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Energy relaxation and thermalization of hot electrons in quantum wires

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Nature Phys. **6**, 489 ('10)

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arXiv:1007.1152

Energy relaxation

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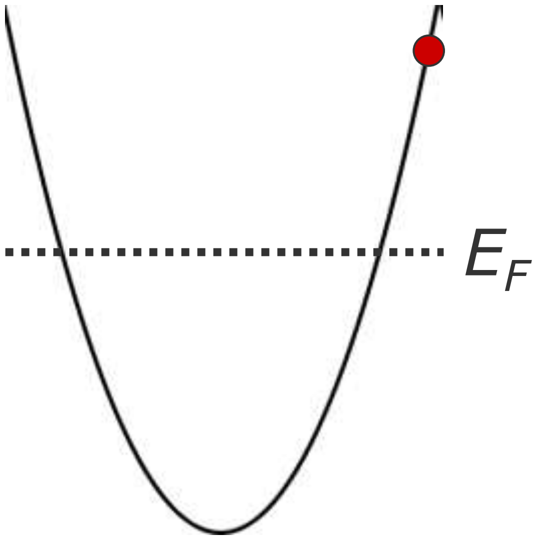
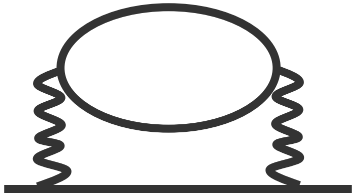
Basic question:

inject hot particle
into one-dimensional
electron liquid



higher dimensions:



$$\boxed{\frac{1}{\tau_p} \sim \varepsilon^2}$$

1d electron systems

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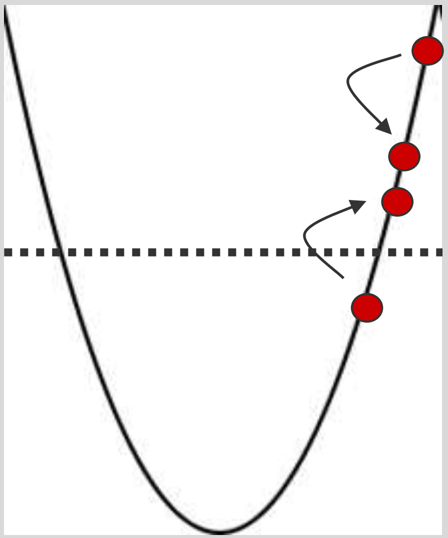
Perturbative approach:



E_F

Momentum & energy conservation

$$k_1 + k_2 = k_3 + k_4$$

$$k_1^2 + k_2^2 = k_3^2 + k_4^2$$

only solutions:

- $k_1 = k_3 \;;\; k_2 = k_4$
- $k_1 = k_4 \;;\; k_2 = k_3$

No relaxation by excitation of p-h pair

1d electron systems

Tomonaga-Luttinger model:

$$H = \frac{\hbar c}{2\pi} \int dx \left\{ \frac{1}{K} (\nabla\phi)^2 + K (\nabla\theta)^2 \right\}$$

- purely quadratic Hamiltonian
- can be mapped to free fermions by rescaling fields

No inelastic processes due to linear dispersion

Outline

- Experiment

  Nature Phys. **6**, 489 (‘10)

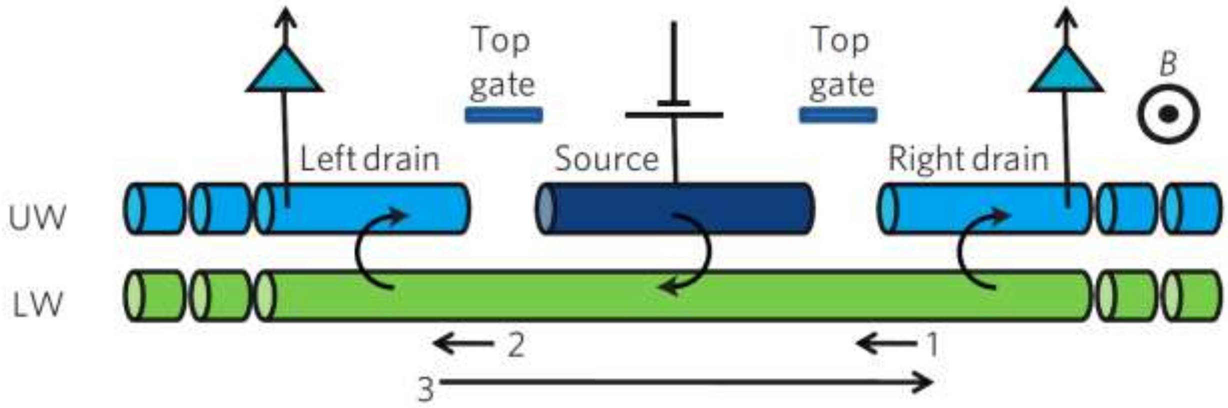
- Fundamental relaxation processes in 1d

  arXiv:1007.1152

- Sketch of derivations

- Conclusions

Experiment

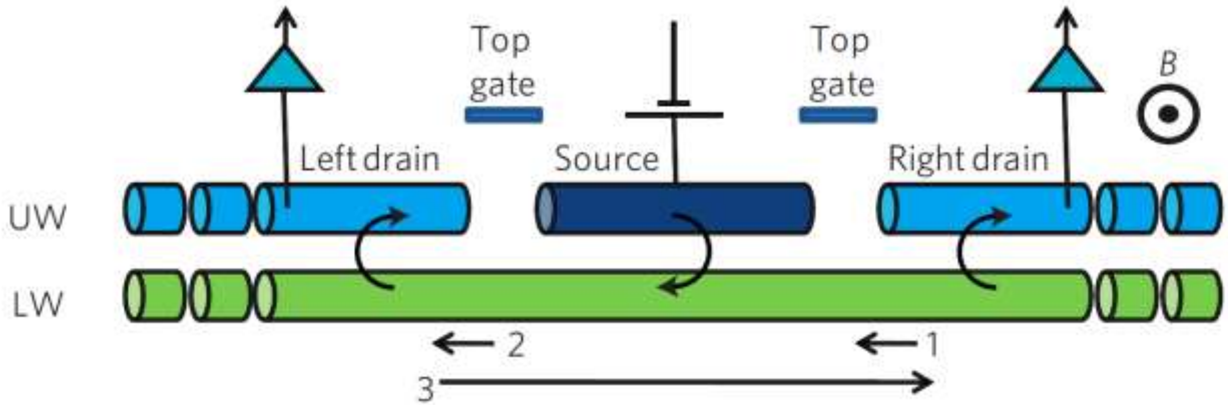


- momentum-conserving tunneling
- voltage drops mostly between source and lower wire

Some numbers:

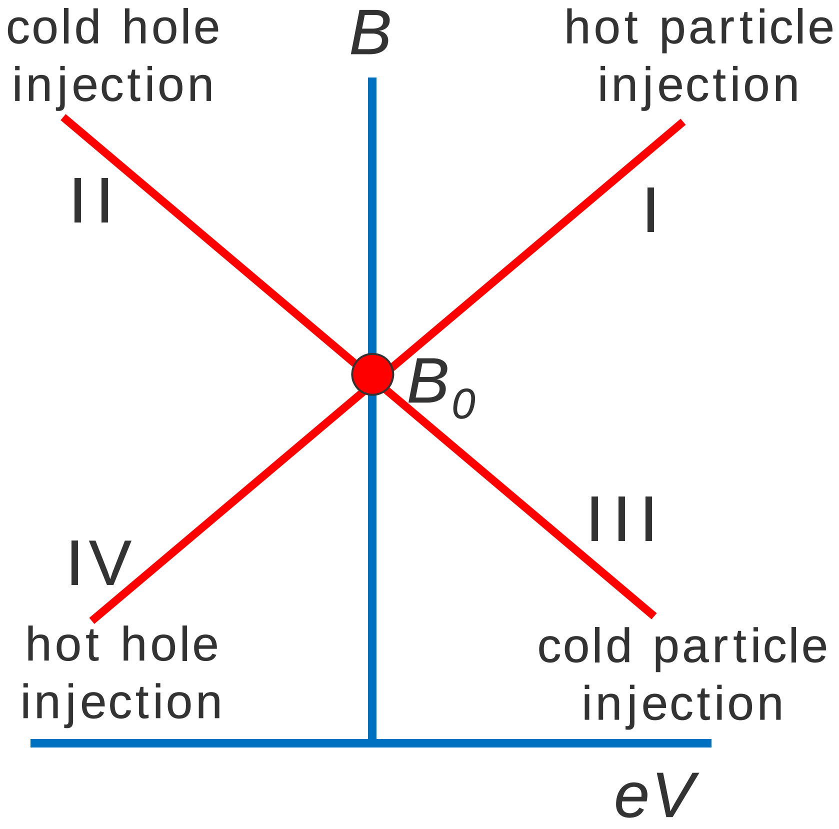
length of source:	*10*μm
distance source/drain:	*2*μm
length of drain:	~*1*mm
Fermi velocity	$2*10^5$m/s

Experiment

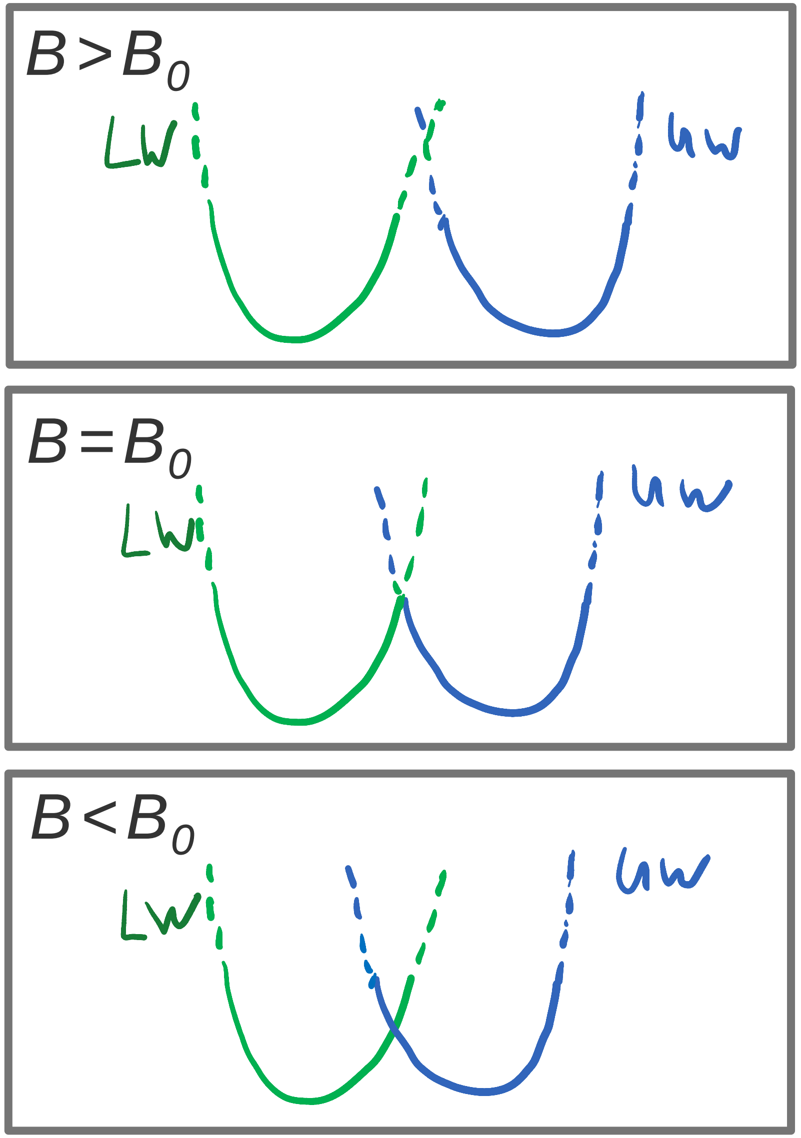


- perpendicular magnetic field: relative shift of dispersions of source and lower wire along momentum direction by $\Delta k = d/{\ell_B}^2$
- voltage: relative shift of dispersions along energy direction by *eV*

Regimes

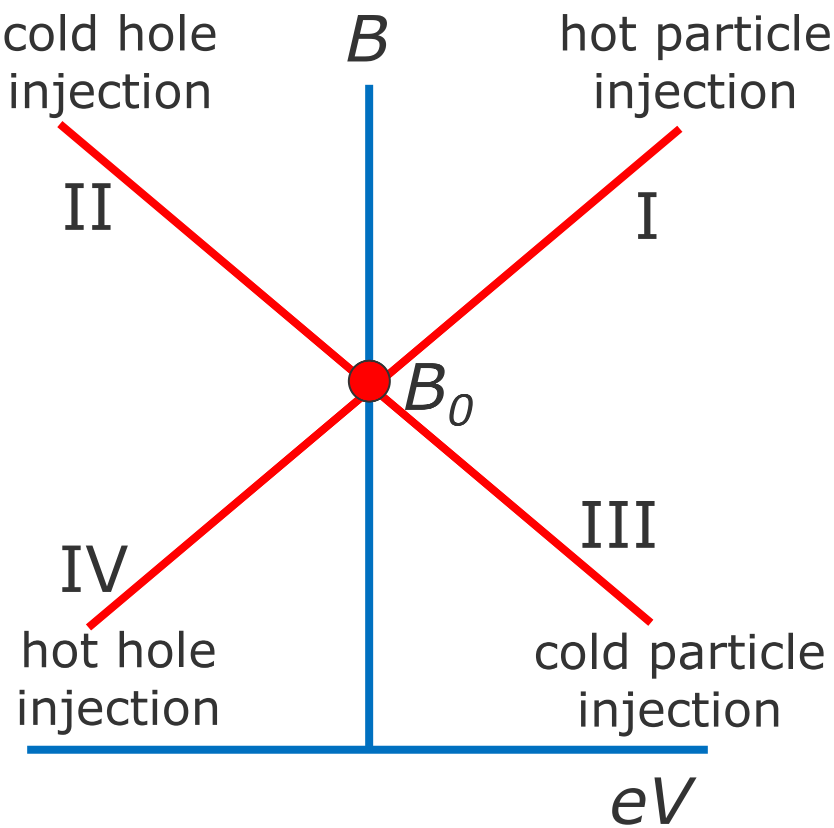


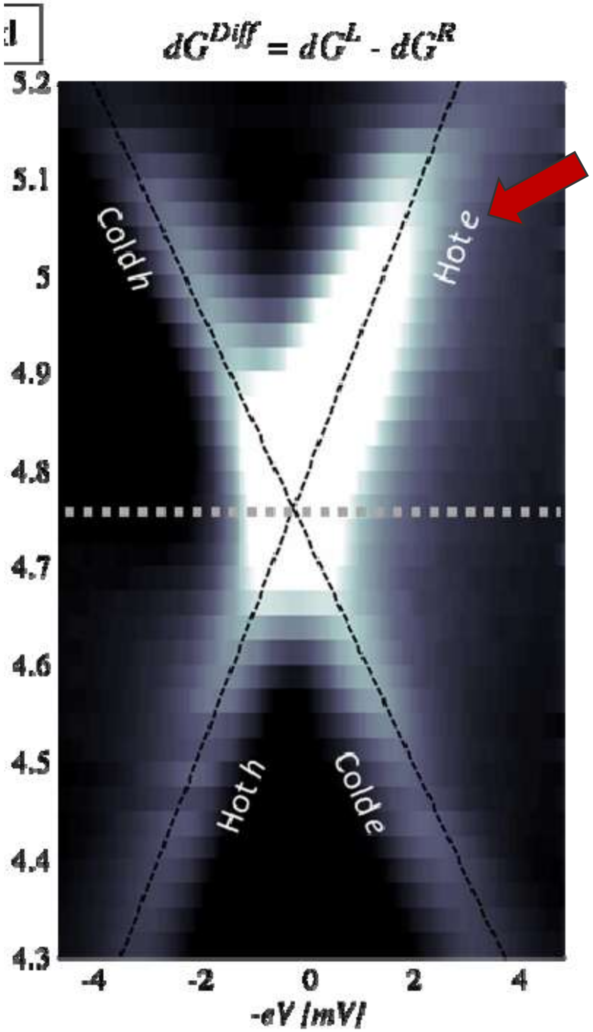
cold hole injection
B
hot particle injection
II
I
B_0
III
IV
hot hole injection
cold particle injection
eV



$B>B_0$
LW
uw
$B=B_0$
LW
uw
$B<B_0$
LW
uw

Measurement





Cold particles and holes

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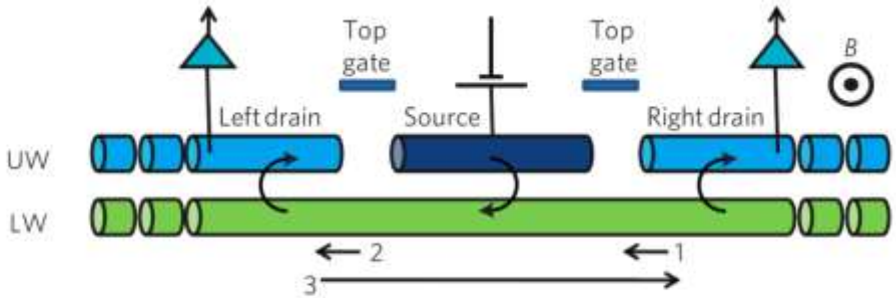
Injection of right movers:

$$I_R \sim T_R + (1-T_R)(1-T_L)T_R + \ldots$$

$$= \frac{T_R}{T_L + T_R - T_L T_R}$$



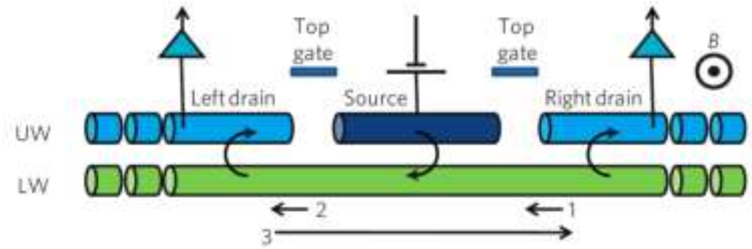
$$I_L \sim (1-T_R)T_L + (1-T_R)(1-T_L)(1-T_R)T_L + \ldots = \frac{(1-T_R)T_L}{T_L + T_R - T_L T_R}$$

asymmetry: $AS = \frac{I_R - I_L}{I_R + I_L} = \frac{T_R - T_L + T_R T_L}{T_L + T_R - T_L T_R}$

$$\boxed{\overline{AS} = \frac{1}{2}\big(AS(B) + AS(-B)\big) = \frac{T_L T_R}{T_L + T_R - T_L T_R}}$$

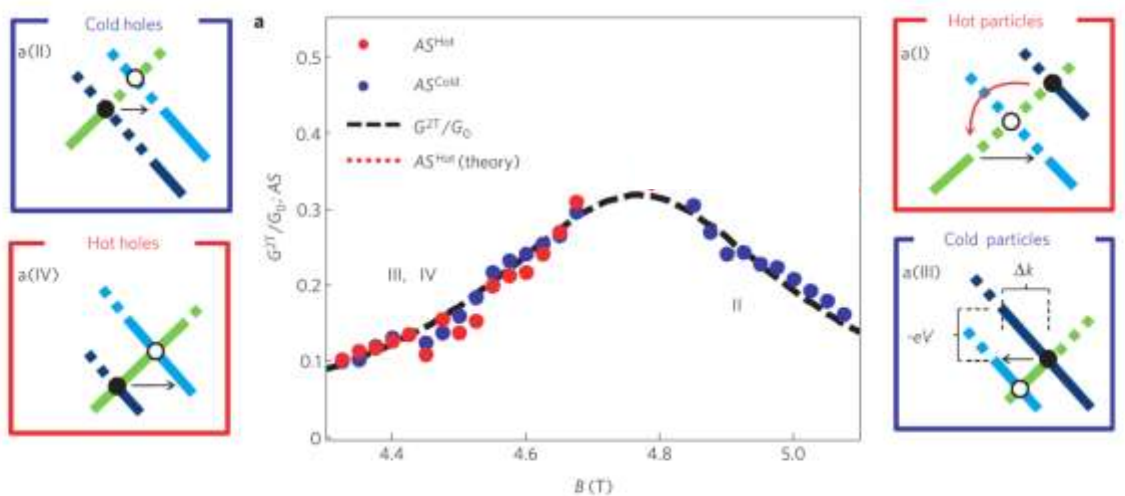
Cold particles and holes

2-terminal conductance:

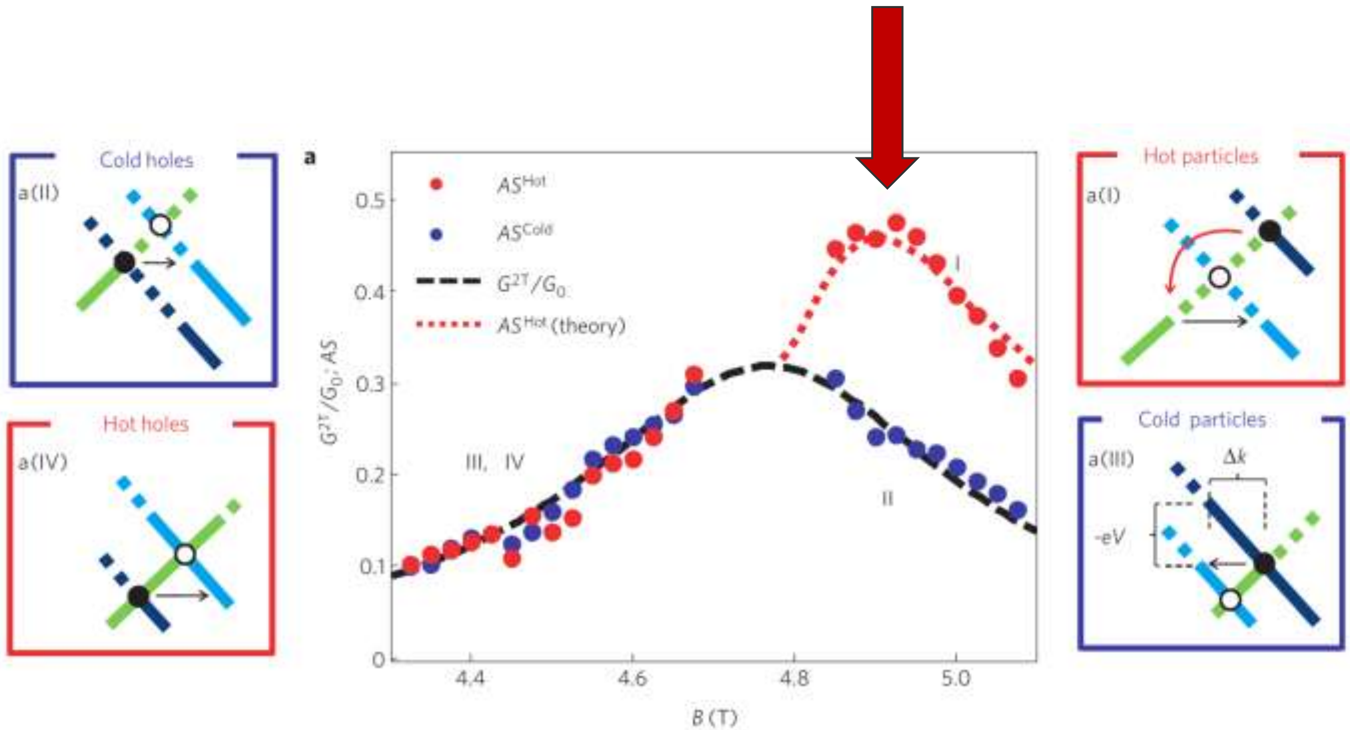


$$G_{2T} \sim T_L T_R + T_L(1 - T_R)(1 - T_L)T_R + \ \dots = \frac{T_L T_R}{T_L + T_R - T_L T_R}$$

$$\boxed{\overline{AS} = G_{2T}}$$



Hot particles



hot particles: asymmetry larger than expected

more than one electron exits to the right
for each injected right-mover

Basic idea

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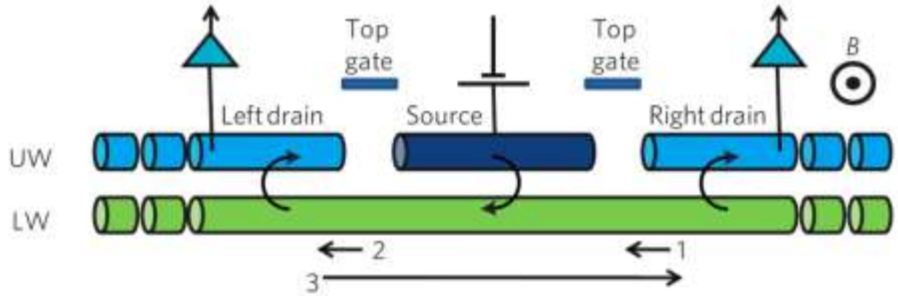
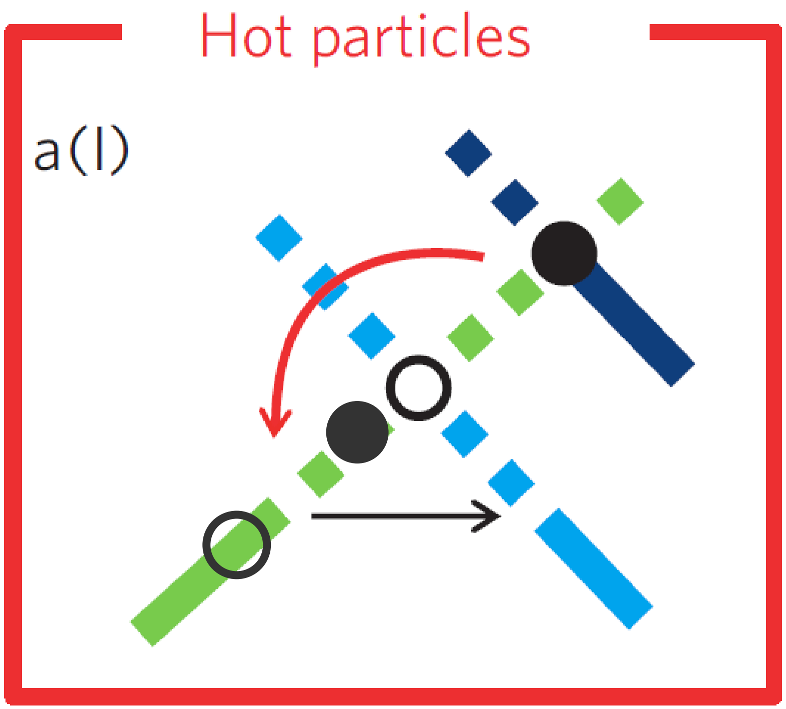
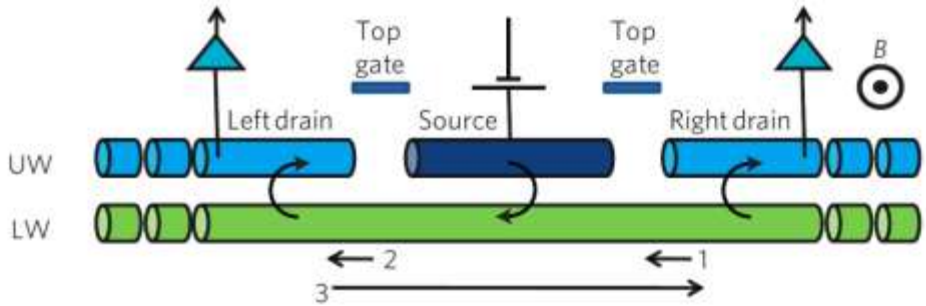
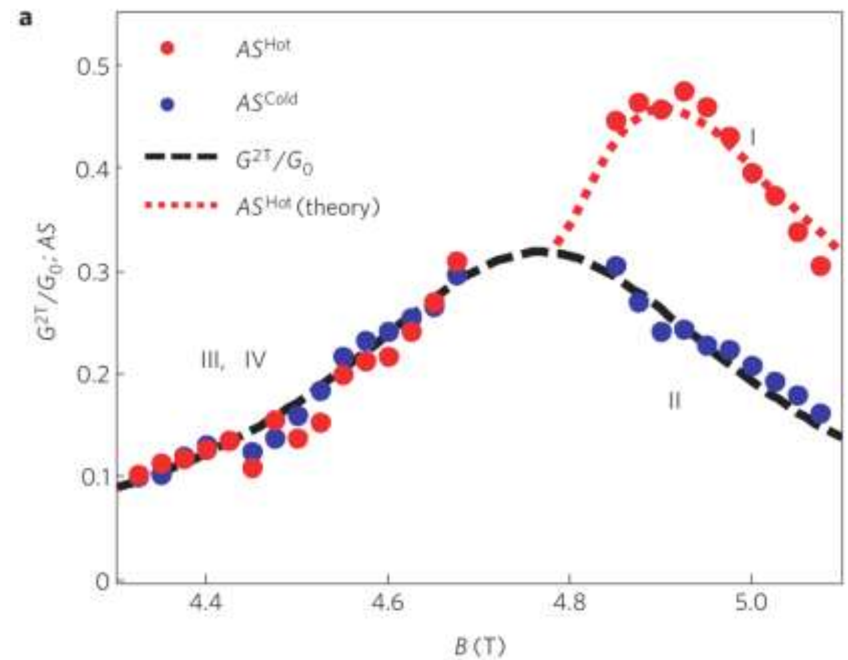




- hot particle excites ph pairs
- particle exits more easily than hole
- additional asymmetry
- **no** equilibration between right- and left-movers
- **no** equilibration of holes

Phenomenological model





- fast equilibration of comoving electrons
- **no** equilibration between right- and left-movers
- determine μ_i and T_i from particle- & energy conservation

relaxation rates

$$\tau_p < 10^{-11} s$$

$$\tau_h > 10^{-11} s$$

Relaxation in perturbative regime

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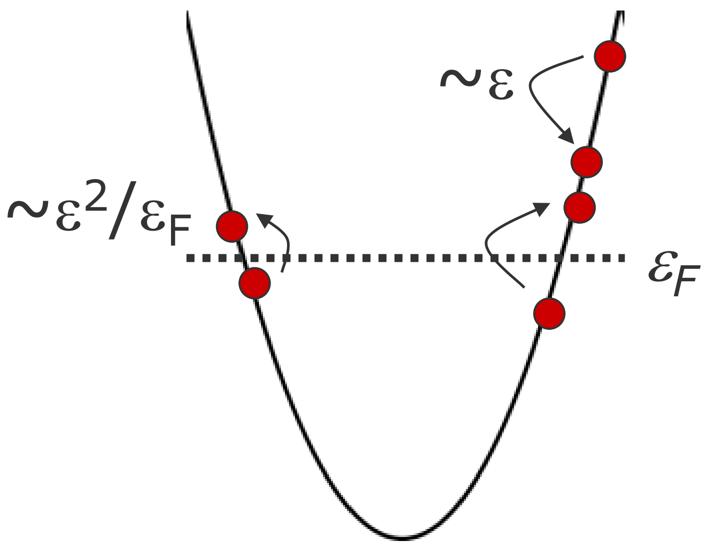
two-particle collisions ineffective

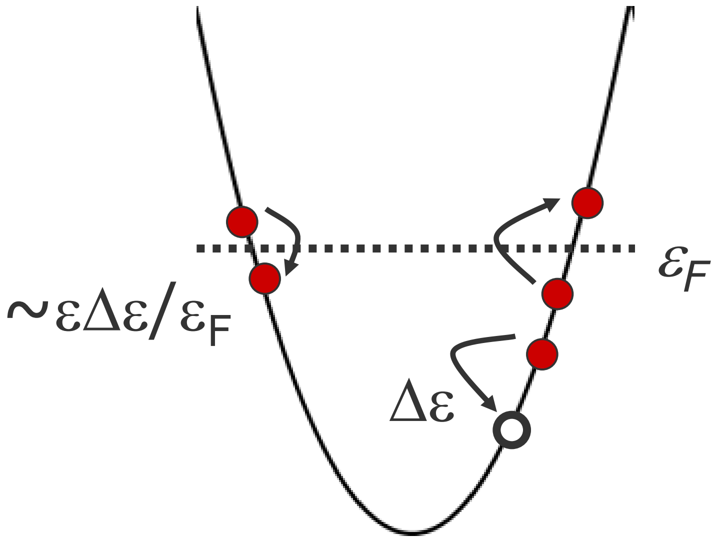
➡ hot-particle relaxation by *three-body* collisions



- energy & momentum conservation due to left-moving p-h pair
- typical energy loss $\sim\varepsilon$
- left moving p-h pair has parametrically smaller energy

phase space $\sim \epsilon^2 \cdot \max(\epsilon^2/\epsilon_F, T)$

Hot-hole relaxation



- all multi p-h pair processes are also forbidden

- holes can*not* relax at zero temperature

finite temperature: process possible for $(\epsilon/\epsilon_F)\Delta\epsilon \sim T$

characteristic energy: $\boxed{\epsilon_T = \sqrt{\epsilon_F T}}$

Hole relaxation at finite T

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$\epsilon \ll \epsilon_T$: hole relaxes by single three-body collision

phase space $\sim \epsilon^2 \cdot T$

relaxation equally fast as for particles

$\epsilon \gg \epsilon_T$: hole floats up to ε_F in many small steps

# of steps : $(\epsilon/\epsilon_T)^2$

phase space : $\epsilon^2 \cdot T$

slower than particle relaxation by $(\epsilon_T/\epsilon)^4$

Matrix elements

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Fermi`s golden rule:

$$\boxed{\frac{1}{\tau} = \sum_{231'2'3'} W^{123}_{1'2'3'} n^0_2 n^0_3 \bar{n}^0_{1'} \bar{n}^0_{2'} \bar{n}^0_{3'}}$$

see, e.g., Lunde et al. PRB 2007

Here:

- Coulomb interaction: $V_q = \begin{cases} \frac{2e^2}{\kappa} \ln(2d/a) & q < 1/d \\ \frac{2e^2}{\kappa} \ln(1/qa) & q > 1/d \end{cases}$
- spin
- energy relaxation rate

Processes

elementary rates

$$W^{123}_{1'2'3'} = \frac{2\pi}{\hbar}|\langle 1'2'3'|VG_0V|123\rangle_c|^2\delta(E-E')$$

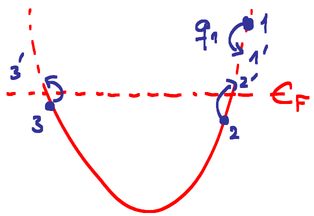
w/ matrix elements

$$\langle 1'2'3'|VG_0V|123\rangle_c = \sum_{P(1'2'3')} (-1)^p \delta_{\sigma_1\sigma_2\sigma_3,P(\sigma_{1'}\sigma_{2'}\sigma_{3'})} T^{123}_{P(1'2'3')}$$

6 different amplitudes

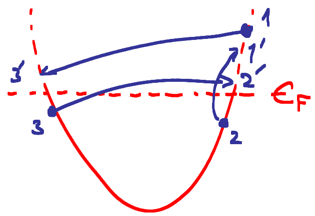
Processes

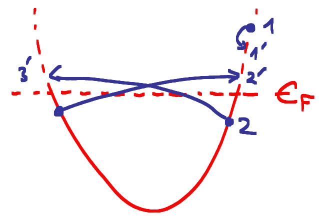
two small-q processes



$$T^{123}_{1'2'3'} = (V_{q_1}/8\epsilon_F L^2)\,(V_{q_1} - V_{q_3})$$

four large-q processes





$$T^{123}_{3'1'2'} = -T^{123}_{2'3'1'}$$

$$= (V_{2k_F}/2\hbar v_F q_1 L^2)\,(V_{2k_F} - V_{p_1})$$

+ exchange 1‘↔2‘

Spinless fermions

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- interaction can be reorganized to involve $V_{2k_F} - V_0$ i.e., low-*q* and *2k$_F$*-processes should contribute the same

- indeed, leading order cancels from *2k$_F$*-processes

- Coulomb: except for logs, energy dependence of rates governed by phase space, e.g., $1/\tau_p \sim \epsilon^4$ for $\epsilon \gg \epsilon_T$

- short-range interaction: no scattering for contact interaction so that $T \sim q^2$ and $1/\tau_p \sim \epsilon^8$

[see, e.g., Khodas et al. PRB (2007); Pereira & Affleck PRB 2009)]

Interpretation: antisymmetric orbital wavefunction of spinless fermions suppresses *2k$_F$*-contribution

Spinful electrons

- symmetric orbital wavefunction for total spin ½
- **no** suppression of *$2k_F$*-processes for Coulomb (only logarithmic suppression of matrix element)
- $T \sim 1/\epsilon$ dependence of rates on energy and temperature involves lower power than phase space

$$\boxed{1/\tau_{\mathrm{p}} = (9\epsilon_F/32\pi^3\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(\epsilon/\epsilon_F)^2}$$

… orders of magnitudes larger
than one may have suspected

Results

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Hot-particle relaxation :

$\epsilon \gg \epsilon_T$: $$1/\tau_\mathrm{p} = (9\epsilon_F/32\pi^3\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(\epsilon/\epsilon_F)^2$$

$\epsilon \ll \epsilon_T$: $$1/\tau_\mathrm{p} = (3c_1\epsilon_F/4\pi^3\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(T/\epsilon_F)$$

Hot-hole relaxation :

$\epsilon \gg \epsilon_T$: $$1/\tau_\mathrm{h} = (2\epsilon_\mathrm{F}/\pi\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(T/\epsilon)^2$$

$\epsilon \ll \epsilon_T$: same as for particles

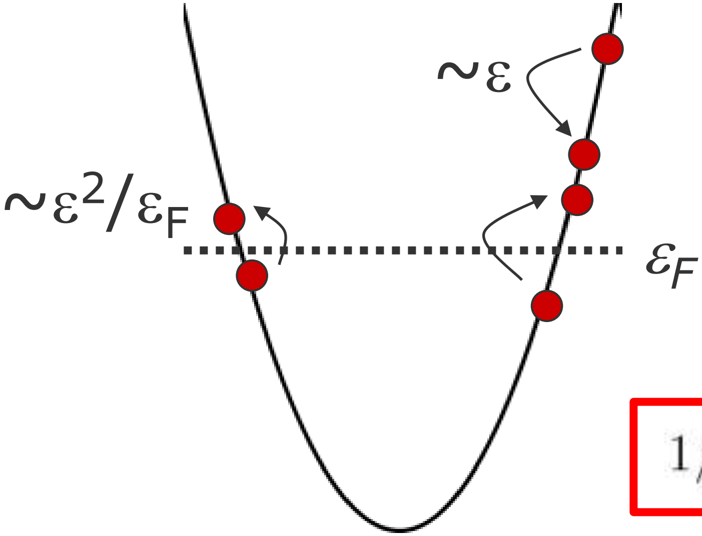
Thermalization

co-moving electrons :

thermalization governed by hole relaxation time, even for injection of hot particles

counter-propagating electrons:

much slower due to small energy transfer per three-body collision between counter-propagating electrons



$$\boxed{1/\tau_{\text{inter}} = (9c_2\epsilon_F/2^8\pi^5\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(T)]^2(T/\epsilon_F)^3}$$

Comparison with experiment

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experiment: $\epsilon \sim \epsilon_F/3$

T=0.25 K

$$1/\tau_p \cong 10^{11}\frac{1}{s} \qquad 1/\tau_h \cong 5\times 10^{9}\frac{1}{s} \qquad 1/\tau_{\mathrm{inter}} \cong 10^{6}\frac{1}{s}$$

… quantitatively consistent w/ experiment

Conclusions

• energy relaxation of electrons and holes in clean quantum wires proceeds via *three*-body collisions

• holes relax at *non*zero temperatures only

• nonzero temperatures introduces characteristic energy scale $(E_F T)^{1/2}$

• very slow equilibration between left & right movers

• energy relaxation rates quantitatively consistent w/ experiment