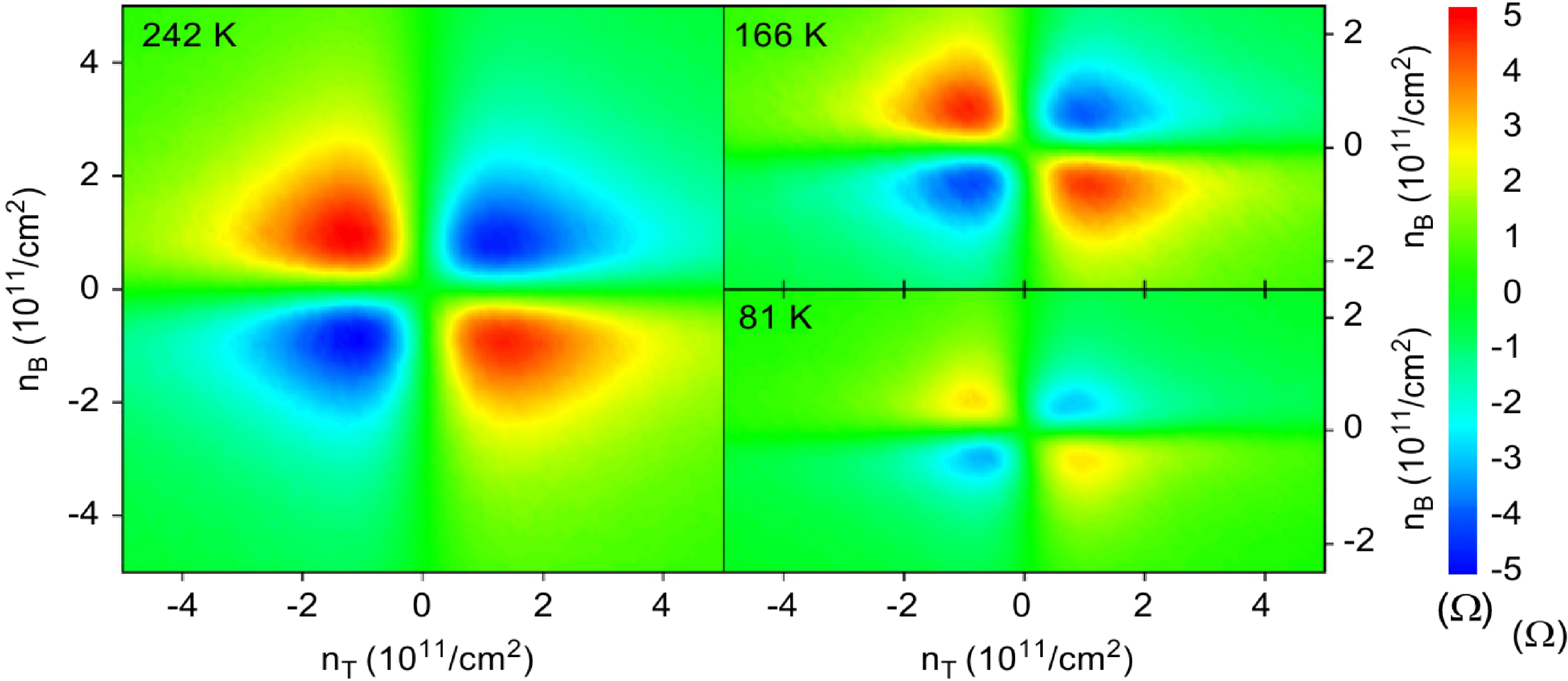
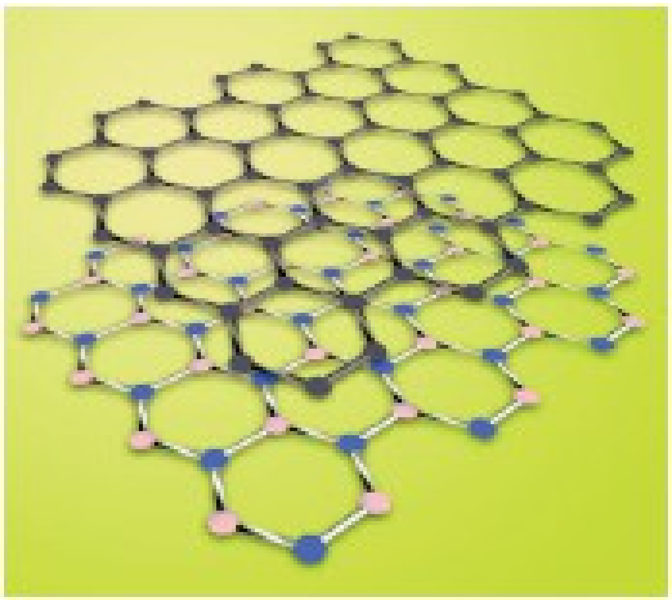


Drag in graphene: experiment

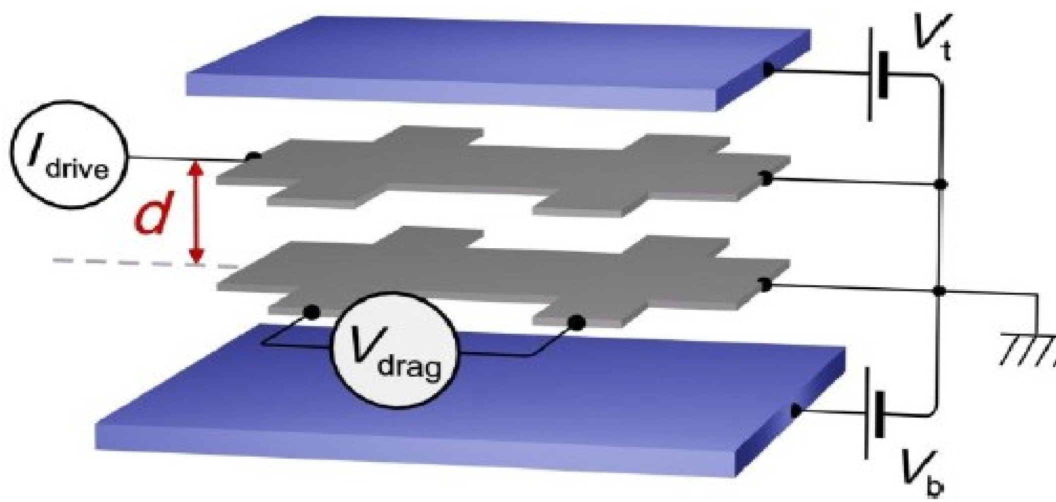
Gorbachev, Geim, Novoselov, Ponomarenko et al. ArXiv:1206.6626



- "clean"
  substrate and spacer – BN

- smaller inter-layer spacing
  d = 1-10 nm

- Double-gate setup:



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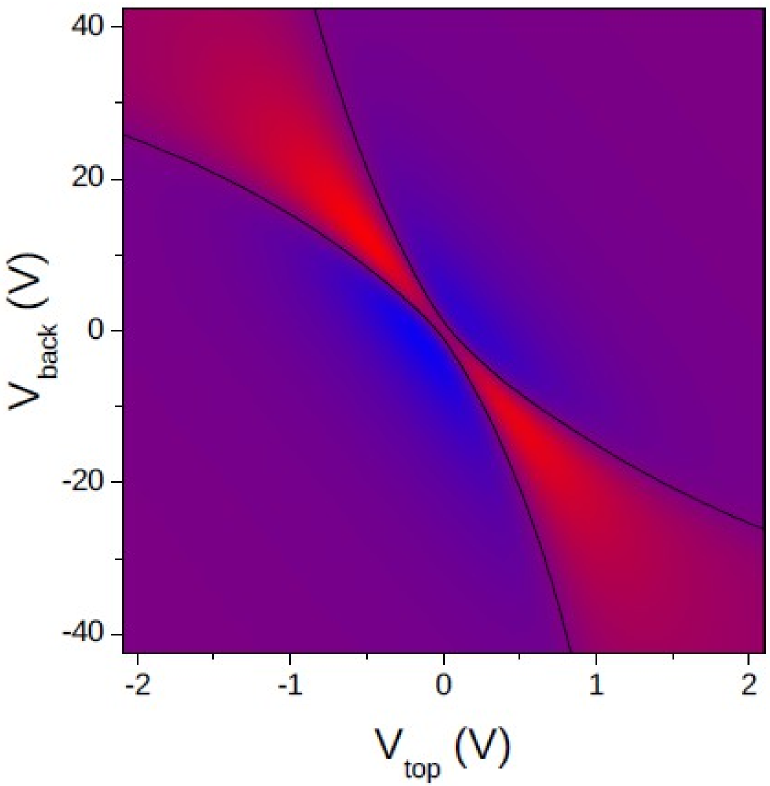

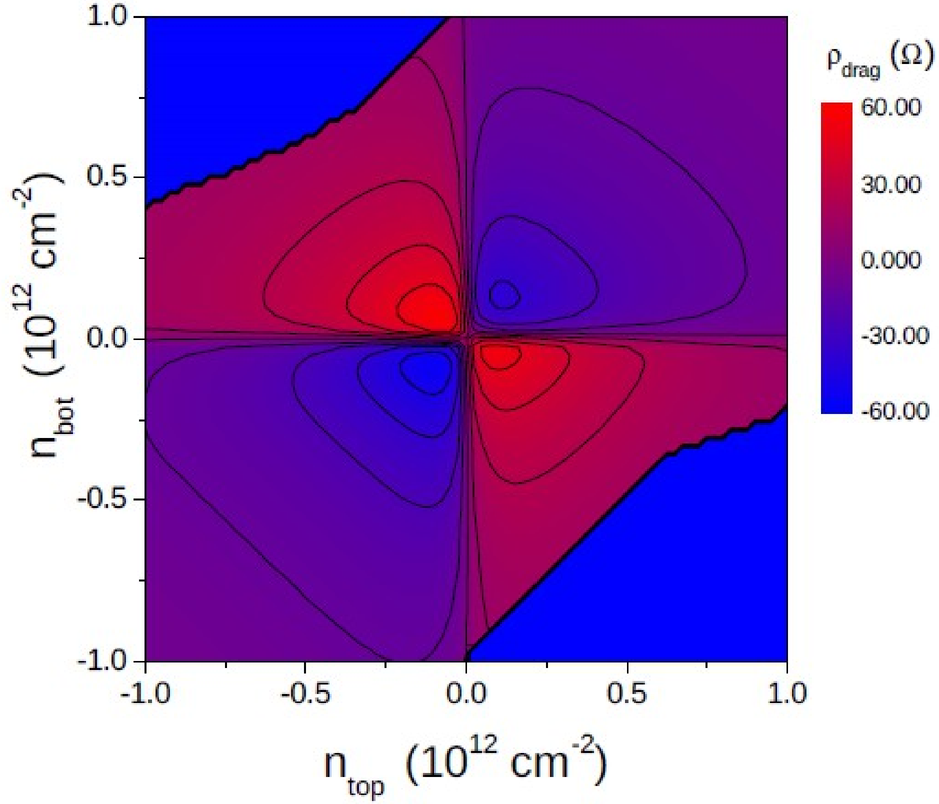

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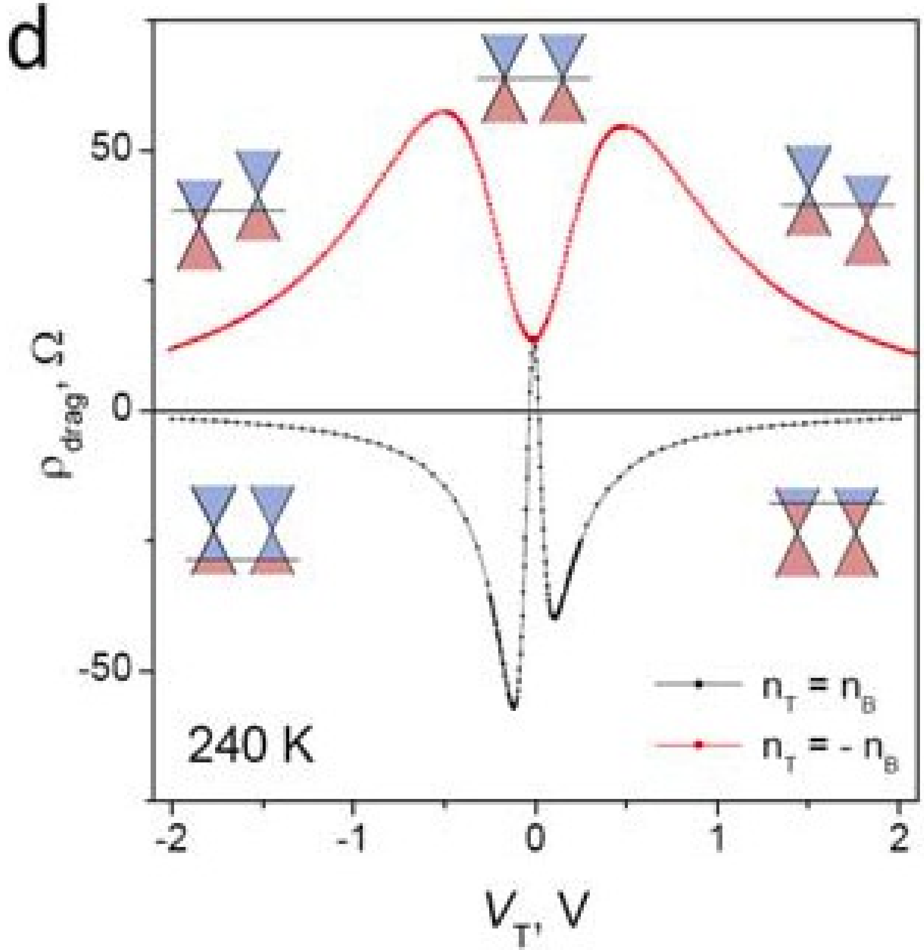

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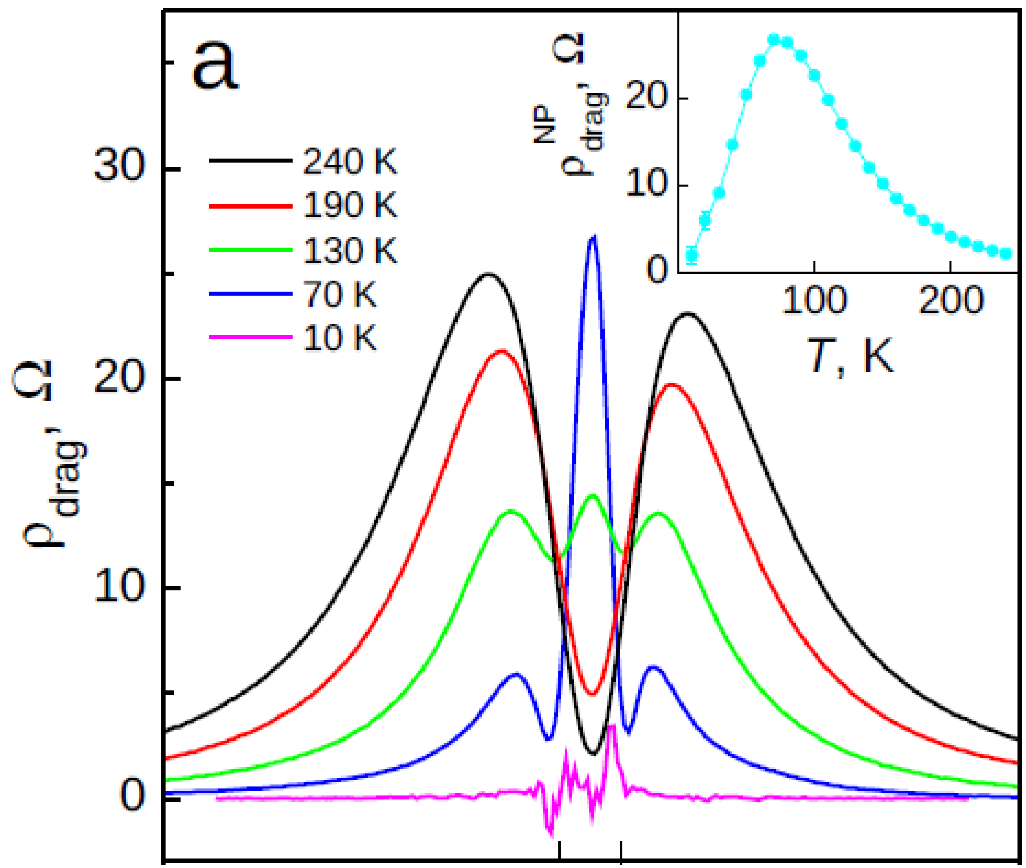
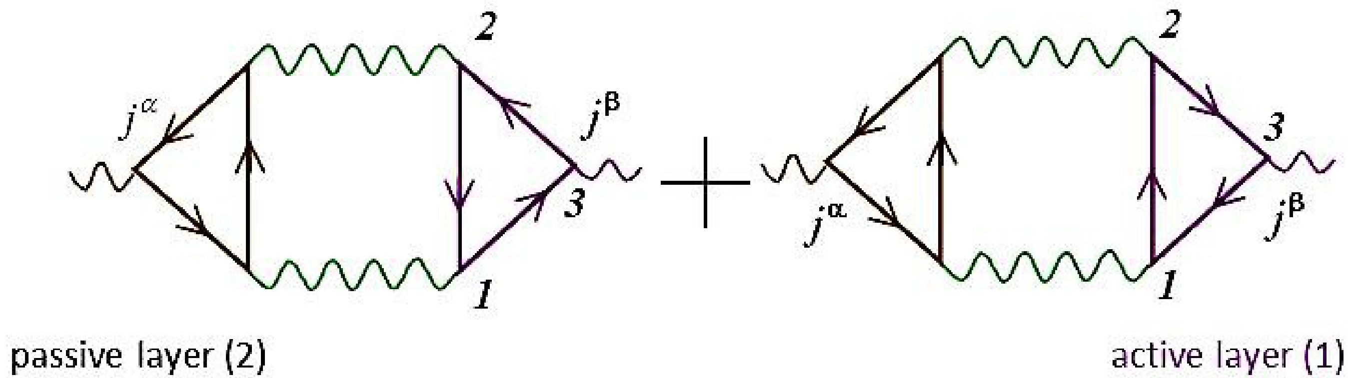


disordered graphene

Second-order perturbation theory

Narozhny, Titov, IG, and Ostrovsky, PRB (2012)



$$\sigma_D^{\alpha\beta} = \frac{1}{16\pi T}\sum_{\boldsymbol{q}}\int \frac{d\omega}{\sinh^2 \frac{\omega}{2T}} \Gamma_1^{\beta}(\omega, \boldsymbol{q})\Gamma_2^{\alpha}(\omega, \boldsymbol{q})|\mathcal{D}_{12}^R|^2;$$

non-linear susceptibility (rectification function)

screened interlayer interaction

$$\rho_D^{(2)}(\mu_1, \mu_2 = 0) = 0$$

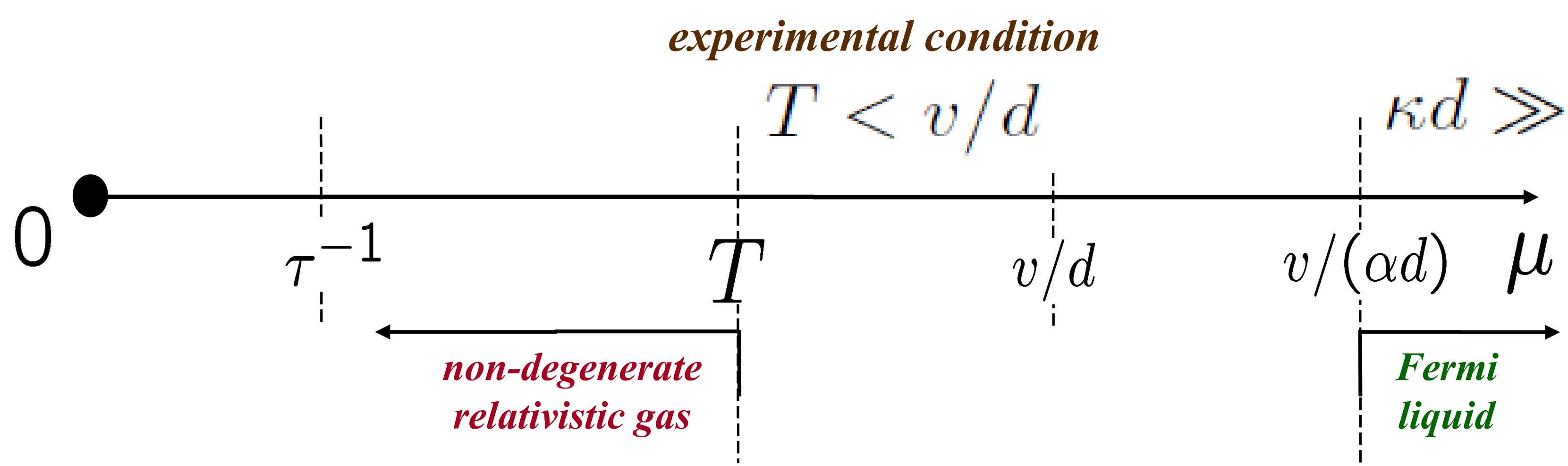
Drag in disordered graphene

controlled perturbation theory $\alpha^2 T\tau \ll 1$

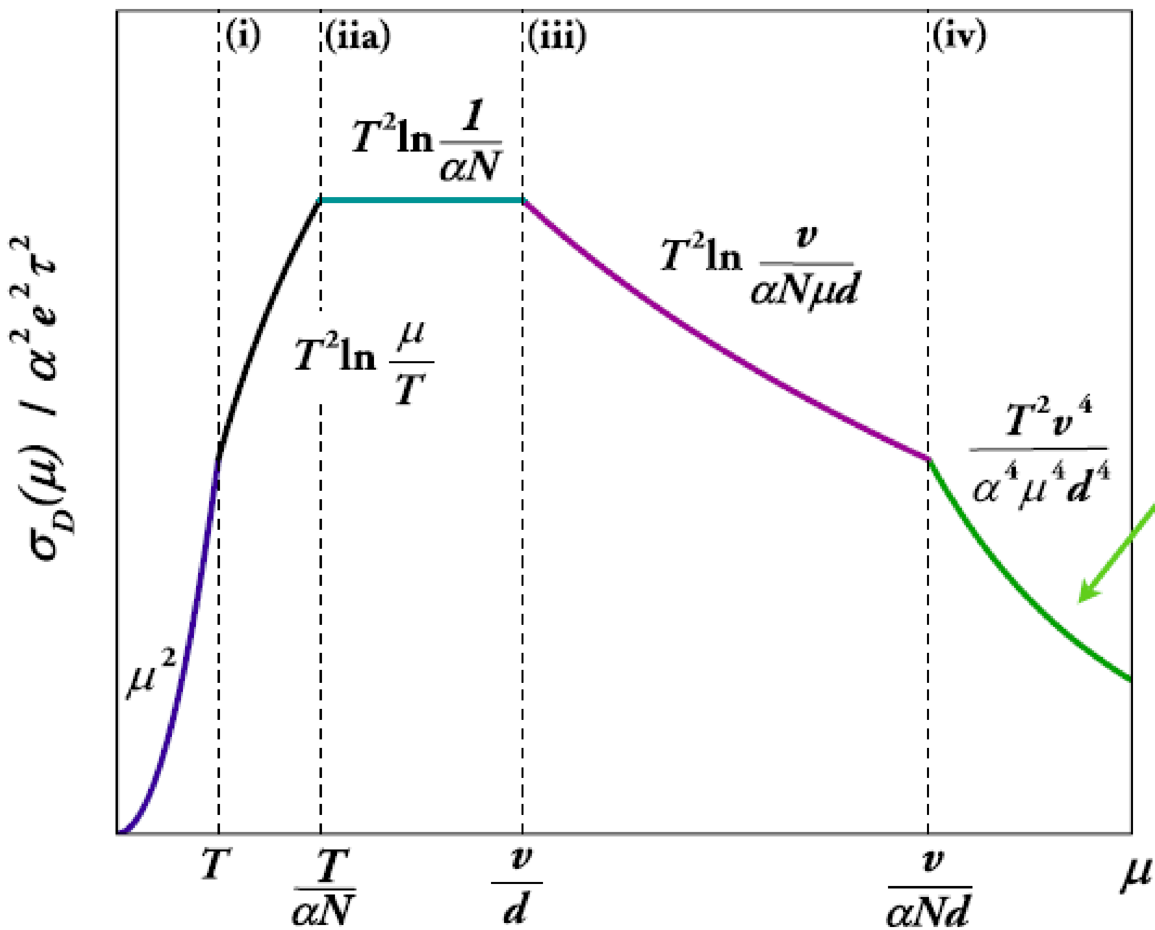
- weak interaction
- dominant scattering mechanism is due to disorder

arbitrary values of chemical potential

- *qualitatively not important:*
  - plasmons
    static screening is sufficient
  - spectrum renormalization
  - energy dependence of disorder scattering time



Results for $\alpha \ll 1$ $\quad d \ll \hbar v/T$



Fermi-liquid asymptotic!

$$\sigma_D = \frac{\zeta(3)}{4} \frac{e^2 \tau^2 T^2}{(k_F d)^2 (\varkappa d)^2}$$

$$\varkappa = 4\alpha k_F$$

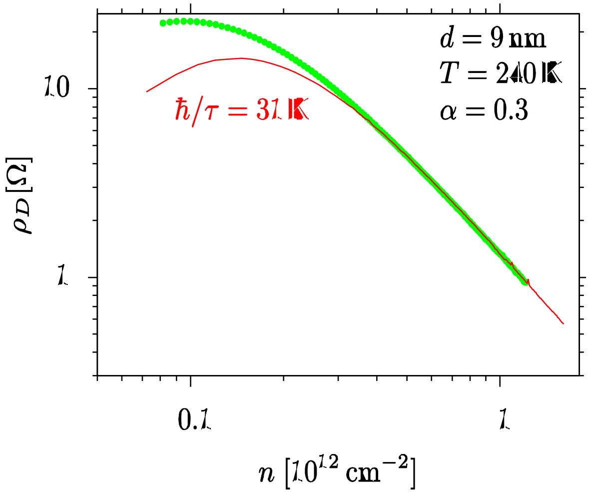
$$\varkappa d \gg 1$$

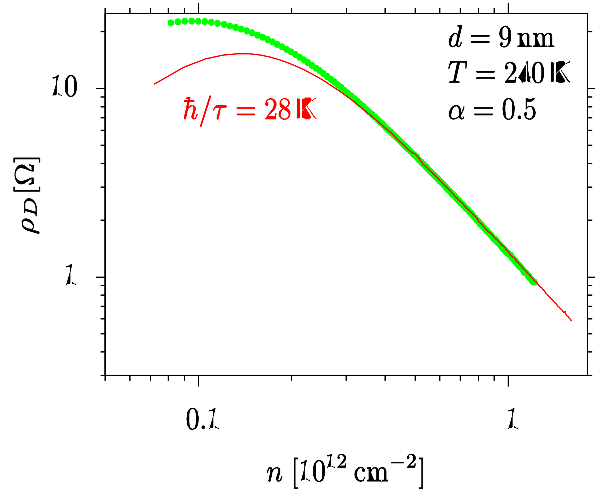
USEFUL to REMEMBER

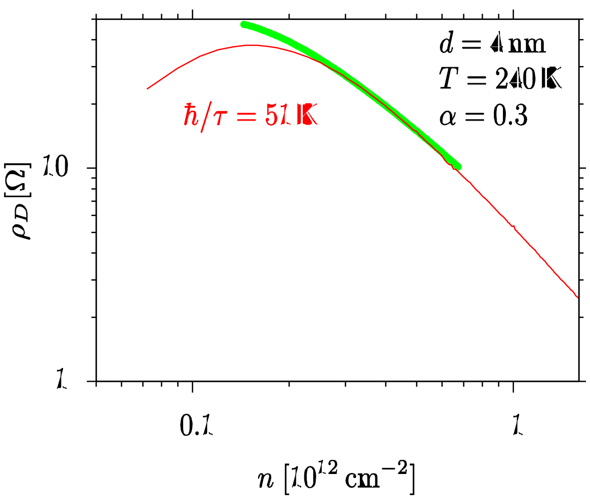
$$\mu \simeq \hbar v \sqrt{\pi\, n}$$

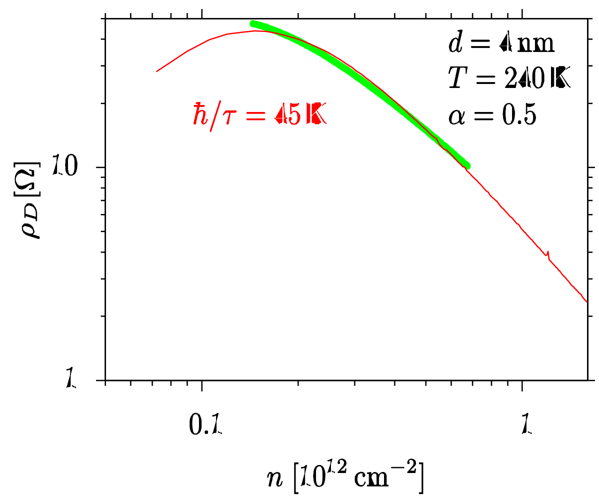
comparison with experiment

courtesy of L. Ponomarenko





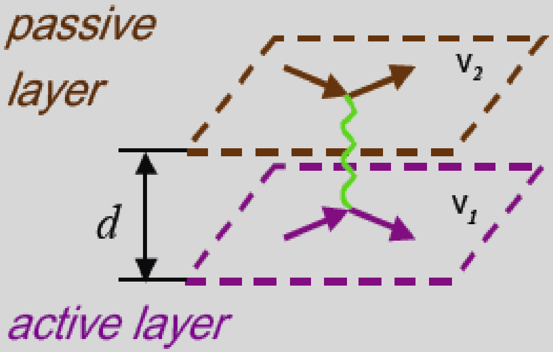




ultra-clean graphene

Drude-like model revisited

momentum transfer rate



$v_2 = 0$

$v_1 = \frac{j}{en}$

$E = \rho_D j$

passive layer

– equation of motion:

$$\frac{d}{dt}v_2 = \frac{e}{m}E + \frac{v_1 - v_2}{\tau_D} - \frac{v_2}{\tau}$$

– trans-resistivity:

$$\rho_D = \left(\frac{e^2 n}{m}\tau_D\right)^{-1}$$

active layer

– equation of motion

$$\frac{d}{dt}v_1 = \frac{e}{m}E + \frac{v_2 - v_1}{\tau_D} - \frac{v_1}{\tau}$$

– disordered sample

$$\frac{1}{\tau} \gg \frac{1}{\tau_D} \qquad \rho_{11} = \rho_0 = \left(\frac{e^2 n}{m}\tau\right)^{-1}$$

– clean sample

$$\frac{1}{\tau} = 0 \qquad \rho_{11} = -\rho_{12} = \rho_D = \left(\frac{e^2 n}{m}\tau_D\right)^{-1}$$

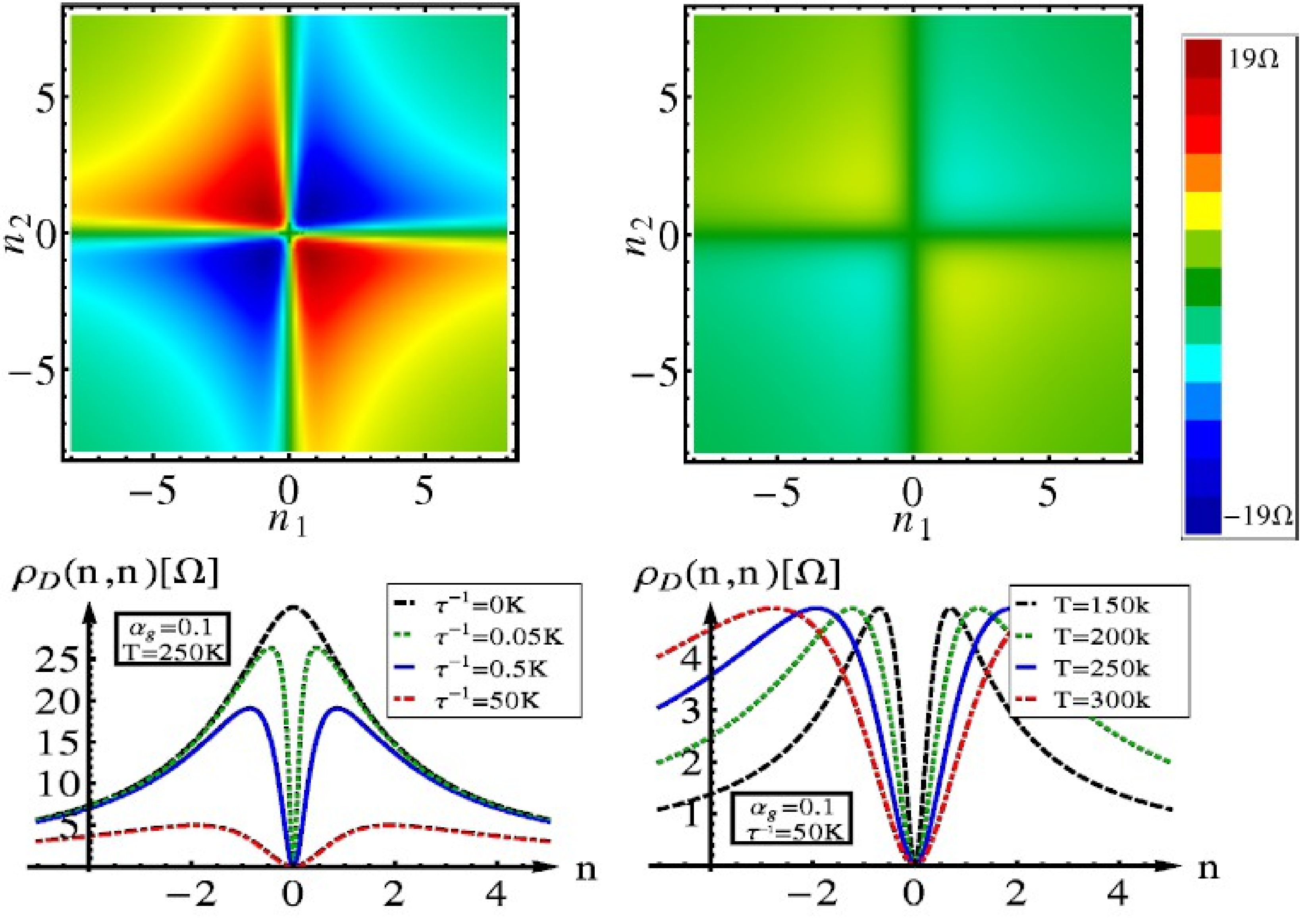
degenerate resistivity matrix

infinite conductivity

resistivity due to inter-layer interaction

Graphene: no Galilean invariance, relativistic dynamics

Clean vs. disordered graphene



Kinetic theory of the drag

Linearized kinetic equation:

$$n_i(\epsilon, \hat{\mathbf{v}}) = n_F^{(i)}(\epsilon) + T\frac{\partial n_F^{(i)}(\epsilon)}{\partial \epsilon} h_i(\epsilon, \hat{\mathbf{v}})$$

$$\frac{\partial h_1}{\partial t} + \frac{e\boldsymbol{E}_1\boldsymbol{v}}{T} = -\frac{h_1}{\tau} + I_{11}\{h_1\} + I_{12}\{h_1, h_2\},$$

$$\frac{\partial h_2}{\partial t} = -\frac{h_2}{\tau} + I_{22}\{h_2\} + I_{21}\{h_2, h_1\},$$

$$I_{ij} = -\int d2\ d3\ d4\ W^{ij}(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4})$$

Inelastic scattering in graphene

Kashuba '08, Fritz, Müller, Schmalian, Sachdev '08

Linear spectrum:

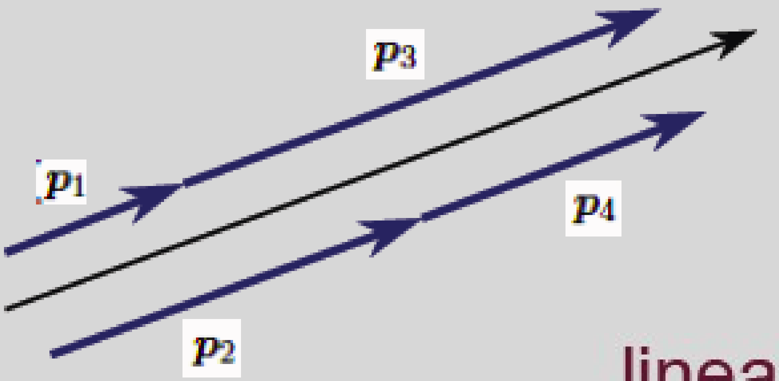
Velocity is not equivalent to momentum:
momentum conservation does not prevent current relaxation
- Finite transport rate due to inelastic e-e scattering

Collinear scattering singularity:
momentum conservation = energy conservation
- Fast thermalization within a given direction

$$n(\epsilon, \hat{\mathbf{v}}) = \frac{1}{1 + \exp\left[\frac{\epsilon - \mu(\hat{\mathbf{v}})}{T(\hat{\mathbf{v}})}\right]} \simeq n_F(\epsilon) - \frac{\partial n_F(\epsilon)}{\partial \epsilon}\left[\frac{\delta\mu(\hat{\mathbf{v}})}{T} + (\epsilon - \mu)\frac{\delta T(\hat{\mathbf{v}})}{T^2}\right]$$

Collinear scattering singularity

collinear scattering



– equivalence of energy and momentum conservation laws

$$p_1 + p_3 = p_2 + p_4$$

$$\epsilon_i = \pm v|p_i|$$

$$\epsilon_1 + \epsilon_3 = \epsilon_2 + \epsilon_4$$

linearized collision integrals

"momentum mode"

$$h_i \propto \boldsymbol{p} \;\Rightarrow\; I_{ij} = 0$$

"velocity mode"

$$h_i \propto \boldsymbol{v} \;\Rightarrow\; I_{ij} = 0$$

if $\boldsymbol{v}_i \| \boldsymbol{v}_j$

$$I_{ij} = -\int d2\, d3\, d4\, W^{ij} \underline{(h_{i,1} - h_{i,2} + h_{j,3} - h_{j,4})}$$

$$W^{ij} = \underline{\delta(\boldsymbol{p}_1 - \boldsymbol{p}_2 + \boldsymbol{p}_3 - \boldsymbol{p}_4)}\, \delta(\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)$$

$$\times \frac{\cosh\frac{\epsilon_1 - \mu_i}{2T}}{2\cosh\frac{\epsilon_2 - \mu_i}{2T}\cosh\frac{\epsilon_3 - \mu_j}{2T}\cosh\frac{\epsilon_4 - \mu_j}{2T}} K^{ij}_{1,2;3,4},$$

Double-layer graphene

Only two modes (velocity and momentum) in each layer:

$$h_i(\epsilon, \hat{\mathbf{v}}) = \left(\chi_v^{(i)} + \chi_p^{(i)} \, \epsilon/T \right) e\boldsymbol{E}\boldsymbol{v}/T^2$$

Fast unidirectional thermalization between layers:

$$\chi_p^{(1)} = \chi_p^{(2)}$$

Kinetic (integral) equation reduces to a 3x3 matrix equation!

Hydrodynamics: total momentum + particle currents

Hydrodynamic equations

$$\boldsymbol{J}_i = -NT \int d\epsilon \nu(\epsilon) \frac{\partial n_F^{(i)}}{\partial \epsilon} \int d\hat{\mathbf{v}} \boldsymbol{v} h_i(\epsilon, \hat{\mathbf{v}})$$

$$\boldsymbol{P} = e\epsilon_0 C_1^2 (\boldsymbol{E}_1 + \boldsymbol{E}_2)\tau$$

$$\boxed{e\epsilon_0 \begin{pmatrix} \boldsymbol{E}_1 \\ \boldsymbol{E}_2 \end{pmatrix} = \left[\frac{1}{\tau} + \widehat{\mathcal{I}}_{ee} - \widehat{\mathcal{I}}_D\right] \begin{pmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \end{pmatrix} + \left[\frac{1}{\tau_D} - \frac{1}{\tau_{ee}}\right] \begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{P} \end{pmatrix}}$$

$$\widehat{\mathcal{I}}_{ee(D)} \;=\; [(\hat{\sigma}_0 + \hat{\sigma}_1) C_1^2 + 2\hat{\sigma}_{0(1)} C_2]/\tau_{ee(D)}$$

$$C_1 = \frac{\langle \epsilon \rangle_\epsilon}{T} \sim \frac{\mu}{T}, \quad C_2 = \frac{\langle \epsilon^2 \rangle_\epsilon - \langle \epsilon \rangle_\epsilon^2}{T^2} \sim const,$$

Scattering rates: Golden Rule

$$\frac{1}{\tau_{ee}^{a}} = \frac{N}{8T^2B_2}\int d\{\epsilon_i\}d\{\hat{\mathbf{v}}_i\}\left[(\mathbf{v}_1-\mathbf{v}_2+\mathbf{v}_3-\mathbf{v}_4)^2\ \mathcal{W}^{aa}\ +2\,(\mathbf{v}_1-\mathbf{v}_2)^2\ \mathcal{W}^{ab}\right]$$

$$\frac{1}{\tau_{ee}^{b}} = \frac{N}{8T^2B_2}\int d\{\epsilon_i\}d\{\hat{\mathbf{v}}_i\}\left[(\mathbf{v}_1-\mathbf{v}_2+\mathbf{v}_3-\mathbf{v}_4)^2\ \mathcal{W}^{bb}\ +2\,(\mathbf{v}_1-\mathbf{v}_2)^2\,\mathcal{W}^{ba}\right]$$

$$\frac{1}{\tau_{D}} = \frac{N}{4T^2B_2}\int d\{\epsilon_i\}d\{\hat{\mathbf{v}}_i\}\ (\mathbf{v}_1-\mathbf{v}_2)\,(\mathbf{v}_4-\mathbf{v}_3)\ \mathcal{W}^{ba}$$

close to the Dirac point

$$\frac{1}{\tau_D} \sim \alpha^2 N \frac{\mu_a\mu_b}{T}$$

$$\frac{1}{\tau_{ee}^{a,b}} \sim \alpha^2 N T$$

away from the Dirac point

$$\frac{1}{\tau_D} \sim \alpha^2 N \frac{T^2}{\mu}\ln\frac{\mu}{T}$$

$$\frac{1}{\tau_{ee}} - \frac{1}{\tau_D} \sim \frac{1}{\tau_{ee}}\frac{T^2}{\mu^2} \ll \frac{1}{\tau_{ee}}$$

Drag resistivity

Equal layers:

$$\rho_D = \frac{\hbar}{e^2}\frac{C_2}{\epsilon_0}\frac{(\tau\tau_D)^{-1} + C_1^2\left[\tau_{ee}^{-2} - \tau_D^{-2}\right]}{\tau^{-1} + C_1^2\left[\tau_{ee}^{-1} - \tau_D^{-1}\right]}$$

Non-equal layers near the Dirac point

$$\rho_D(\mu_i \ll T) \approx 2.87\frac{h}{e^2}\,\alpha^2\frac{\mu_1\mu_2}{\mu_1^2 + \mu_2^2 + 0.49T/(\alpha^2\tau)}$$

Finite drag at the Dirac point in the clean case!

neutrality point

Drag rate: beyond Golden Rule

$$\left|M_{ab}^{(1)} + M_{ab}^{(2)}\right|^2 \simeq \left|M_{ab}^{(1)}\right|^2 + 2\mathrm{Re}\left\{M_{ab}^{(1)}\left[M_{ab}^{(2)}\right]^*\right\}$$

$$\frac{1}{\tau_D} \sim \alpha^2 N \frac{\mu^2}{T^2} + \alpha^3 NT$$

$$\rho_D \sim \frac{\hbar}{e^2}\frac{\alpha^3 T + \alpha^4\mu^2\tau N}{T + \alpha^2\mu^2\tau N}, \qquad \mu \ll \alpha^{1/2}T, \quad \frac{1}{\tau} \ll T$$

Exactly at the Dirac point: finite third-order drag

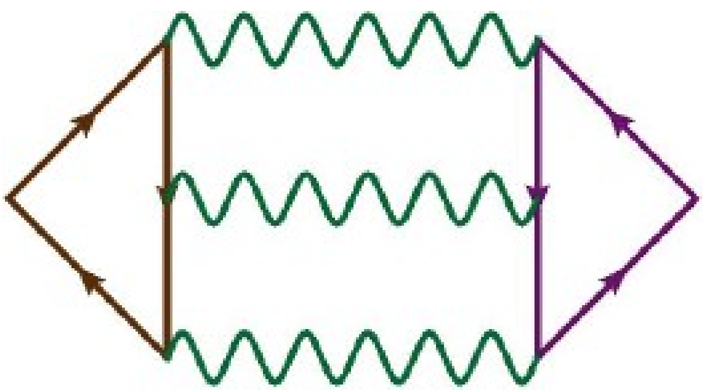
$$\rho_D^{(3)}(\mu = 0) \sim (\hbar/e^2)\alpha^3$$

Diffusive regime

Conventional (second-order) drag vanishes at the Dirac point

$$\rho_D^{(2)}\left(\mu_i \ll T \ll \tau^{-1}\right) \sim (\hbar/e^2)\alpha^2\mu_1\mu_2 T\tau^3$$

Third-order (Levchenko & Kamenev 2008) drag dominates



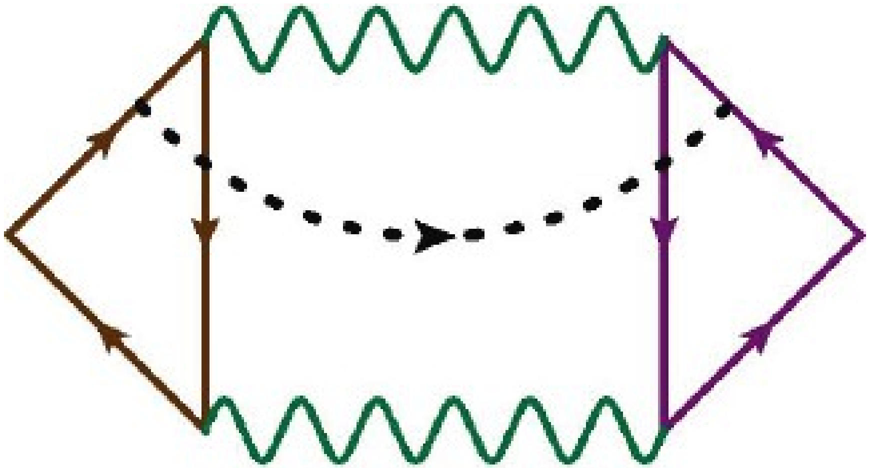
$$\varkappa d \ll 1$$

$$\rho_D^{(3)} \sim \frac{\alpha^3}{(T\tau)^{3/2}}$$

$$\rho_D^{(3)} \sim h/e^2 \text{ for } \tau^{-1} \gg \alpha^{-2}T$$

Correlated disorder

Correlated elastic scattering in the two layers with common impurities



Ballistic regime:

$$\frac{1}{\tau_D^{\mathrm{corr}}} \sim \alpha^2 T (T\tau_{12})^{-1} = \frac{\alpha^2}{\tau_{12}}$$

$$\rho_D^{\mathrm{corr}} \sim \frac{\alpha^2}{T\tau_{12}}$$

Diffusive regime:

interlayer Cooper mode (IG, Yashenkin, Khveshchenko '99)

$$\rho_D^{\mathrm{corr}} \sim \frac{\hbar}{e^2}\frac{\alpha^2}{[1-\alpha\ln(T\tau)]^2}, \qquad \tau_g^{-1} \ll T \ll \tau^{-1}$$

$$\rho_D^{\mathrm{corr}} \sim \frac{\hbar}{e^2}\frac{\alpha^2 (T\tau_g)^2}{[1-\alpha\ln(T\tau)]^2}, \qquad T \ll \tau_g^{-1}.$$

Correlated disorder

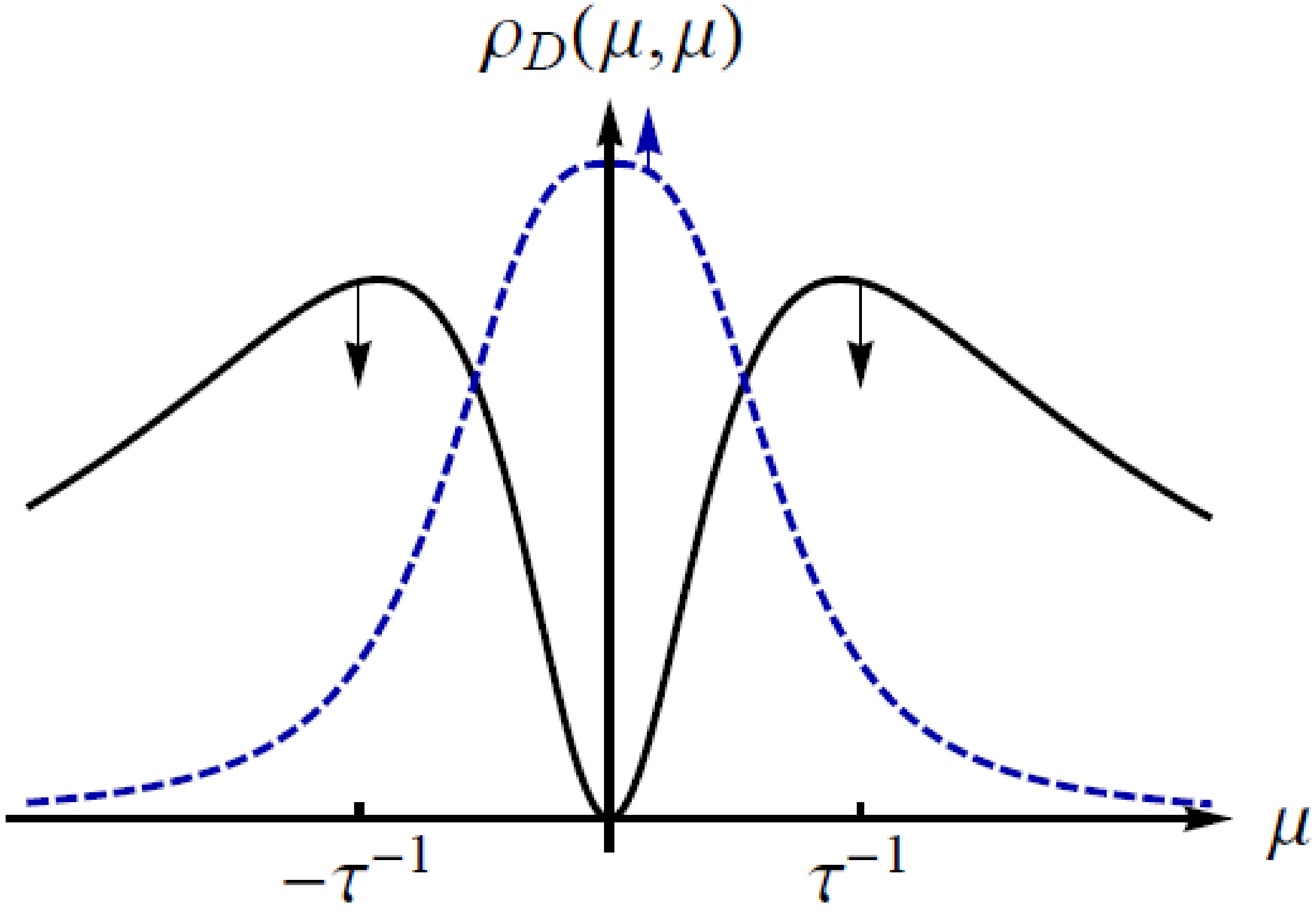
Correlated elastic scattering in the two layers with common impurities

$$\frac{1}{\tau_g} = \frac{\tau_{12} - \tau}{\tau^2}$$

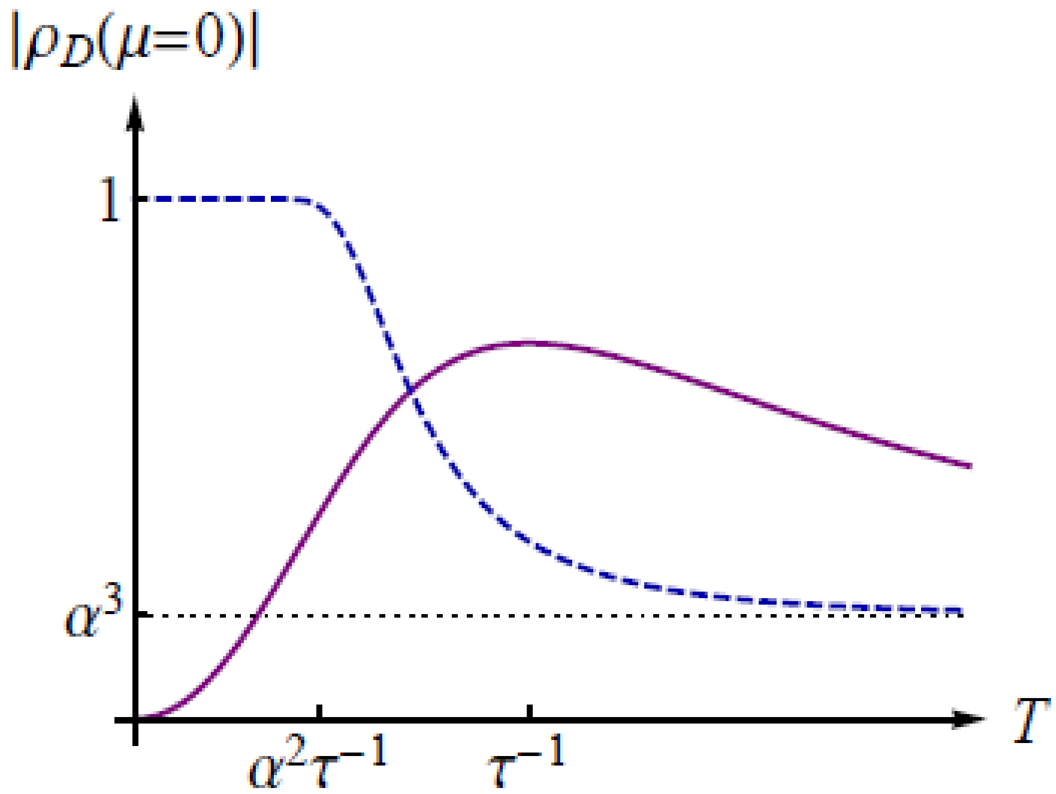
Moderately correlated disorder: $\tau_g \sim \tau \sim \tau_{12}$

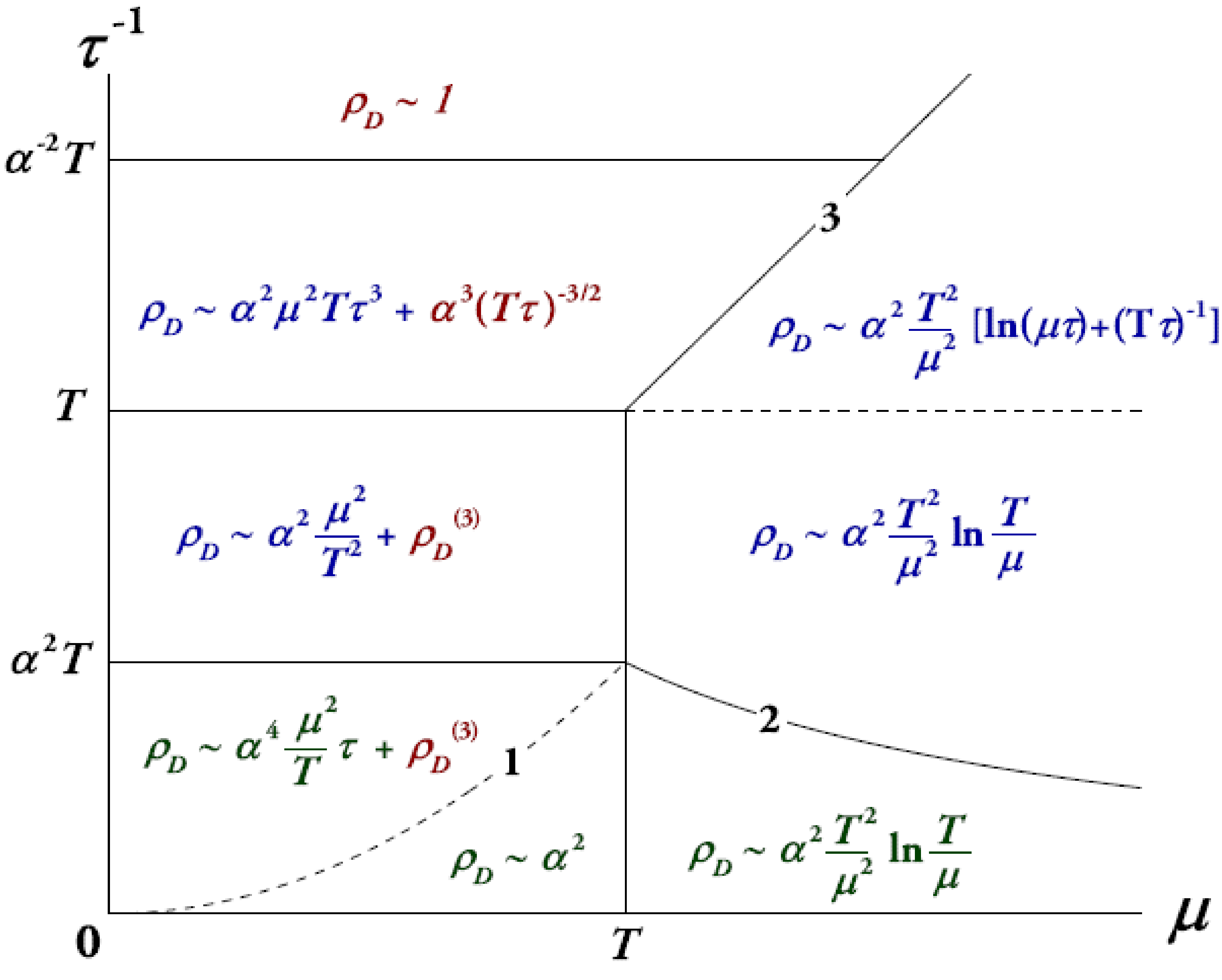
$$\rho_D^{\mathrm{corr}} \sim \frac{\hbar}{e^2}\alpha^2 \begin{cases} (T\tau)^{-1}, & T\tau \gg 1 \\ (T\tau)^2, & T\tau \ll 1 \end{cases}$$

2^{nd} vs. 3^{rd} order drag



Drag at the Dirac point

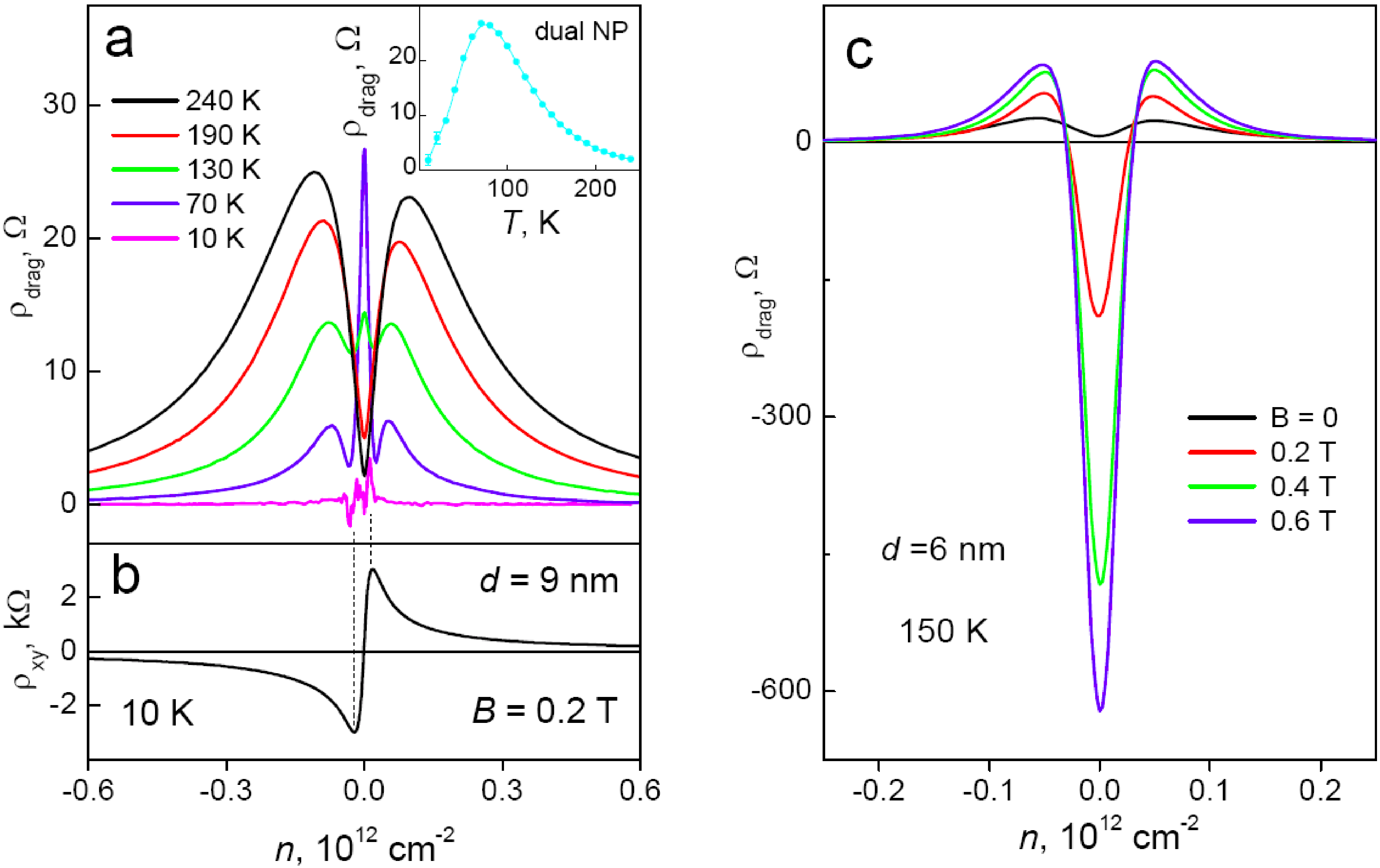




τ^{-1}
$\rho_D \sim 1$
$\alpha^{-2}T$
3
$\rho_D \sim \alpha^2\mu^2 T\tau^3 + \alpha^3(T\tau)^{-3/2}$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2}[\ln(\mu\tau)+(T\tau)^{-1}]$
T
$\rho_D \sim \alpha^2 \frac{\mu^2}{T^2} + \rho_D^{(3)}$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu}$
$\alpha^2 T$
2
$\rho_D \sim \alpha^4 \frac{\mu^2}{T}\tau + \rho_D^{(3)}$
1
$\rho_D \sim \alpha^2$
$\rho_D \sim \alpha^2 \frac{T^2}{\mu^2} \ln \frac{T}{\mu}$
0
T
μ

Magnetodrag

Gorbachev, Geim, Novoselov, Ponomarenko et al. ArXiv:1206.6626



Summary

Coulomb drag in graphene:

- Perturbation theory

- Kinetic theory (3 mode hydrodynamics)

- Clean graphene (equilibrated drag)

- Peak at the Dirac point
  3rd order drag, drag with correlated disorder