Quantum dots and Majorana Fermions

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$$\gamma = \gamma^\dagger$$

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and

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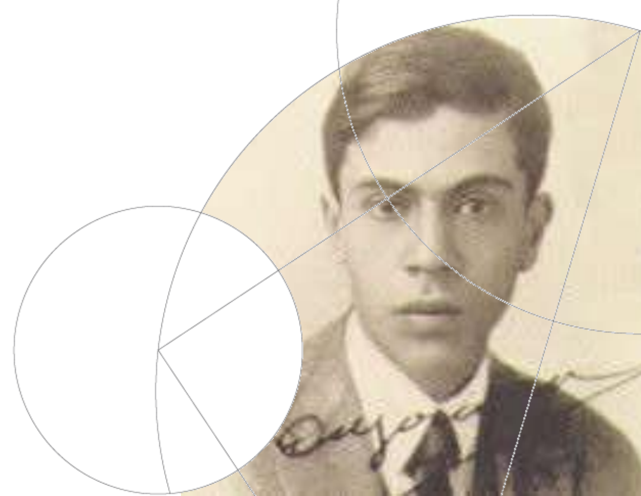


Outline:

- Introduction to Majorana fermions

- 1D topological superconductor and Majorana bound states: using circulating magnetic field

- Detection of Majorana bound states: resonant Andreev reflection

- Poor's man Majorana in double dot system

- Non-abelian manipulation via single electron control

- Outlook

Majorana fermions briefly:

$$\gamma = \gamma^\dagger$$

Majorana fermions are their own antiparticles

Hence: carries no charge and no spin

Must have the form

$$\gamma = \int d\mathbf{r} \left(f(\mathbf{r})\Psi_\uparrow(\mathbf{r}) + g(\mathbf{r})\Psi_\downarrow(\mathbf{r}) + f^*(\mathbf{r})\Psi_\uparrow^\dagger(\mathbf{r}) + g^*(\mathbf{r})\Psi_\downarrow^\dagger(\mathbf{r}) \right)$$

i.e. superposition of particle & anti-particle

so we look in superconductors, where quasiparticles are mixtures of holes and electrons

How many particles?

Majorana fermions cannot be ”counted”: $\gamma_1^\dagger \gamma_1 = \gamma_1^2 = 1$

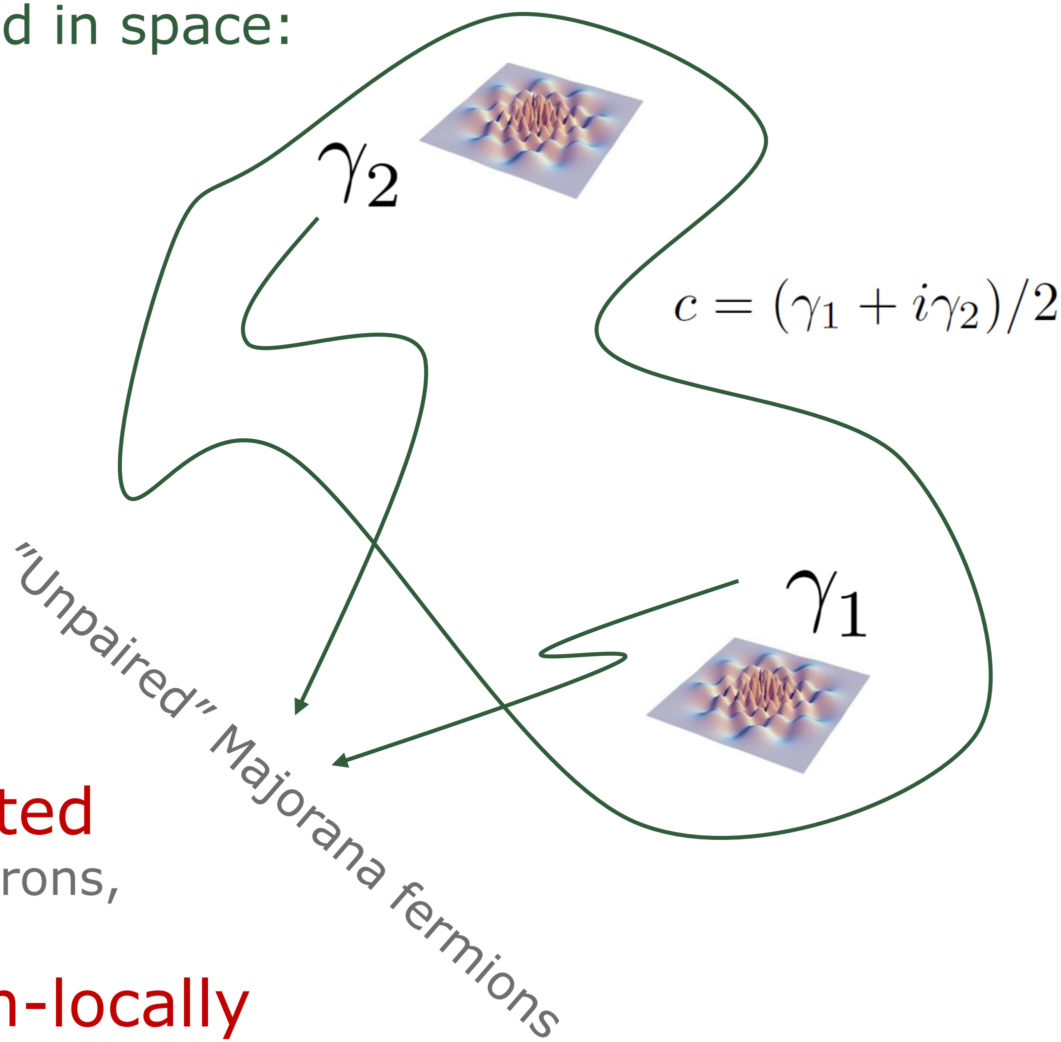
But out of two Majorana fermions, we can make one usual fermion:

$$c = (\gamma_1 + i\gamma_2)/2$$

Obeys usual Fermion relations, i.e. *c* is ”Dirac” fermion.

Two Majorana fermions equivalent to one two-level system: empty or full

Majorana fermions separated in space:



Groundstate degenerated

Even and odd number of electrons, has the same energy (parity)

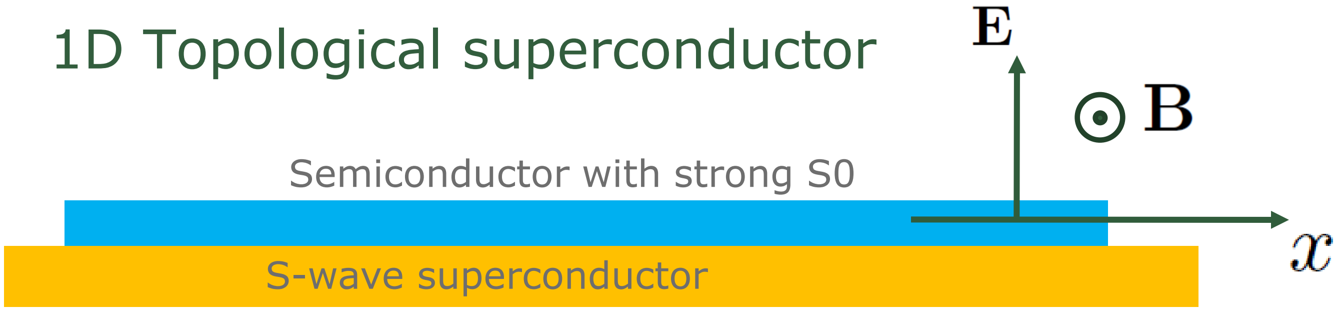
Information stored non-locally

Allows topological quantum computing

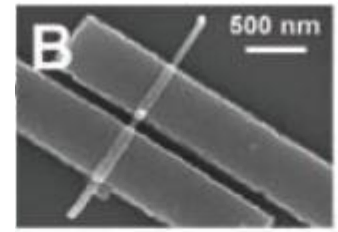
BUT NOT UNIVERSAL ☹

How to make Majorana Fermions in **hybrid** structures

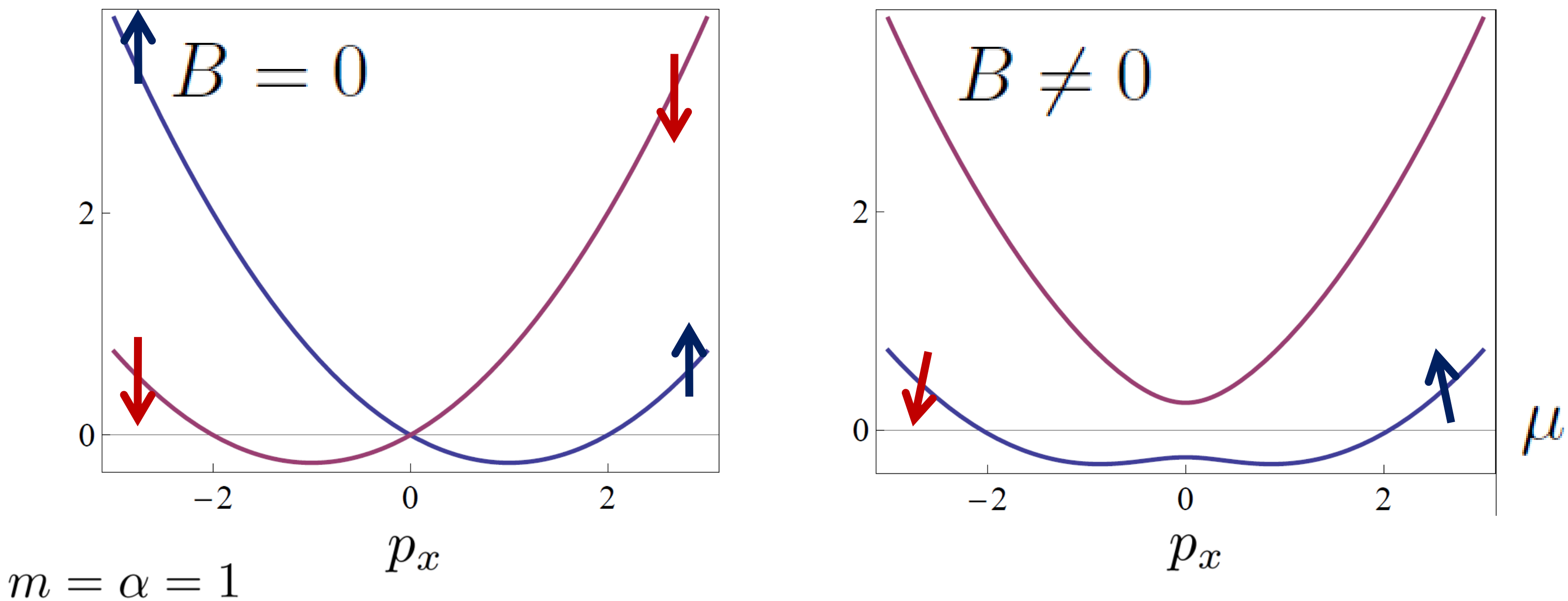
1D Topological superconductor



Oreg et al. 2010
Alicea 2010
Lutchyn et al 2010
Sau et al. 2010
Alicea et al. 2010



$$H_0 = \frac{p_x^2}{2m} + \alpha(\mathbf{E} \times \mathbf{p}_x) \cdot \sigma + \frac{1}{2} g\mu_B \mathbf{B} \cdot \sigma$$



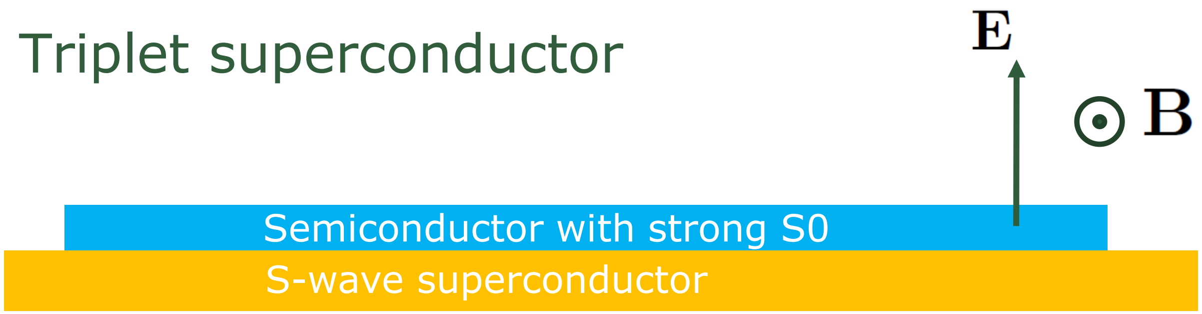
With B: can couple to s-wave superconductor

Bogoliubov-de Gennes

$$\mathcal{H} = \begin{pmatrix} \mathcal{H} & \Delta \\ \Delta^\dagger & -\mathcal{H}^T \end{pmatrix}$$

Triplet superconductor

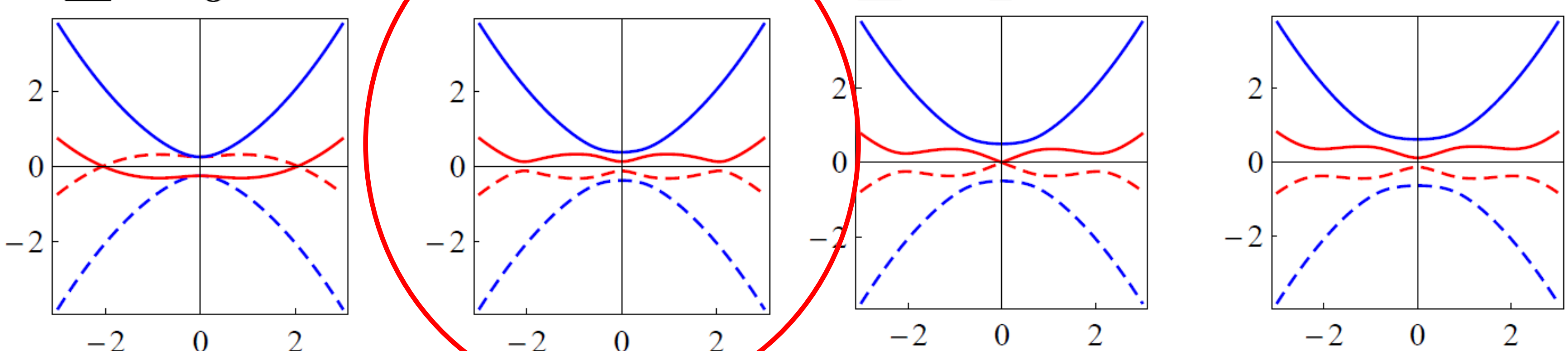
Lutchen et al. PRL 2010
Oreg et al. PRL 2010



$$H = \frac{1}{2}\sum_k \bar{c}_k^\dagger \begin{pmatrix} \xi_{k\uparrow} & B_\perp & \Delta & 0 \\ B_\perp & \xi_{k\downarrow} & 0 & \Delta \\ \Delta & 0 & -\xi_{k\downarrow} & B_\perp \\ 0 & \Delta & B_\perp & -\xi_{k\uparrow} \end{pmatrix} \bar{c}_k \qquad \bar{c}_k = \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \\ c_{k\downarrow}^\dagger \\ -c_{k\uparrow}^\dagger \end{pmatrix}$$

Pairing in semiconductor induced by proximity effect:

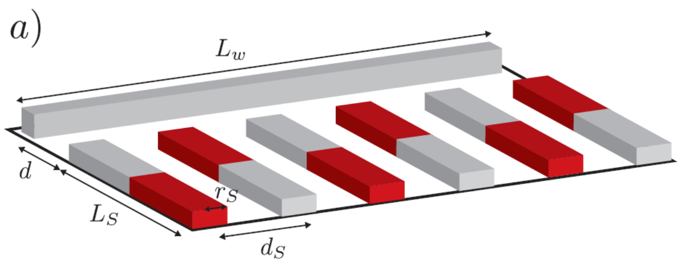
$\Delta = 0$ $\quad \Delta = 0.5B$ $\quad \Delta = B$ $\quad \Delta = 1.5B$

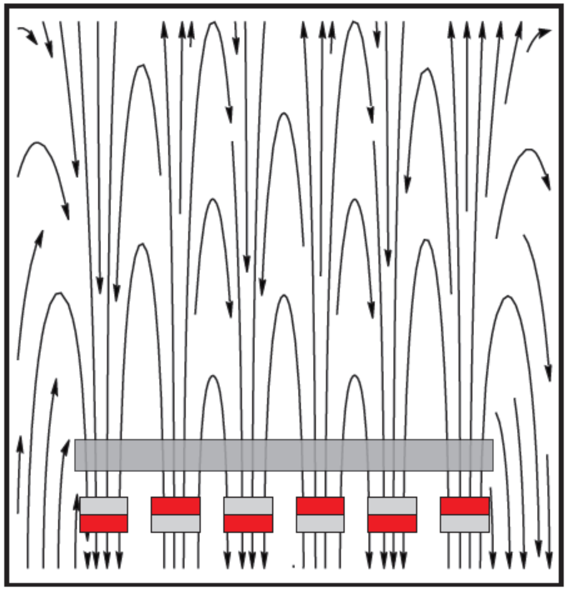


Boundstates at the ends

Alternative methods to make topologically non-trivial superconductor **without** spin-orbit coupling

Braunecker et al. , PRB (2010)
Choy et al., PRB (2011)
Martin, F. Morpurgo, PRB (2012)
Kupferschmidt, Brouwer, PRB (2011)
…





$$\mathbf{B} \approx B_0(\sin(\xi/R), 0, \cos(\xi/R))$$

$$H = \frac{p_\xi^2}{2m} + \frac{1}{2} g\mu_B \mathbf{B}(\xi) \cdot \boldsymbol{\sigma}$$

Rotate spin to align with **B**

$$U = \exp(i\sigma_y \xi/2R)$$

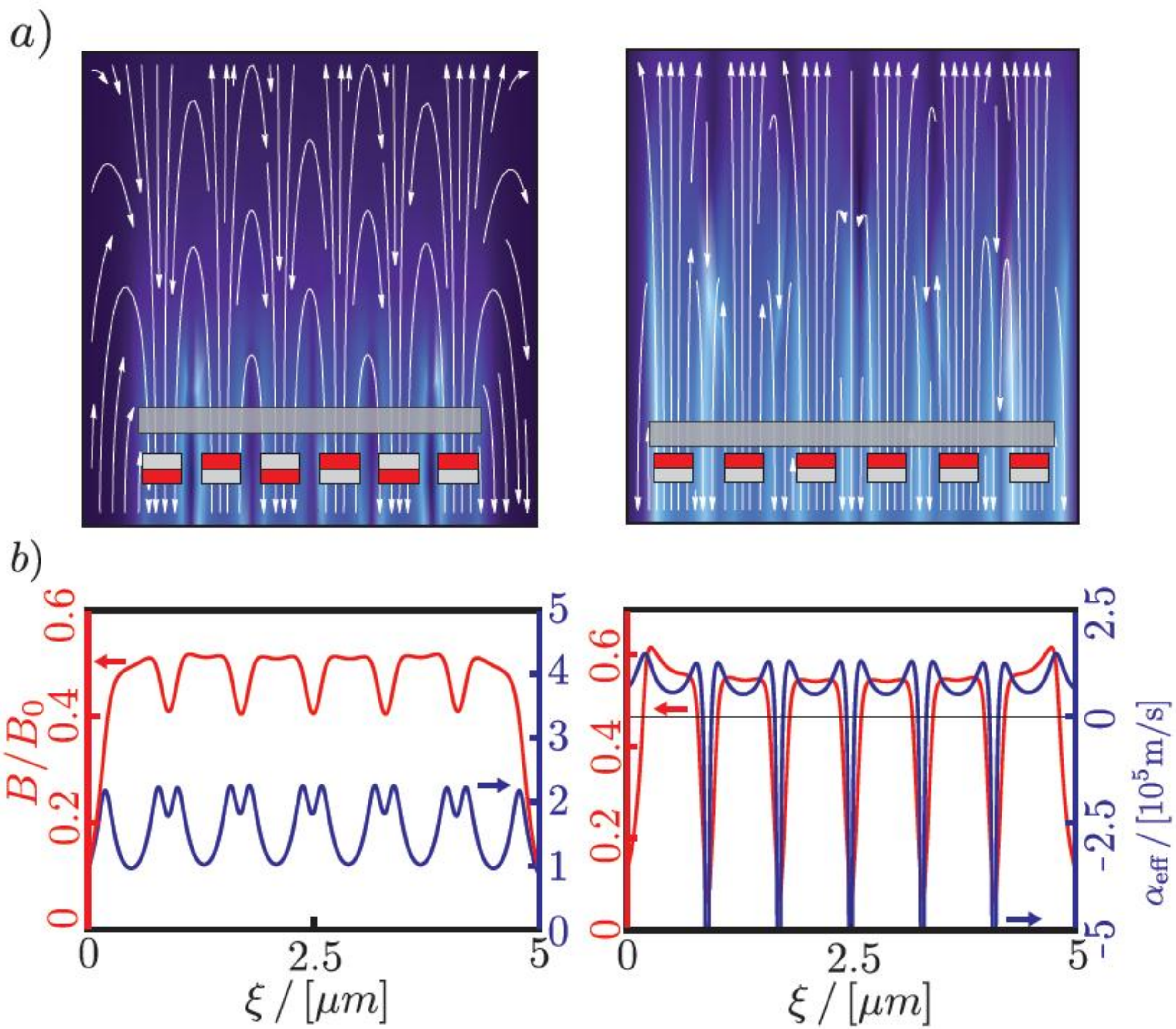
Gives spin-orbit coupling in local frame:

$$\tilde{H} = U^\dagger H U = \frac{p_\xi^2}{2m} + \frac{\hbar}{2mR}\sigma_2 p_\xi + \frac{\hbar^2}{8mR^2}$$

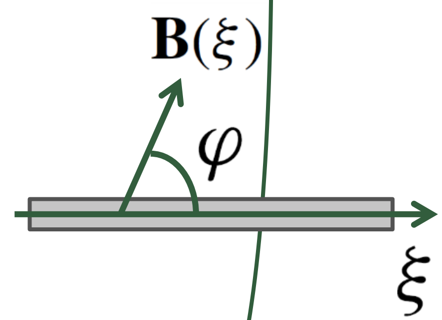
= spin-orbit interaction

Kjærgaard, Wölms, Flensberg, PRB 2012.

"Optimized" geometries: nanowire



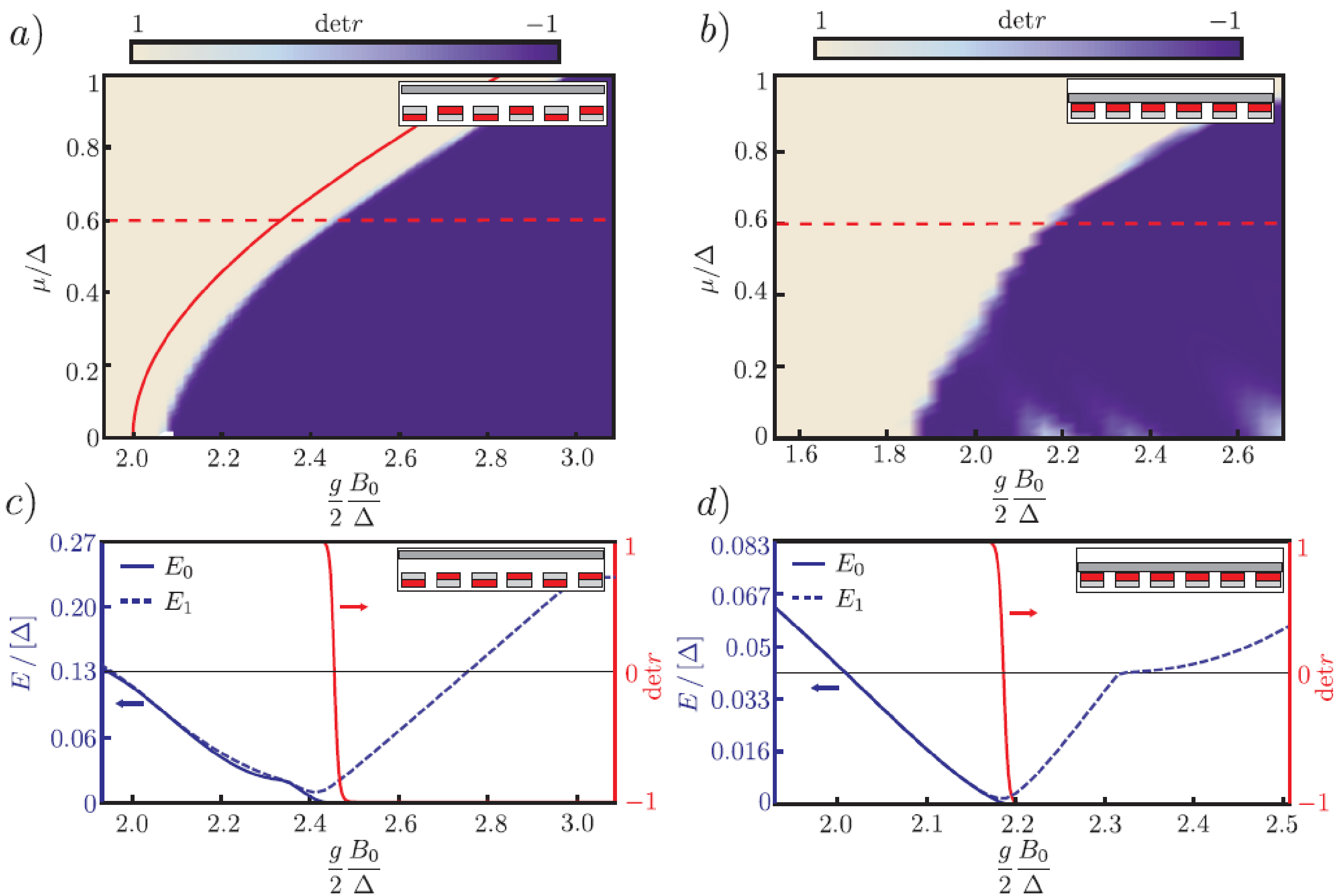
Comparable to strong intrinsic SOI



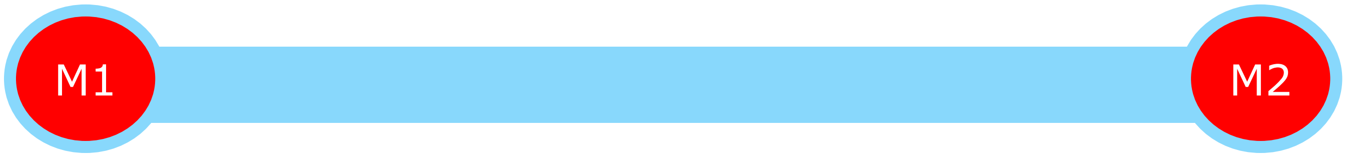
$$\alpha_{\mathrm{eff}} = \frac{\hbar}{2m}\frac{d\varphi}{d\xi}$$

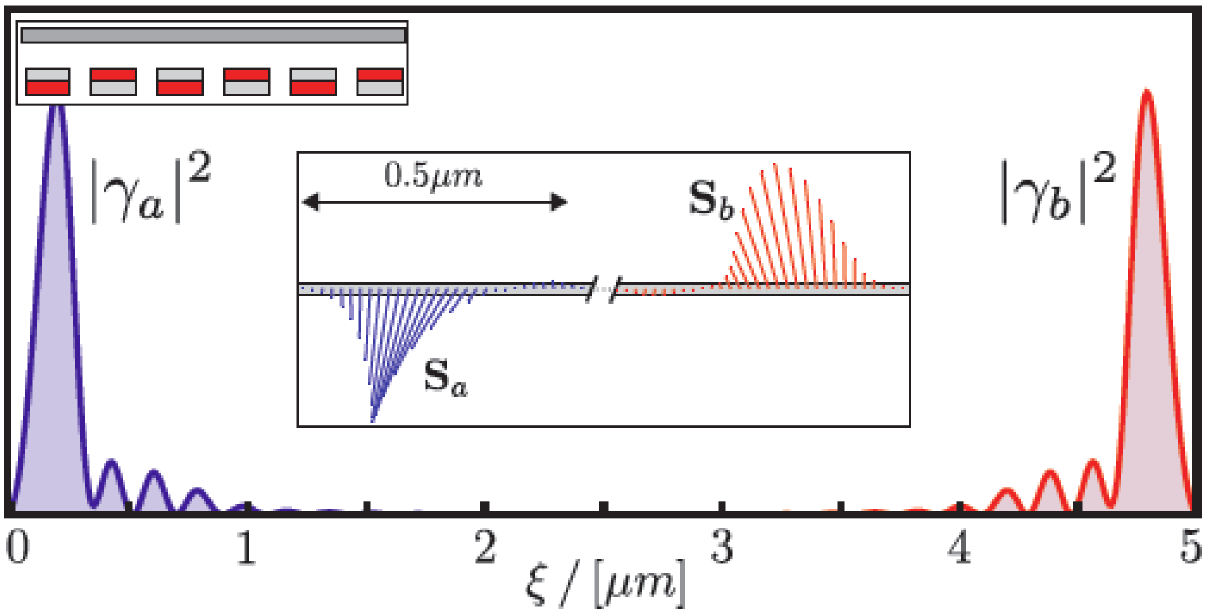
Akhmerov, Hassler, M. Wimmer, Beenakker (2011)

Topological quantum number



Majorana state with spin texture





$$\psi_M(\mathbf{r}) = \begin{pmatrix} f(\mathbf{r}) \\ g(\mathbf{r}) \\ g^*(\mathbf{r}) \\ -f^*(\mathbf{r}) \end{pmatrix}$$

= electron spin up

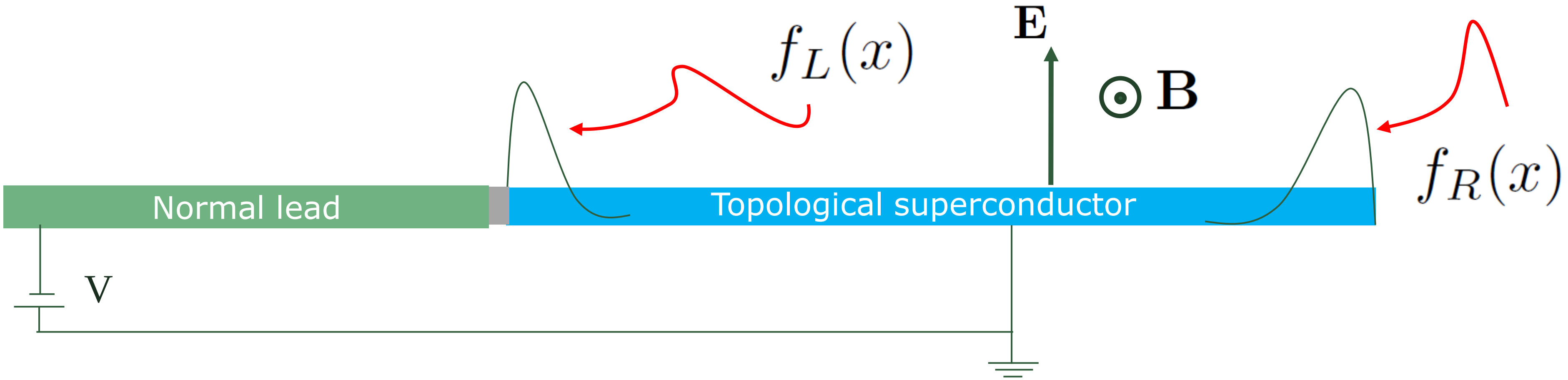
= electron spin down

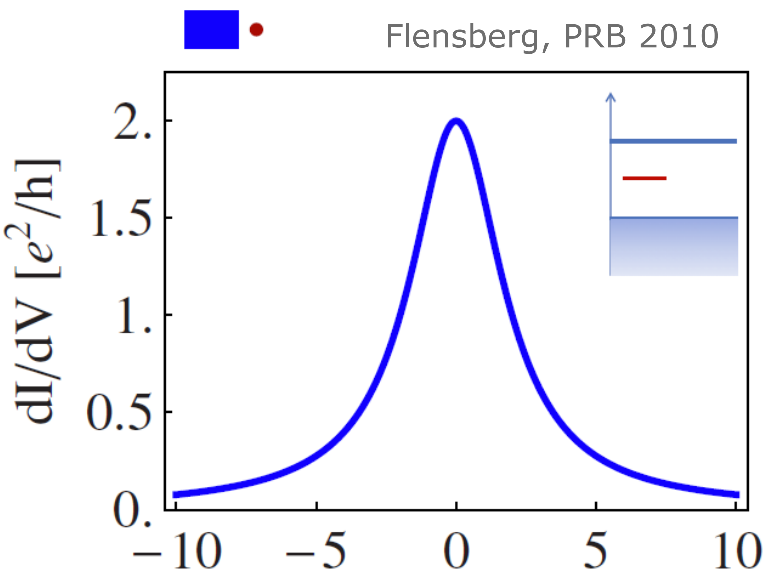
= hole spin down

= hole spin up

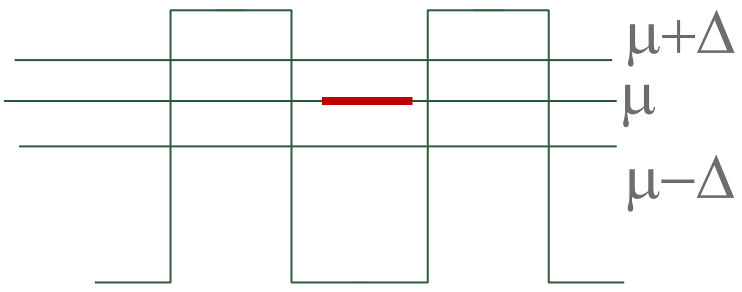
Kjærgaard, Wölms, Flensberg, PRB 212

How to detect Majorana end bound states





Andreev channel: resonant



Bolech and Demler, PRL 2007.
Law, Lee, and Ng, PRL 2009

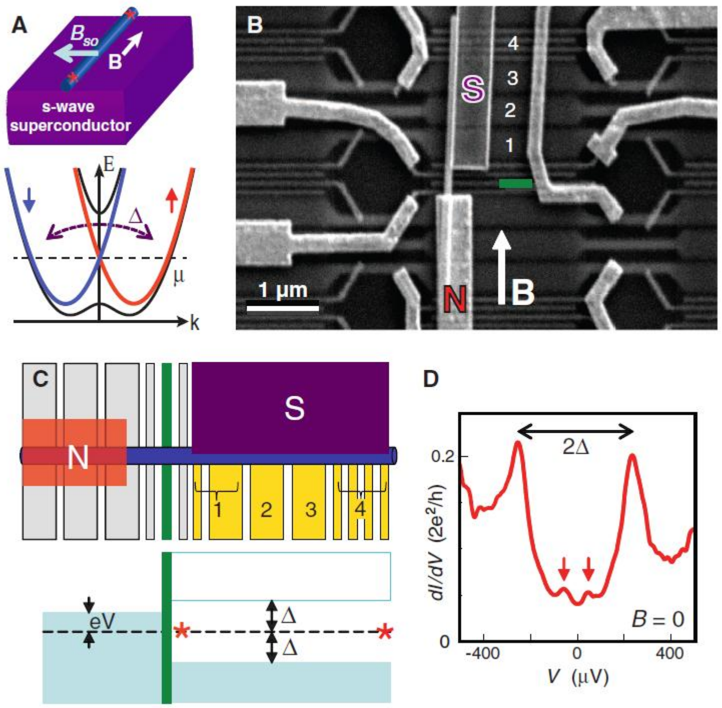
Experimental progress ?

Delft experiments

25 MAY 2012 VOL 336 SCIENCE www.sciencemag.org

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†



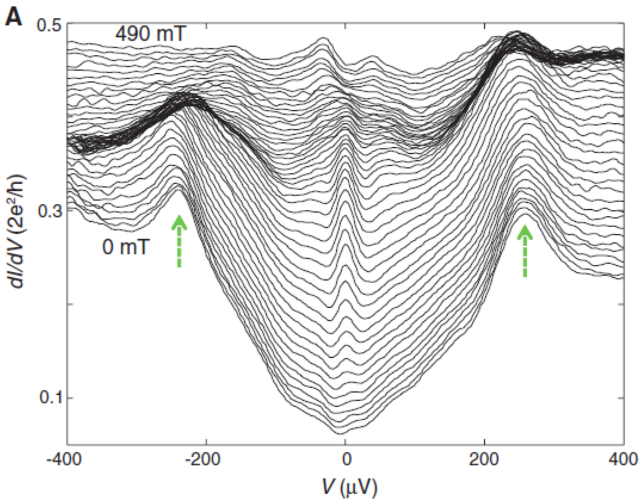


Fig. 2. Magnetic field–dependent spectroscopy. (**A**) *dI/dV* versus *V* at 70 mK taken at different *B* fields (from 0 to 490 mT in 10-mT steps; traces are offset for clarity, except for the lowest trace at *B* = 0). Data are from device 1. Arrows indicate the induced gap peaks. (**B**) Color-scale plot of *dI/dV* versus *V*

Weizmann experiments

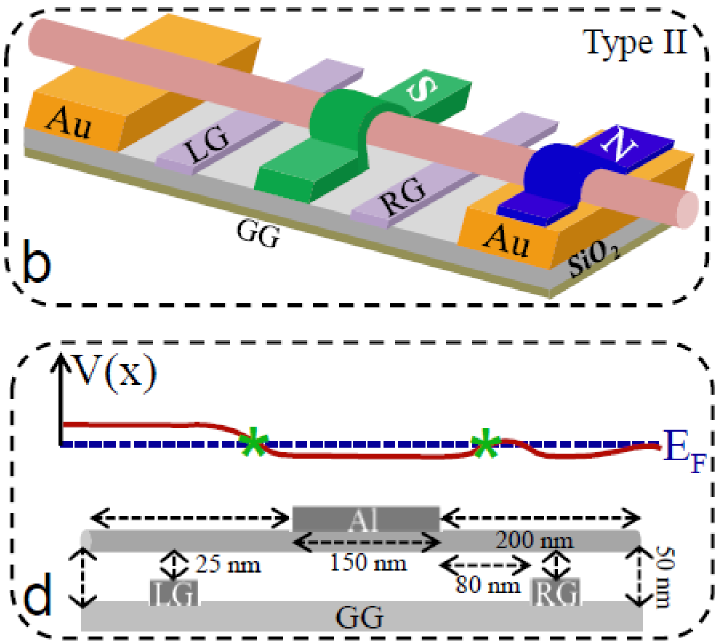
Evidence of Majorana fermions in an

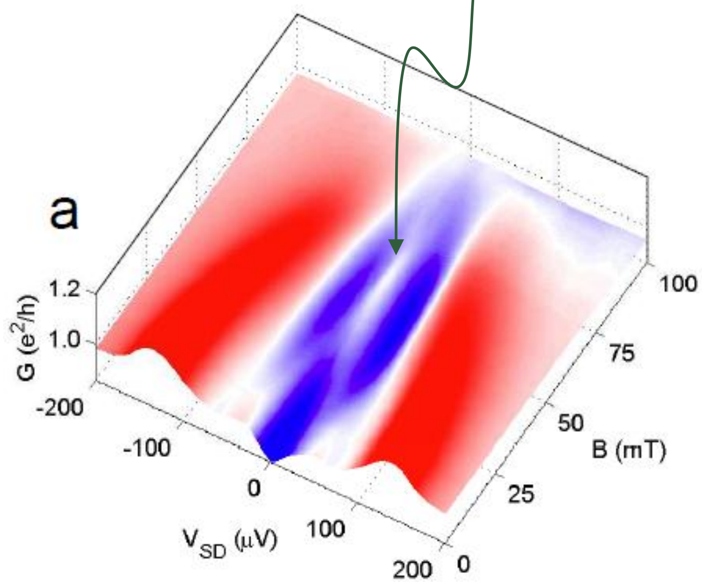
Al – InAs nanowire topological superconductor

Anindya Das*, Yuval Ronen*, Yonatan Most, Yuval Oreg, Moty Heiblum#, and Hadas Shtrikman

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Majorana peak ?





While we are weating for the real thing ...

poor man's Majorana ...
(Leijnse & Flensberg, arXiv: arXiv:1207.4299)

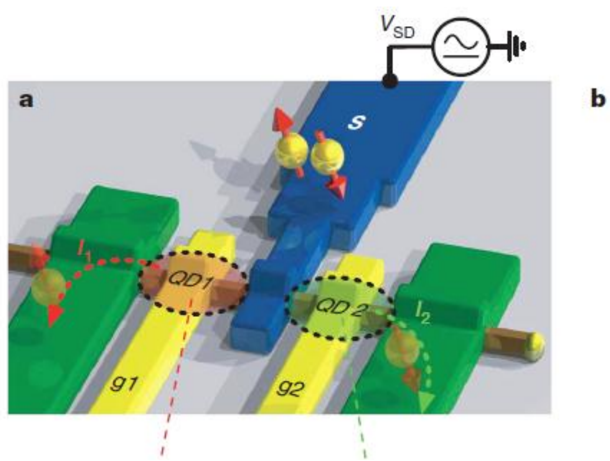
Geometry similar to:

nature Vol 461|15 October 2009|doi:10.1038/nature08432

LETTERS

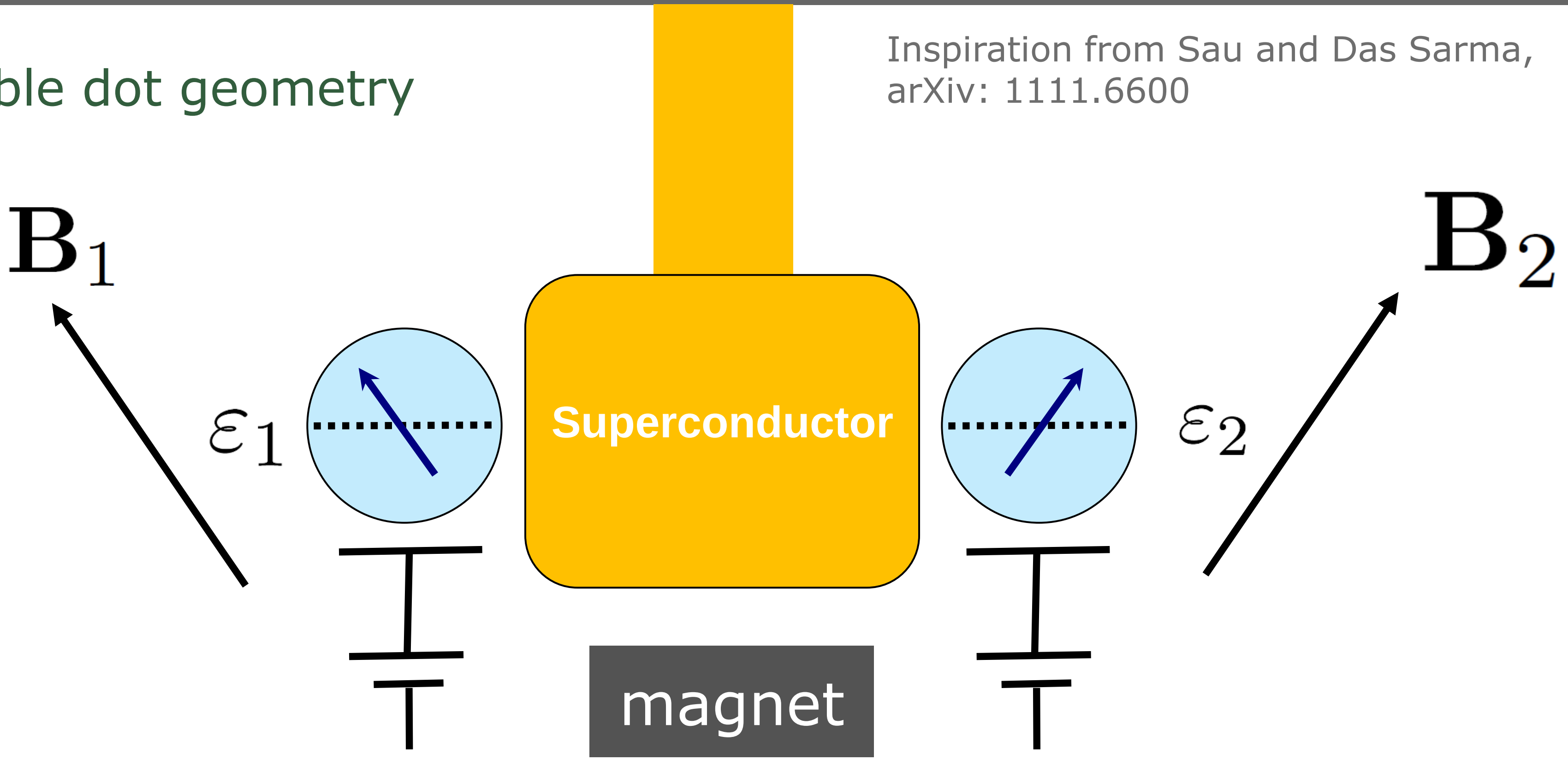
Cooper pair splitter realized in a two-quantum-dot Y-junction

L. Hofstetter[1]*, S. Csonka[1,2]*, J. Nygård[3] & C. Schönenberger[1]



Double dot geometry

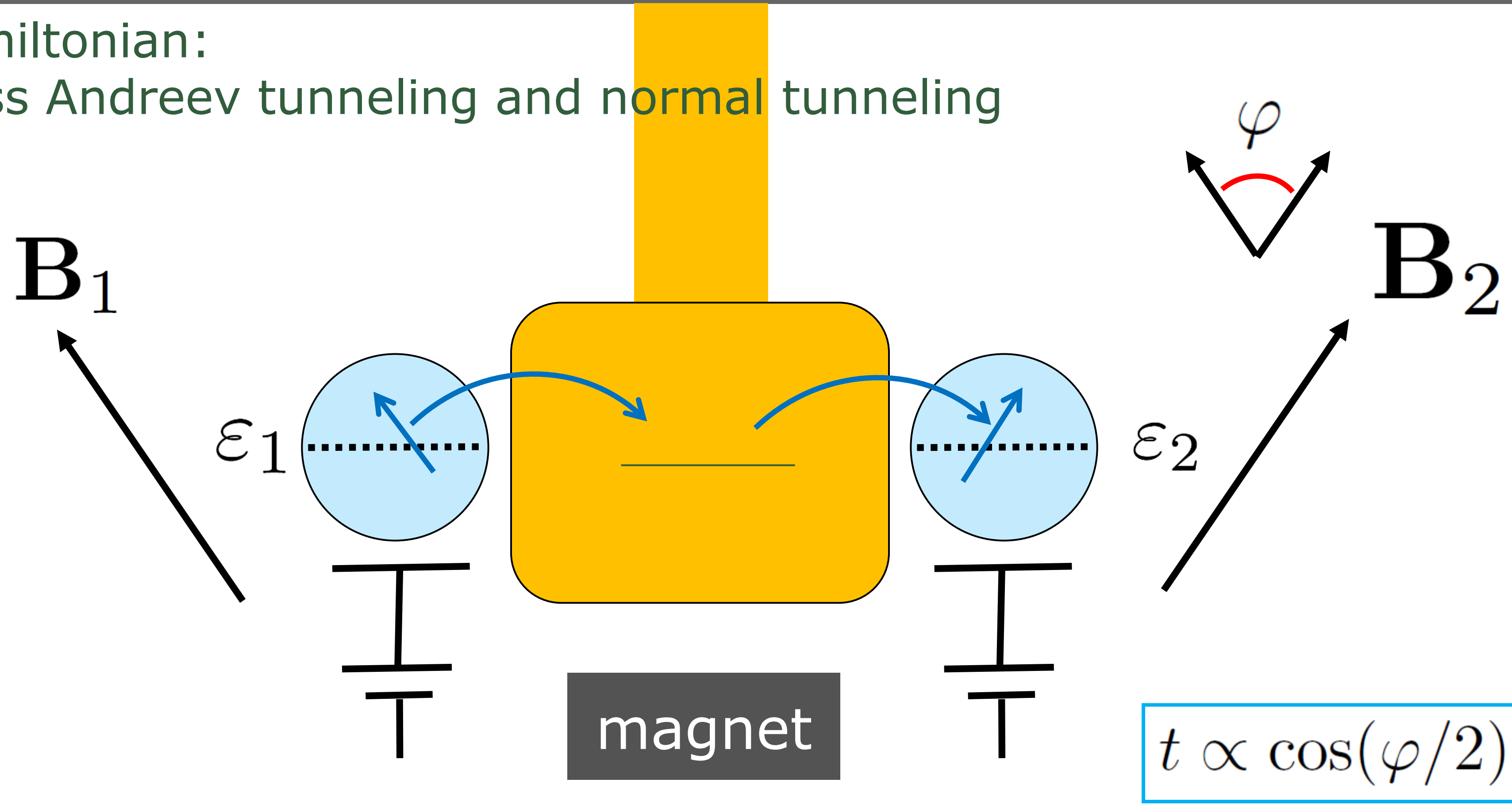
Inspiration from Sau and Das Sarma, arXiv: 1111.6600



- Two quantum dots with tunable onsite energy
- Strong non-collinear magnetic field (Spin-orbit not needed !)
- Cross Andreev reflection (This is a very bad Cooper-pair splitter)

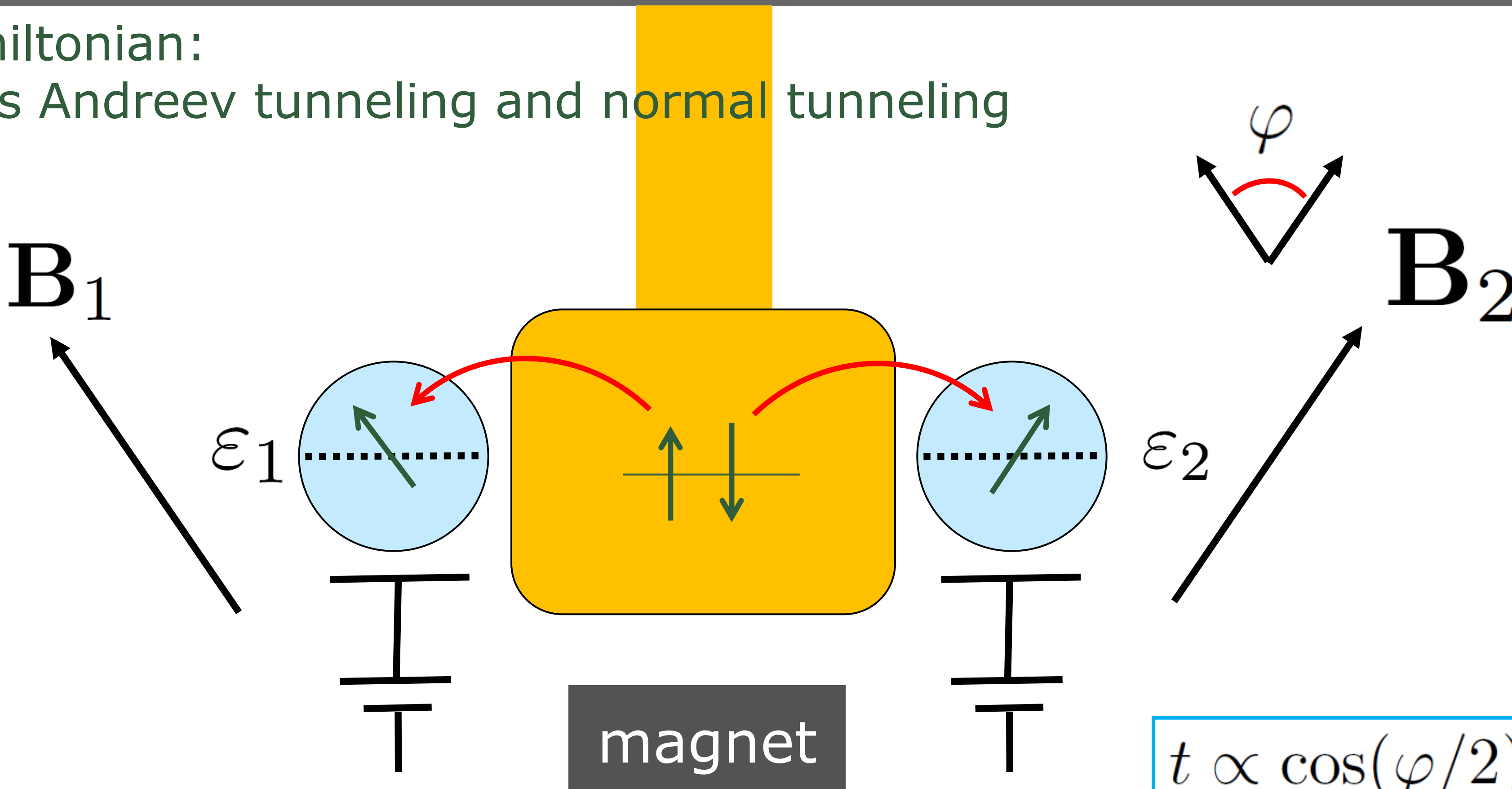
Summary:
- Majorana modes localized to the dots
- Somewhat robust
- Easy tuning of normal and Andreev tunneling by angle between $\mathbf{B}_1$ and $\mathbf{B}_2$
- Allows testing of properties of a parity qubit

Hamiltonian:
cross Andreev tunneling and normal tunneling



$$H = \varepsilon_1 d_1^\dagger d_1 + \varepsilon_2 d_2^\dagger d_2 + t\left(d_1^\dagger d_2 + \mathrm{h.c.}\right)$$

Hamiltonian: cross Andreev tunneling and normal tunneling



$\boxed{t \propto \cos(\varphi/2)}$

$\boxed{\Delta \propto \sin(\varphi/2)}$

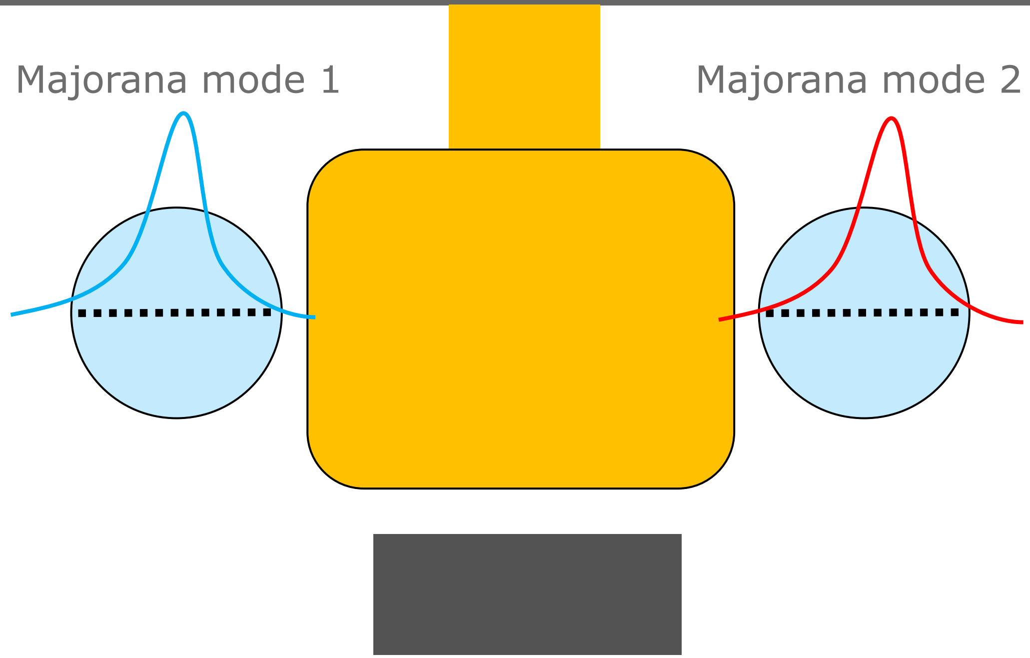
Control ratio : $\Delta/t \propto \tan(\varphi/2)$

$$H = \varepsilon_1 d_1^\dagger d_1 + \varepsilon_2 d_2^\dagger d_2 + t\left(d_1^\dagger d_2 + \text{h.c.}\right) + \Delta\left(d_1^\dagger d_2^\dagger + \text{h.c.}\right)$$

BdG equations

$$H = \frac{1}{2}\Psi^\dagger h \Psi \qquad \Psi = \begin{pmatrix} d_1 \\ d_2 \\ d_1^\dagger \\ d_2^\dagger \end{pmatrix}$$

$$h = \begin{pmatrix} \epsilon_1 & t & 0 & \Delta \\ t & \epsilon_2 & -\Delta & 0 \\ 0 & -\Delta & -\epsilon_1 & -t \\ \Delta & 0 & -t & -\epsilon_2 \end{pmatrix}$$



sweet spot: $\epsilon_1 = \epsilon_2 = 0, \quad t/\Delta = 1$

Zero energy solutions: $\psi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \psi_2 = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

Operators:

$$\gamma_1 = \psi_1 \cdot \Psi = \frac{1}{\sqrt{2}}\left(d_1^\dagger + d_1\right)$$

$$\gamma_2 = \psi_2 \cdot \Psi = \frac{i}{\sqrt{2}}\left(d_2^\dagger - d_2\right)$$

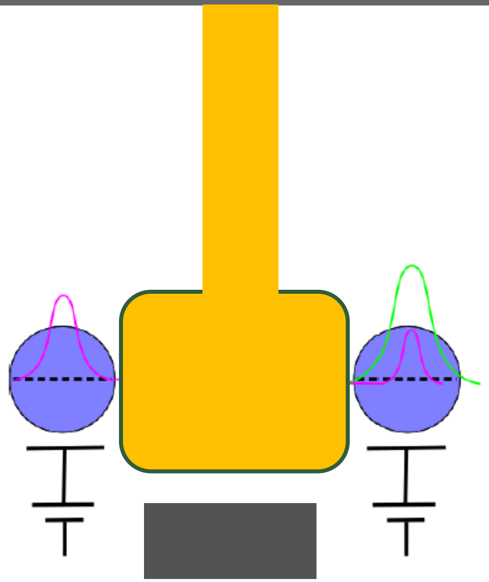
- Non-overlapping Majorana Fermions,
- Localized to each dot

Quadratic “Protection”

$\epsilon_1 \neq 0,\ \epsilon_2 = 0$

$$\Rightarrow \begin{cases} \gamma_1 = \frac{1}{N}\left(d_1^\dagger + d_1 - \frac{\epsilon_1}{2t} d_2 - \frac{\epsilon_1}{2t} d_2^\dagger\right) \\ \gamma_2 = \frac{i}{\sqrt{2}}\left(d_2^\dagger - d_2\right) \end{cases}$$

Remain at zero energy!



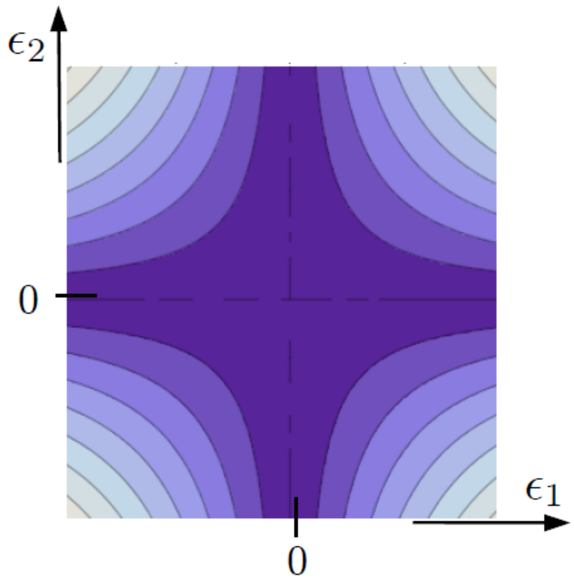
$\epsilon_1 \neq 0,\ \epsilon_2 \neq 0$

➡ *Quadratic* splitting

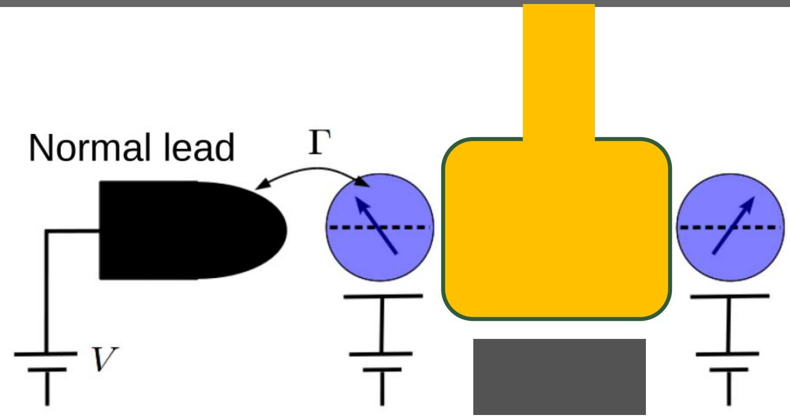
$t \neq \Delta$ or $U_{12} \neq 0$

➡ *Linear* splitting

(Can be fixed by adjusting ϵ_1, ϵ_2)



Transport properties

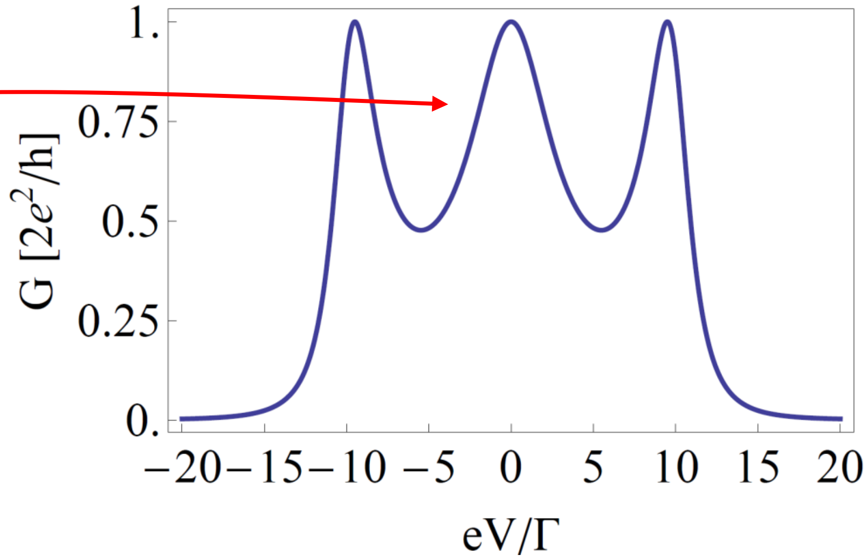


$$I = \frac{2e}{h}\int_{-\infty}^{\infty} d\omega |a(\omega)|^2 [f(\omega - eV) - f(\omega + eV)]$$

$$S = \begin{pmatrix} r & a \\ a^* & r^* \end{pmatrix} = 1 + 2i\pi W[h - \omega + i\pi W^\dagger W]^{-1} W^\dagger$$

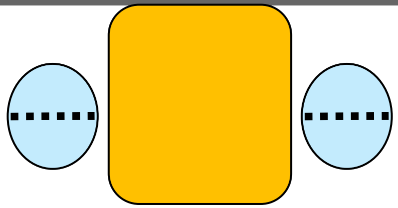
$$W = \sqrt{\frac{\Gamma}{2\pi}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Sweet spot and **t= Δ=5**Γ



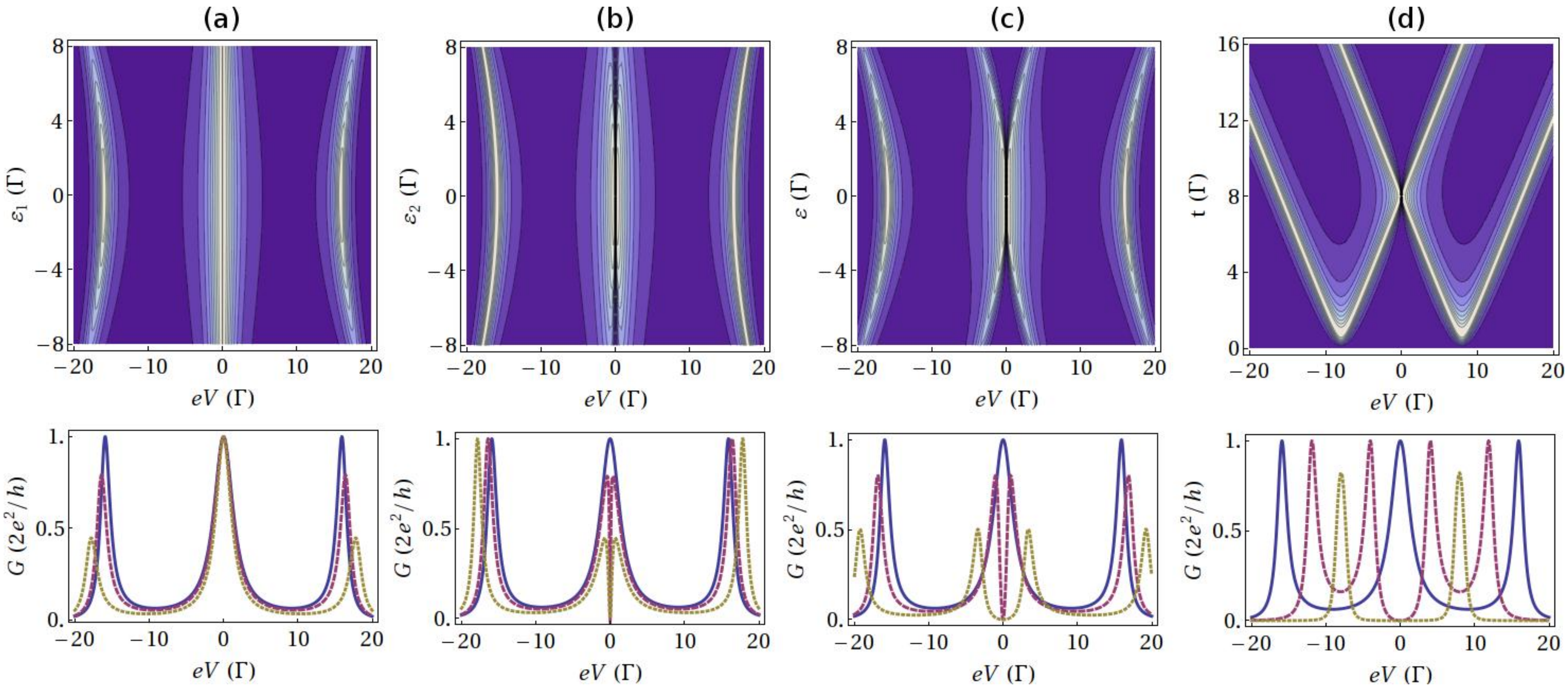
Resonant Andreev tunneling peak !

Tuning ε_1, ε_2 and t via transport spectroscopy

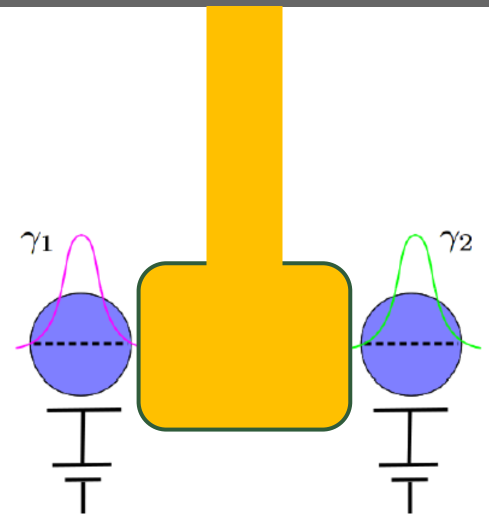


$$t = \Delta = 8\Gamma$$

$\varepsilon_2 = 0$, Changing ε_1	$\varepsilon_1 = 0$, Changing ε_2	$\varepsilon_1 = \varepsilon_2 = \varepsilon$	$\varepsilon_1 = \varepsilon_2 = 0$, Changing t



Many-body formulation:



Number states: $|n_1 n_2\rangle,\ n_i = d_i^\dagger d_i$

In basis: $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$

$$H = \begin{pmatrix} 0 & 0 & 0 & \Delta \\ 0 & \epsilon_1 & t & 0 \\ 0 & t & \epsilon_2 & 0 \\ \Delta & 0 & 0 & \epsilon_1 + \epsilon_2 + U_{12} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 0 & 0 & t \\ 0 & 0 & t & 0 \\ 0 & t & 0 & 0 \\ t & 0 & 0 & U_{12} \end{pmatrix}$$

At *sweet spot*: degenerate g.s. with even/odd particle number

$$\begin{cases} |e\rangle = \dfrac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) \\ |o\rangle = \dfrac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right) \end{cases}$$

Non-local fermion: $f = \frac{1}{2}\left(\gamma_1 - i\gamma_2\right),\ n_f = f^\dagger f$

➡ $n_f|e\rangle = 0,\ n_f|o\rangle = |o\rangle$

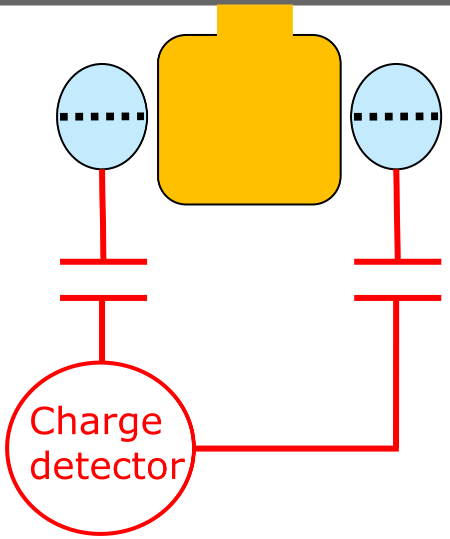
And: $\gamma_1|e\rangle = |o\rangle$ etc

Majorana degeneracy in number state basis

Note

Interactions: $U_{12} \neq 0$ remove the quadratic protection

Non-locality



Parity qubit = $\begin{cases} |0\rangle \equiv |e\rangle = \dfrac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right), \quad (n_f = 0) \\ |1\rangle \equiv |o\rangle = \dfrac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right), \quad (n_f = 1) \end{cases}$

Non-local because one cannot determine qubit state by measuring charge on a single dot $\langle o|n_1|o\rangle = \langle e|n_1|e\rangle = \frac{1}{2}$

Non-local measurement can determine qubit state:

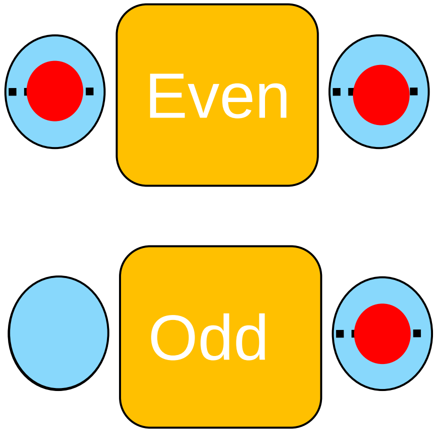
$$\langle o|(n_1 + n_2)^2|o\rangle = 1$$

Different fluctuations!

$$\langle e|(n_1 + n_2)^2|e\rangle = 2$$

Fluctuations can be measured with a non-linear charge detector

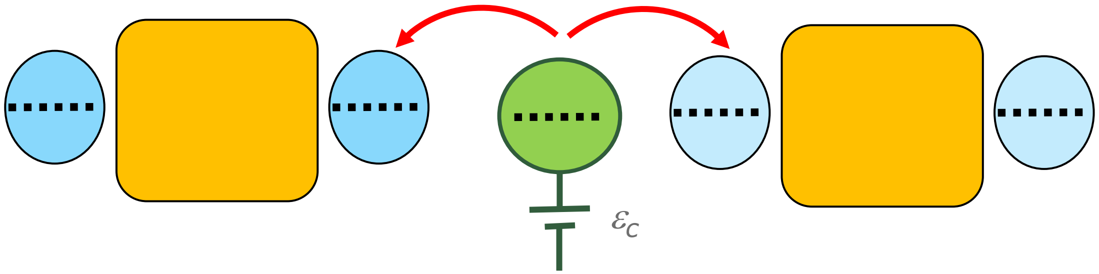
Parity qubit



$$|e\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|o\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right)$$

Parity qubit dephasing measurement

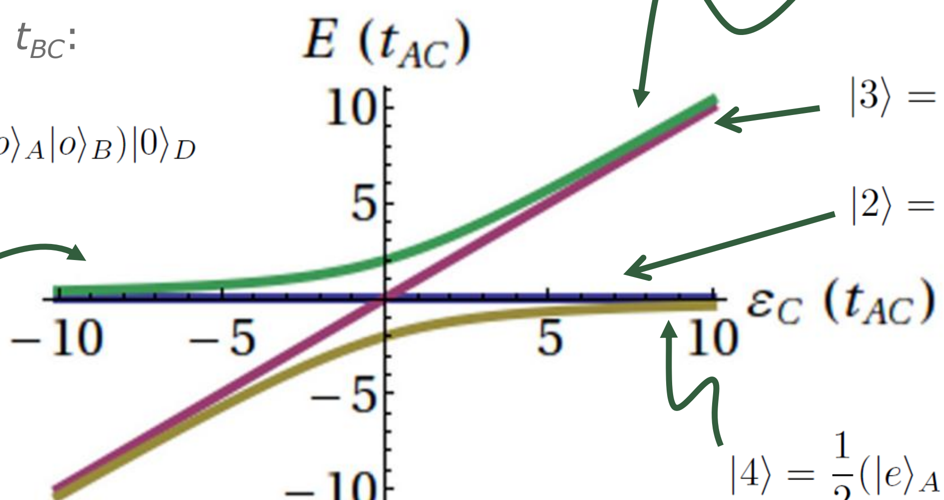


Eigenstates for 8x8

Even sector and $t_{AC} = t_{BC}$:

$$|1\rangle = \frac{1}{2}(|e\rangle_A|e\rangle_B + |o\rangle_A|o\rangle_B)|0\rangle_D$$

$$|1\rangle = \frac{1}{2}(|e\rangle_A|o\rangle_B + |o\rangle_A|e\rangle_B)|1\rangle_D$$



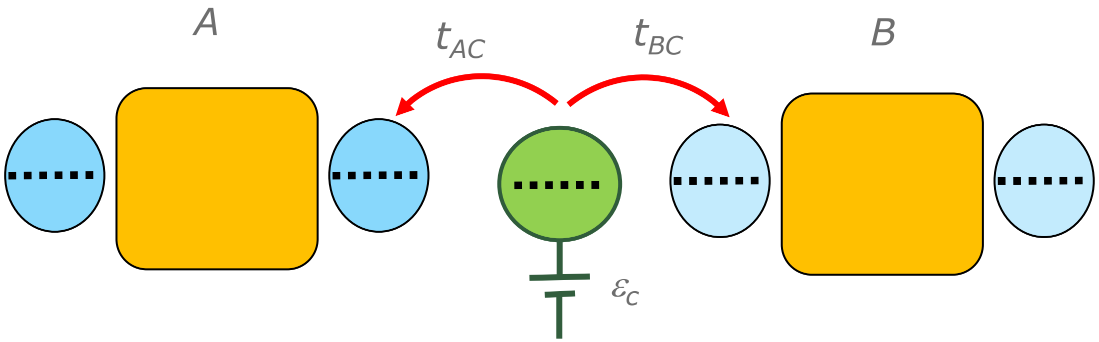
$$|3\rangle = \frac{1}{2}(|e\rangle_A|o\rangle_B - |o\rangle_A|e\rangle_B)|1\rangle_D$$

$$|2\rangle = \frac{1}{2}(|e\rangle_A|e\rangle_B - |o\rangle_A|o\rangle_B)|0\rangle_D$$

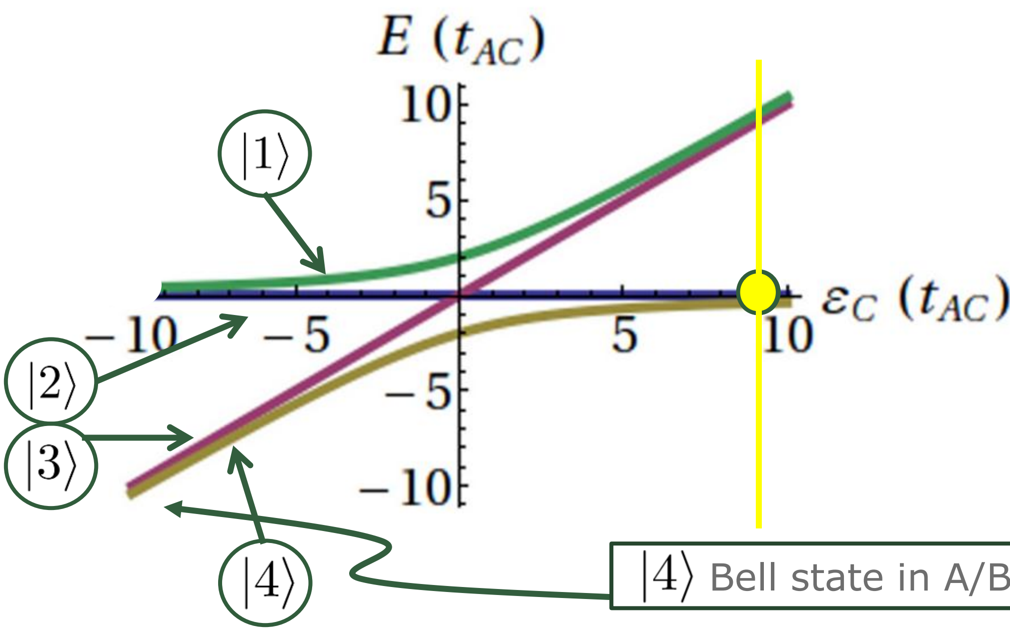
$$|4\rangle = \frac{1}{2}(|e\rangle_A|e\rangle_B + |o\rangle_A|o\rangle_B)|0\rangle_D$$

$$|4\rangle = \frac{1}{2}(|e\rangle_A|o\rangle_B + |o\rangle_A|e\rangle_B)|1\rangle_D$$

Parity qubit dephasing measurement



Always full dot: $|3\rangle$. Always empty dot: $|2\rangle$



$|4\rangle$ Bell state in A/B and e/o

Sequence:

- Relax system at large ε_c to:

  $$|i\rangle = |e\rangle_A |e\rangle_B |0\rangle_D = |2\rangle + |4\rangle$$

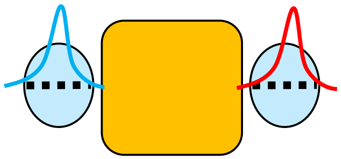
- Move to negative ε_c
- Dot charge "collapses"

  Empty: $|e\rangle_A|e\rangle_B - |o\rangle_A|o\rangle_B$

  Full: $|e\rangle_A|o\rangle_B + |o\rangle_A|e\rangle_B$

- Move back to positive ε_c :
  No dephasing: full with P = 1
  Dephased: full with P=1/2

Summary poor man's Majorana



- Localized Majorana modes
- Quadractically protected (onsite energy)
- Resonant Andreev tunneling can be tested
- Dephasing and lifetime of parity qubit can be tested

BACK TO RICH MAN'S MAJORANAS

Ways to manipulate the groundstate manifold of the Majorana bound states

Real space exchange:

ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NPHYS1915
nature physics

Non-Abelian statistics and topological quantum information processing in 1D wire networks

Jason Alicea[1]*, Yuval Oreg[2], Gil Refael[3], Felix von Oppen[4] and Matthew P. A. Fisher[3,5]

ARTICLES NATURE PHYSICS DOI: 10.1038/NPHYS1915

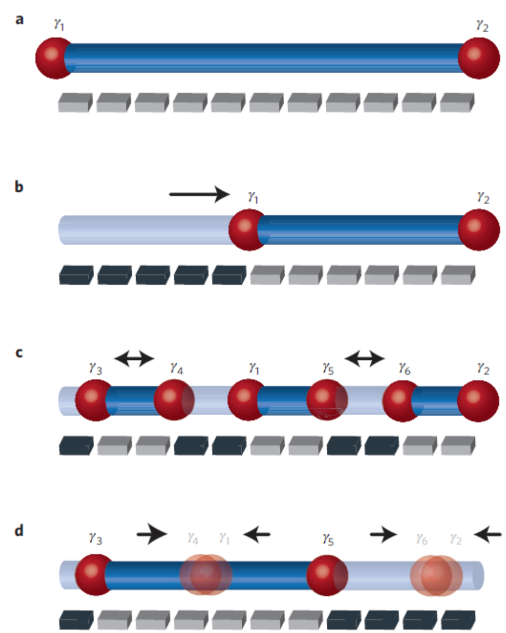


Figure 2 | Applying a 'keyboard' of individually tunable gates to the wire allows local control of which regions are topological (dark blue) and non-topological (light blue), and hence manipulate Majorana fermions while maintaining the bulk gap. As **a** and **b** illustrate, sequentially applying the leftmost gates drives the left end of the wire non-topological, thereby transporting γ_1 rightward. Nucleating a topological section of the wire from an ordinary region or vice versa creates pairs of Majorana fermions out of the vacuum as in **c**. Similarly, removing a topological region entirely or connecting two topological regions as sketched in **d** fuses Majorana fermions into either the vacuum or a finite-energy quasiparticle.

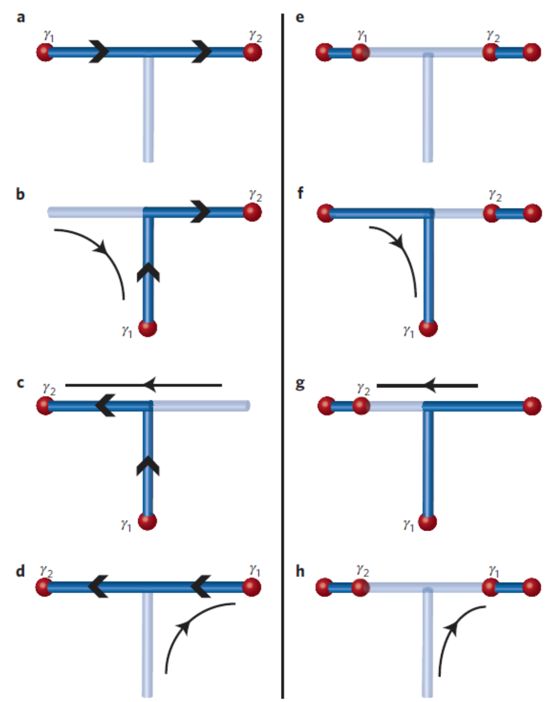


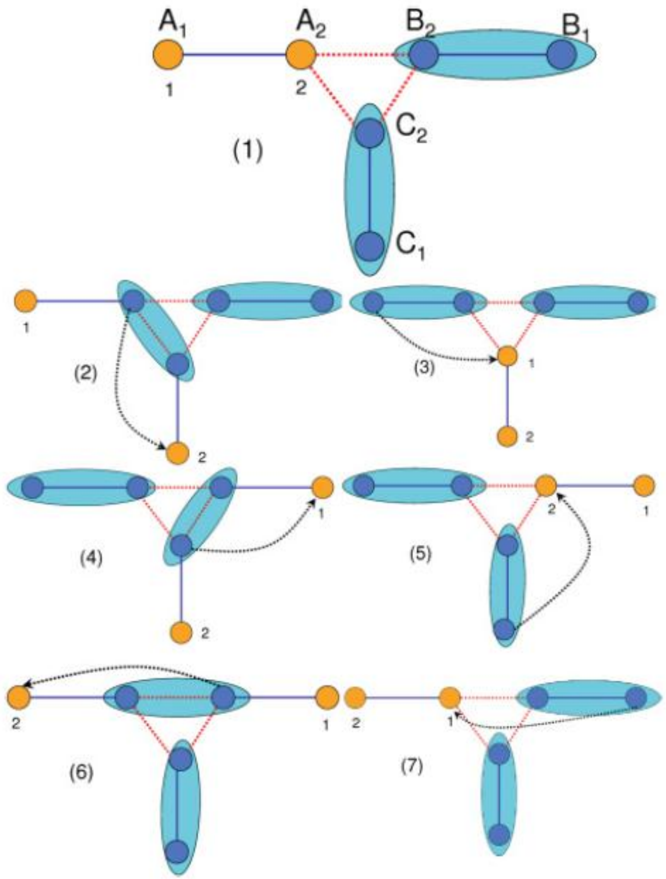
Figure 3 | A T-junction provides the simplest wire network that enables meaningful adiabatic exchange of Majorana fermions. Using the methods of Fig. 2, one can braid Majoranas bridged by either a topological region (dark blue lines) as in **a–d**, or a non-topological region (light blue lines) as in **e–h**. The arrows along the topological regions in **a–d** are useful for understanding the non-Abelian statistics, as outlined in the main text.

Braid by changing tunnel coupling between MBS:

PHYSICAL REVIEW B **84**, 094505 (2011)

Controlling non-Abelian statistics of Majorana fermions in semiconductor nanowires

Jay D. Sau,[1] David J. Clarke,[2] and Sumanta Tewari[3]



Combining MBS and Josephson junctions:

New Journal of Physics
The open-access journal for physics

Coulomb-assisted braiding of Majorana fermions in a Josephson junction array

B van Heck[1,3], A R Akhmerov[1], F Hassler[2], M Burrello[1]
and C W J Beenakker[1]

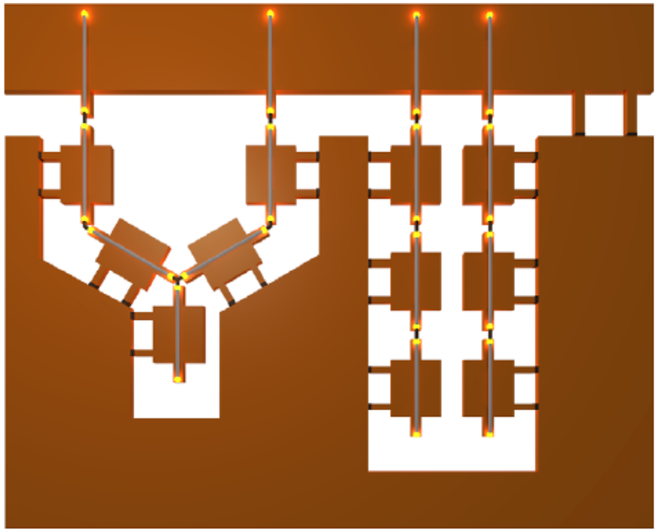


Figure 5. A Josephson junction array containing Majorana fermions. The magnetic flux through a split Josephson junction controls the Coulomb coupling on each superconducting island. This device allows one to perform the three types of operations on topological qubits needed for a universal quantum computer: readout, rotation and braiding. All operations are controlled magnetically; no gate voltages are needed.

Using quantum dots:

PRL 106, 090503 (2011) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2011

Non-Abelian Operations on Majorana Fermions via Single-Charge Control

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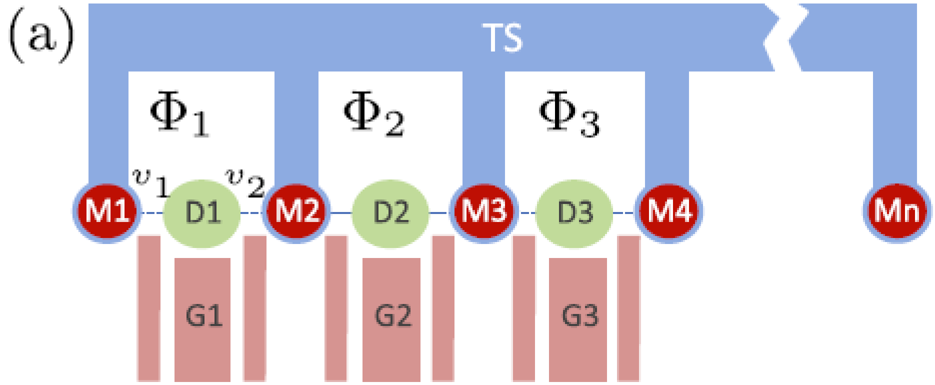
(Received 24 November 2010; published 2 March 2011)

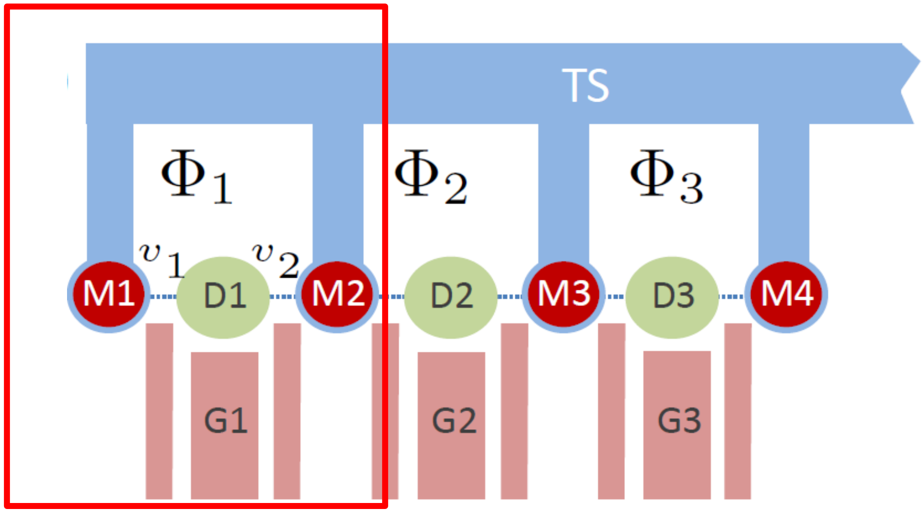


By changing the charge on a dot by one electron:

$$P_{12} : |i\rangle \mapsto \left(|v_1|\gamma_1 + |v_2|\gamma_2\right)|i\rangle$$

Coupling to two Majorana bound states

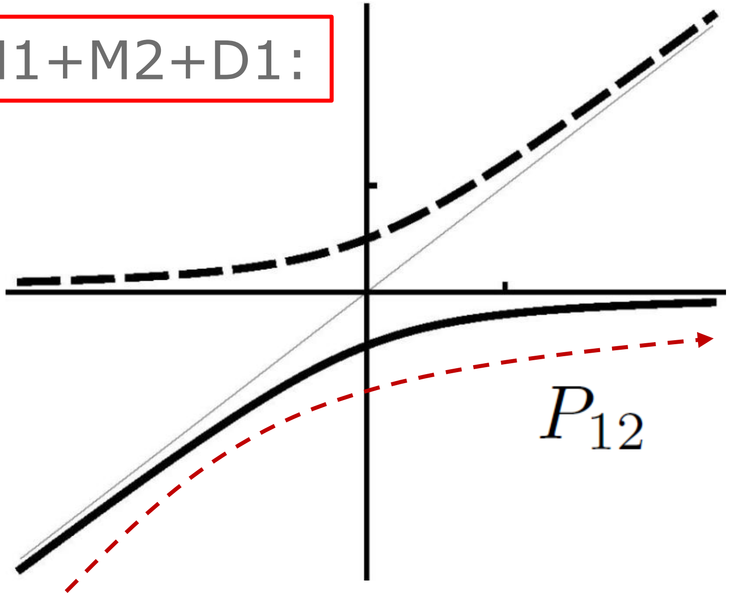
Flensberg, PRL (2011)



$$\gamma_{12} = \frac{1}{\sqrt{|v_1|^2 + |v_2|^2}}\left(|v_1|\gamma_1 + |v_2|\gamma_2\right)$$

$$P_{12}: \quad |\mathrm{i}\rangle \mapsto \gamma_{12}|i\rangle$$

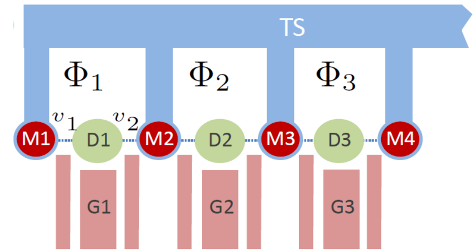
M1+M2+D1:

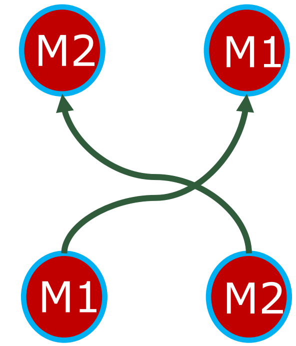


Requires:
-Constant tunneling amplitudes
-Constant flux

But no dependence on timing

Compare to braiding





$$\gamma_{12} = \frac{1}{\sqrt{|v_1|^2 + |v_2|^2}} \left(|v_1|\gamma_1 + |v_2|\gamma_2 \right)$$

With $v_1 = v_2$:

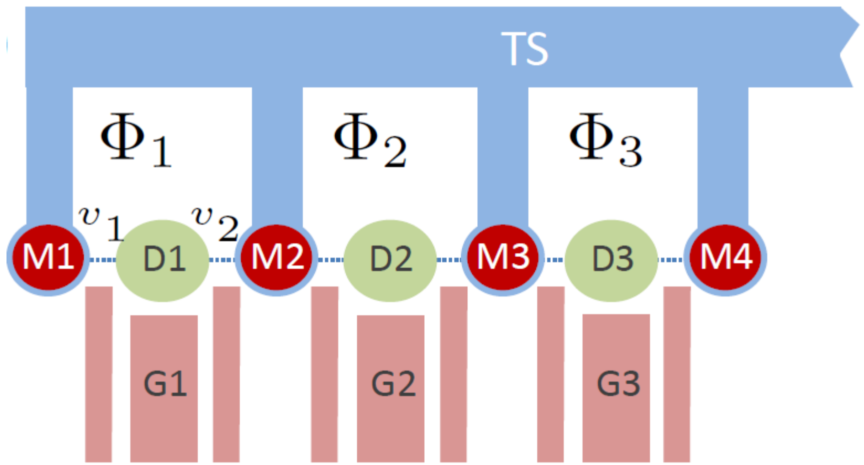
$$P_{i,i+1} \Rightarrow F_i = \frac{1}{\sqrt{2}} (\gamma_i + \gamma_{i+1})$$

$$B_i = \frac{1}{\sqrt{2}}(1 + \gamma_{i+1}\gamma_i)$$

$$B_i = F_i \gamma_i = \gamma_{i+1} F_i$$

”Tunnel braid” can mimic real space braiding, but also more

Demonstration of non-Abelian operations



Two fermions:

$$d_1 = (\gamma_1 + i\gamma_2)/2$$

$$d_2 = (\gamma_3 + i\gamma_4)/2$$

Basis: $|n_1 n_2\rangle$

$$F_i = \frac{1}{\sqrt{2}}(\gamma_i + \gamma_{j+1})$$

Initialize: $|00\rangle$

$$F_1 F_2 F_2 F_3 |00\rangle = |11\rangle$$

$$F_2 F_1 F_3 F_2 |00\rangle = |00\rangle$$

Read out!

Outlook

Majorana fermions exist in system that combine

- Ordinary s-wave superconductors
- Semi-conductors with few channels (low density!)
- Strong spin-orbit coupling, or spatially varying B-field

Once we have such systems, we can start to investigate:

- Perfect Andreev reflection?
- Non-Abelian nature of Majorana quasiparticles?
- What is the phase coherence of the parity degree of freedom?
- Can these systems be used for topological quantum computing? (in hybrid structures!)
- ?

Thank you!