Quantum Dots Talking through a Quantum Wire

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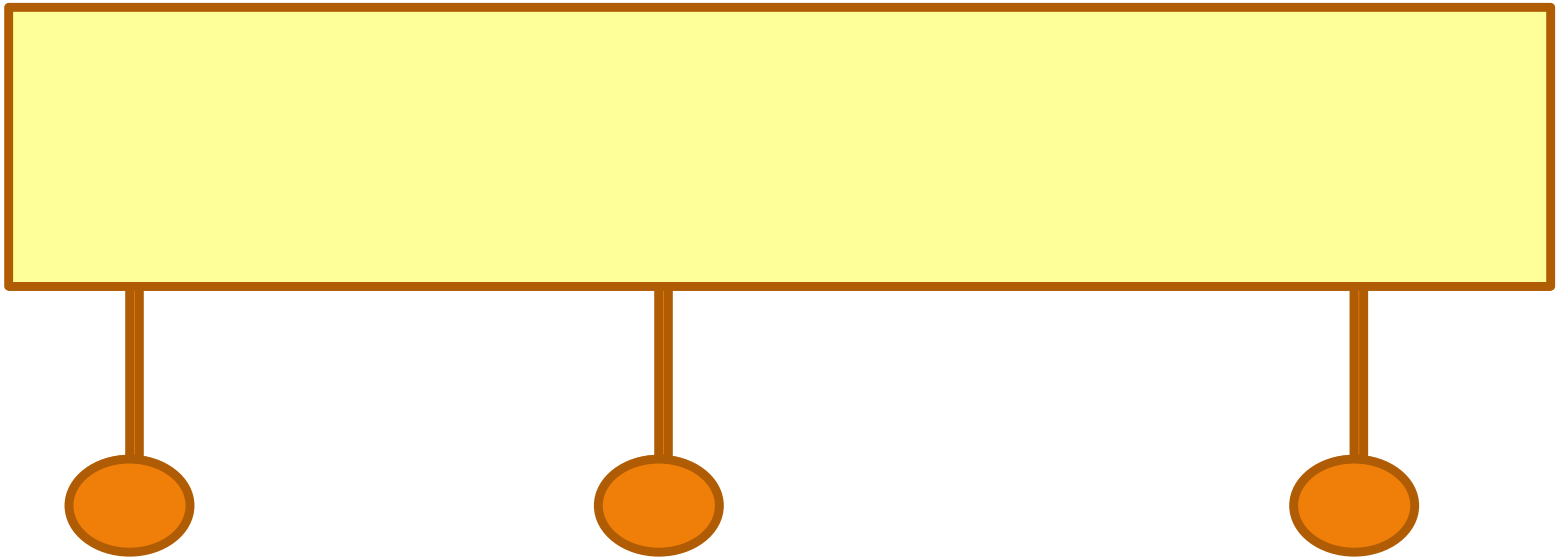
"The Science of Nanostructures: New Frontiers in the Physics of Quantum Dots"

Chernogolovka, Russia

Warming up…

System:

Quantum Dots Coupled with a Wire



States of QDs become correlated

Limiting case: a Single Channel Wire + Single Level QDs
(spinless/spin polarized electrons)

Quantities of interest:

averaged populations: $\langle n_j \rangle$ (depend on QD positions)

correlation functions: $S_{jl}(t-t') \simeq \langle n_j(t) n_l(t') \rangle_c$

can be straightforwardly calculated for

non-interacting electrons

(equilibrium, non-equilibrium, quenches, etc.)

If not the interaction…

A Single Channel Wire with Interacting electrons

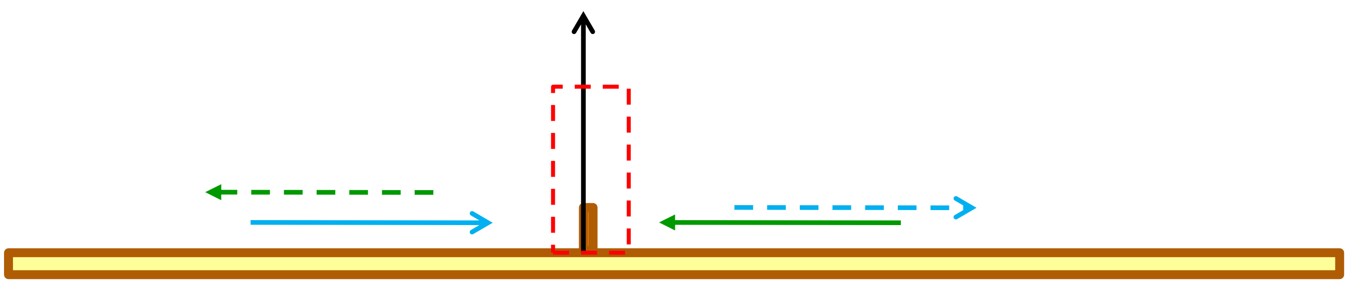
(no interaction between QD's and wire electrons)

Luttinger Liquid + Inhomogeneities

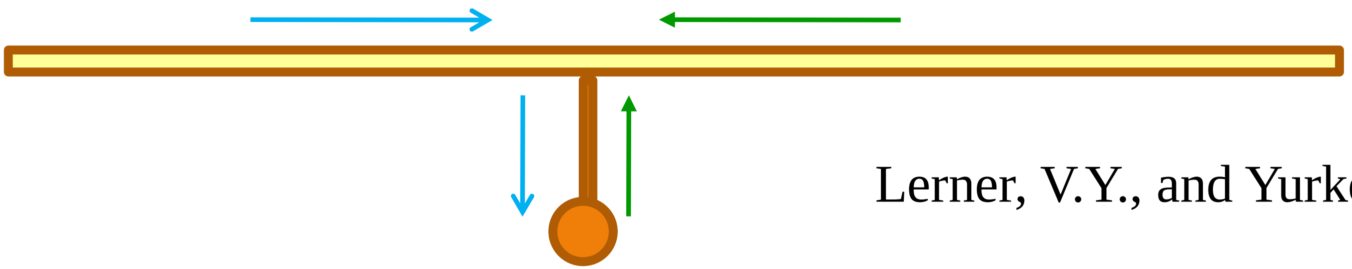
A brief reminder:

1. LL + *a single static* impurity

$$U_0(x) \rightarrow U_{eff}(x) \rightarrow \infty$$ Kane & Fisher (92)

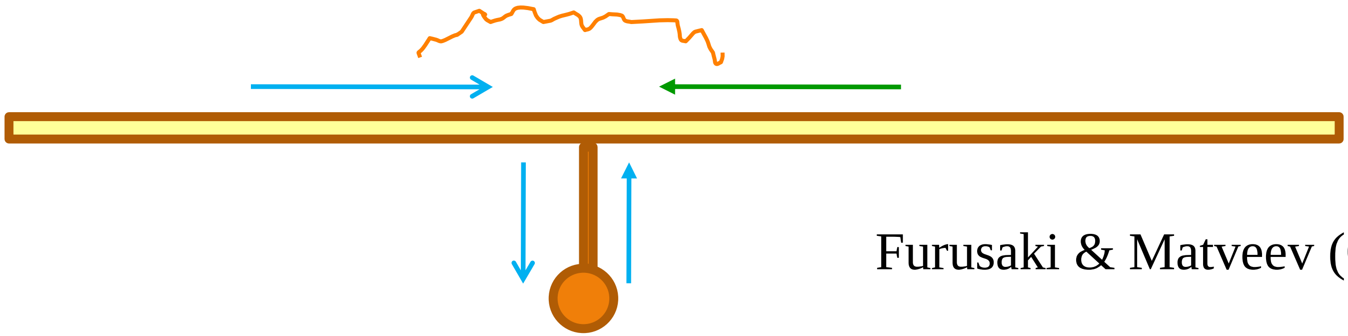


2. Luttinger Liquid + a single level QD



Lerner, V.Y., and Yurkevich (08)

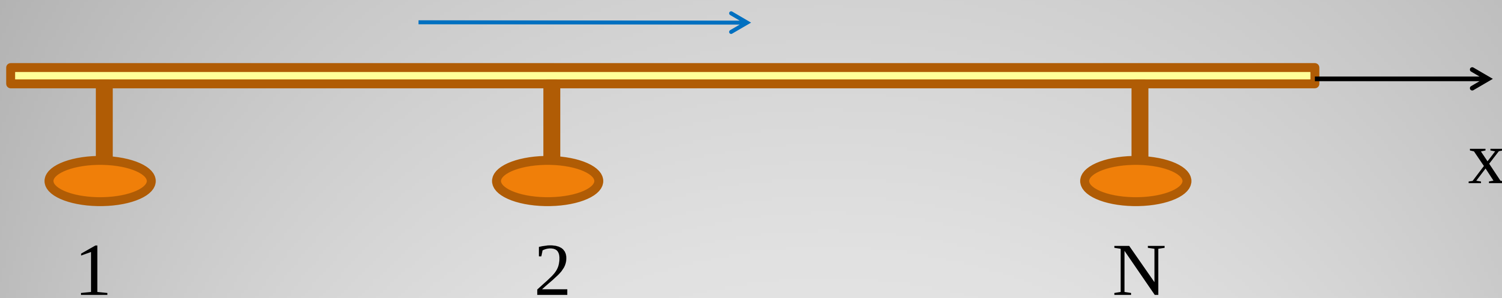
3. Chiral Luttinger Liquid + a single level QD



Furusaki & Matveev (02)

Chiral Toy Model:

(only right movers)



e-e interaction: $$\frac{1}{2}\int dx[\bar{\psi}_R(x)\psi_R(x)+0]V_0(x-x')[\bar{\psi}_R(x')\psi_R(x')+0]$$

Common wisdom:

The only effect – renormalization of Fermi-velocity: $v_F \to v$

To begin with...

Non-interacting electrons
(for comparison, notations, etc.)

Generating functional integral (for Matsubara technique):

$$Z[J] = \int D[\overline{\psi},\psi]D[\overline{a},a]e^{-S[\overline{\psi},\psi]-S[\overline{a},a;J]-S_{hybr}[\overline{\psi},\psi;\overline{a},a]}$$

$$S[\overline{\psi},\psi] = \int d\tau \int dx\, \overline{\psi}(x,\tau)\underbrace{[\partial_\tau + iv_F\partial_x]}_{}\psi(x,\tau)$$

$$-g_0^{-1} \Longleftrightarrow \quad g_0(p,i\omega_n) = \frac{1}{i\omega_n - v_F p}$$

$$S[\overline{a},a;J] = \sum_j \int d\tau\, \overline{a}_j(\tau)\,[\partial_\tau + \varepsilon_j - J_j(\tau)]\,a_j(\tau)$$

$$S_{hybr}[\overline{\psi},\psi;\overline{a},a] = \sum_j t_j \int d\tau [\overline{a}_j(\tau)\psi(x_j,\tau) + \overline{\psi}(x_j,\tau)a_j(\tau)]$$

QD action (after integration over wire electrons):

$$S[\bar{a}, a; J] = \sum_{j,l} \int d\tau\, \bar{a}_j(\tau) \underbrace{[\delta(\tau-\tau')\delta_{jl}(\partial_\tau + \varepsilon_j - J_j(\tau)) + t_j g_0(x_j,\tau; x_l,\tau')t_l]}_{-G_{jl}^{(0)-1}} a_l(\tau')$$

$$G_{jl}^{(0)-1}(i\omega_n > 0) = \begin{pmatrix} i\omega_n - \varepsilon_1 + i\Gamma_1 & 0 & 0 \\ i\Gamma_{21}(i\omega_n) & i\omega_n - \varepsilon_2 + i\Gamma_2 & 0 \\ i\Gamma_{31}(i\omega_n) & i\Gamma_{32}(i\omega_n) & i\omega_n - \varepsilon_3 + i\Gamma_3 \end{pmatrix}$$

Here:

$$\Gamma_j = \frac{t_j^2}{2v_F} \qquad \Gamma_{jl}(i\omega_n) = \frac{t_j t_l}{v_F} e^{-|\omega_n(x_j - x_l)|} \equiv \Gamma_{jl} e^{-|\omega_n(x_j - x_l)|}$$

Some consequences:

Equilibrium populations $\langle n_j \rangle$ do not depend on positions of QDs (in contrast to the non-chiral model)

Triangular structure of the correlation function:

$$S^R_{jl}(\Omega) = \left\langle T_\tau [n_j(\tau) n_l(\tau')] \right\rangle_{i\Omega_n(>0)\to\Omega} \neq 0, \quad at \quad j > l$$

Example (two QDs), low-temperature limit:

$$S^R_{21}(\Omega) = \frac{i\Gamma^2_{21}}{\pi} e^{i\Omega(x_2-x_1)/v_F} \left\{ \frac{\Omega}{\Omega^2 - (\varepsilon_0 - i\Gamma)^2} - \frac{1}{\Omega + 2i\Gamma} \ln\left[\frac{\varepsilon_0^2 - (\Omega + i\Gamma)^2}{\varepsilon_0^2 + \Gamma^2} \right] \right\}$$

Accounting for interaction

Elements of functional bosonization

(Grishin, Yurkevich, & Lerner (04))

1. H-S decoupling: $$e^{-\frac{1}{2}\int d\tau\, dx \overline{\psi}\psi V_0 \overline{\psi}\psi} = \int D\phi\, e^{-\frac{1}{2}\int d\tau\, dx\, \phi V_0^{-1}\phi - \int d\tau\, dx \overline{\psi}\,(-i\phi)\psi}$$

Fermionic action:

$$S[\overline{\psi},\psi;\phi] = \int d\tau \int dx\, \overline{\psi}(x,\tau)[\partial_\tau + iv_F\partial_x - i\phi(x,\tau)]\,\psi(x,\tau)$$

2. Gauge transformation: $\overline{\psi} \to \overline{\psi}\, e^{-i\theta} \quad ; \quad \psi \to e^{i\theta}\psi$

$$[\partial_\tau + iv_F\partial_x]\theta = \phi$$

3. Accounting for the Jacobian: $V_0^{-1} \to V^{-1} = V_0^{-1} + \Pi$

QDs functional (after integration over wire electrons):

$$Z[J] = \int D[\phi] e^{-S[\phi]} \int D[\bar{a}, a] e^{-S[\bar{a}, a; \theta; J]}$$

$$S[\bar{a}, a; J] = \sum_{j,l} \int d\tau \, \bar{a}_j(\tau) \, [\delta(\tau - \tau')\delta_{jl}(\partial_\tau + \varepsilon_j - J_j(\tau)) +$$

$$t_j e^{i\theta(x_j, \tau)} g_0(x_j, \tau; x_l, \tau') e^{-i\theta(x_l, \tau')} t_l \,] \, a_l(\tau')$$

Bosonic field correlation functions (for the toy model):

$$\left\langle \theta(q, i\Omega_n) \, \theta(-q, -i\Omega_n) \right\rangle = \frac{V_0}{(i\Omega_n - vq)(i\Omega_n - v_F q)}$$

$$v = v_F \left[1 + \frac{V_0}{\pi v_F} \right] \equiv v_F g$$

In low-temperature limit:

$$-\frac{1}{2}\left\langle[\theta(x_j,\tau)-\theta(x_l,\tau')]^2\right\rangle = \ln\left[\frac{v_F(\tau-\tau')-i(x_j-x_l)}{v(\tau-\tau')-i(x_j-x_l)}\right]$$

$$g_0(x_j,\tau;x_l,\tau') = \frac{1}{2\pi[v_F(\tau-\tau')-i(x_j-x_l)]}$$

I. Wire Green's function for the chiral model:

$$g(x_j,\tau;x_l,\tau') = \left\langle e^{i\theta(x_j,\tau)} g_0(x_j,\tau;x_l,\tau') e^{-i\theta(x_l,\tau')}\right\rangle_{S[\theta]} = [g_0(x_j,\tau;x_l,\tau')]_{v_F\to v}$$

As a result, DOS of the "chiral toy model":

$$\nu(\omega) = \frac{v_F}{v}\nu_0(\omega) = Const$$

The "toy model" is not the Luttinger Liquid

("Common wisdom")

II. One QD coupled with a toy chiral mode:

$$\left\langle e^{-\int d\tau d\tau' \overline{a}(\tau) t^2 e^{i\theta(0,\tau)} g_0(0,\tau;0,\tau') e^{-i\theta(0,\tau')} a_l(\tau')} \right\rangle_{S[\theta]}$$

In the n-th order of a formal expansion

$$-\frac{1}{2}\left\langle [\theta(\tau_1)+\ldots+\theta(\tau_n)-\theta(\tau'_1)-\ldots-\theta(\tau'_n)]^2 \right\rangle = n\ln\left[\frac{v_F}{v}\right]$$

Exactly as many as required to renormalize the product of n bare Green's functions!

Resume:

A system of a single QD coupled to an toy chiral mode is trivial (the only effect: $v_F \to v$)

III. Several QD's coupled with a chiral mode:

$$\left\langle e^{-\int d\tau d\tau' \overline{a}_j(\tau) t^2 e^{i\theta(x_j,\tau)} g_0(x_j,\tau;x_l,\tau') e^{-i\theta(x_l,\tau')} a_l(\tau')} \right\rangle_{S[\theta]}$$

Already in the 2nd order of a formal expansion

$$-\frac{1}{2}\left\langle [\theta(x_1\tau_1)+\theta(x_2,\tau_2)-\theta(x_2,\tau'_1)-\theta(x_1,\tau'_2)]^2 \right\rangle =$$

$$\ln\left[\frac{v_F(\tau_1-\tau'_1)-i(x_1-x_2)}{v(\tau_1-\tau'_1)-i(x_1-x_2)}\right]+\ln\left[\frac{v_F(\tau_2-\tau'_2)-i(x_2-x_1)}{v(\tau_2-\tau'_2)-i(x_2-x_1)}\right]+$$

$$+\ln\left[\frac{v_F}{v}\right]-\ln\left[\frac{v_F(\tau_1-\tau_2)-i(x_1-x_2)}{v(\tau_1-\tau_2)-i(x_1-x_2)}\right]+\ln\left[\frac{v_F}{v}\right]-\ln\left[\frac{v_F(\tau'_1-\tau'_2)-i(x_1-x_2)}{v(\tau'_1-\tau'_2)-i(x_1-x_2)}\right]$$

Too many terms!

System of several QD coupled to an interacting toy chiral mode is apparently "non-trivial".

In fact, this is the property of the chiral model itself: averaging of a product of several Green's functions over bosons generates more additional factors than the number of Green's functions.

In case of the chiral toy model, cancellation of extra terms might be provided by an "almost complete" vanishing of point interaction effects for a single mode. For the chiral Luttinger model (two types of movers) no cancellation is expected.

System of several QDs coupled with a chiral mode needs a proper field-theoretical treatment.
One of ways – a gauge transformation in QD action.

Gauge transformation in QD's action:

$$S[\bar{a}, a; J] = \sum_{j,l} \int d\tau\, \bar{a}_j(\tau)\, [\delta(\tau-\tau')\delta_{jl}(\partial_\tau + \varepsilon_j - J_j(\tau)) +$$

$$t_j e^{i\theta(x_j,\tau)} g_0(x_j,\tau; x_l,\tau') e^{-i\theta(x_l,\tau')} t_l]\, a_l(\tau')$$

$$\bar{a}_j \rightarrow \bar{a}_j\, e^{-i\theta(x_j,\tau)} \quad ; \quad a_j \rightarrow e^{i\theta(x_j,\tau)} a_j$$

Transformed action:

$$S[\bar{a}, a; J] = \sum_{j} \int d\tau\, \bar{a}_j(\tau)\, [-G_0^{-1} + \partial_\tau \theta(x_j,\tau)]\, a_j(\tau)$$

Correlations of bosonic field $A_j(\tau) \equiv \partial_\tau \theta(x_j,\tau)$:

$$\left\langle A_j(q, i\Omega_n)\, A_l(-q, -i\Omega_n) \right\rangle = 2\pi\, |\Omega_n|\, [e^{-|\Omega_n (x_j - x_l)|/v} - e^{-|\Omega_n (x_j - x_l)|/v_F}]\, \vartheta[\Omega_n (x_j - x_l)]$$

(for the toy chiral model with only right movers)

Correlators of bosonic field $A_j(\tau) \equiv \partial_\tau \theta(x_j, \tau)$: for chiral Luttinger model (both right and left interacting movers):

$$\left\langle A_j(q, i\Omega_n)\, A_l(-q, -i\Omega_n) \right\rangle = \frac{V_0 |\Omega_n|}{v} \{ \vartheta[-\Omega_n (x_j - x_l)] \frac{v - v_F}{2(v + v_F)} e^{-|\Omega_n (x_j - x_l)|/v}$$

$$+ \vartheta[\Omega_n (x_j - x_l)] \left[\frac{v + v_F}{2(v - v_F)} e^{-|\Omega_n (x_j - x_l)|/v} - \frac{2 v v_F}{v^2 - v_F^2} e^{-|\Omega_n (x_j - x_l)|/v} \right] \}$$

Perturbation Theory. Lowest order (in V_0).

TWO QDS:

$$\left\langle A_2(q, i\Omega_n)\, A_1(-q, -i\Omega_n) \right\rangle = \frac{V_0 \Omega_n}{v} \vartheta[\Omega_n] \left[\frac{v + v_F}{2(v - v_F)} e^{-\Omega_n r / v} - \frac{2 v v_F}{v^2 - v_F^2} e^{-\Omega_n r / v} \right]$$

Triangular structure!

As a consequence:

Triangular structure of

Self-Energy and Green's function:

$$\Sigma_{21}(i\omega_n),\, G_{21}(i\omega_n) \propto \vartheta(\omega_n)$$

Population correlation function:

$$S_{21}^{R}(\Omega) = \left\langle T_\tau [n_2(\tau) n_1(\tau')] \right\rangle_{i\Omega_n (>0) \to \Omega} \neq 0,$$

$$S_{12}^{R}(\Omega) = \left\langle T_\tau [n_1(\tau) n_2(\tau')] \right\rangle_{i\Omega_n (>0) \to \Omega} = 0$$

Summary

1. System of several QDs coupled with a single chiral branch of an interacting electron wire allows a simpler (as compared to coupling with the both branches) analysis with the use of a gauge field in the QD's action.
2. This allows one to develop a perturbation theory in the gauge field to calculate various correlation functions of QDs.
3. The triangular structure of Green's functions and correlation function of population in different QDs is preserved for the chiral LL in the first order in interaction (at least).

4. **What about higher orders?**

Thank you!