A Tale of Two Crossovers

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Acknowledgements

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Outline

- Ultrasmall Aluminium grains
- Aluminium Films: a possible new crossover regime
- Interactions: The Universal Hamiltonian
- Two tunnel-coupled dots above T_c with Φ (tame)
- One dot below T_c with Φ (wild, in progress)
- Conclusions and open questions

Ultrasmall Aluminium grains

What happens in a superconducting grain when the level spacing δ becomes comparable to the superconducting gap Δ? This question was answered by

Black, Ralph, and Tinkham PRL 74, 3241 (1995); PRL 76, 688 (1996)

who looked at the parity effect, namely the difference in the addition energy between when the dot has an even number of electrons versus an odd number.

$$\Delta_{ML} = E_{N+1} - \frac{1}{2}(E_N + E_{N+2})$$

Matveev and Larkin, PRL 78, 3749 (1997); von Delft and Ralph Phys. Rep. 345, 61 (2001).

Spin vs orbital effect of B

Consider an ultrasmall grain r nanometers in radius. $l \gg r$, so the grain is ballistic. Putting in the parameters for Aluminium, measuring energies in eV and magnetic field in Tesla

$$\delta \simeq \frac{10^{-2}}{r^3} \qquad E_T \simeq \frac{1}{r}$$

$$\Delta_{Bulk} = 0.035 \qquad E_{Zeeman} \simeq 10^{-2} B$$

$$E_X \simeq E_T \left(\frac{\phi}{\phi_0}\right)^2 \simeq 10^{-6} r^3 B^2$$

So for ultrasmall grains

$$E_X \ll E_{Zeeman}$$

An Aluminium Pancake

Now consider a thin film in the shape of a pancake of thickness $t \simeq 2nm$ and radius r. For $l \ll r$, we can increase $r \simeq 100nm$ and obtain for $B = 0.1\,Tesla$

$$\delta \simeq 10^{-6} \qquad E_T \simeq 10^{-2}$$

$$E_{Zeeman} \simeq 10^{-3} \qquad E_X \simeq 10^{-2}$$

So it is possible to achieve

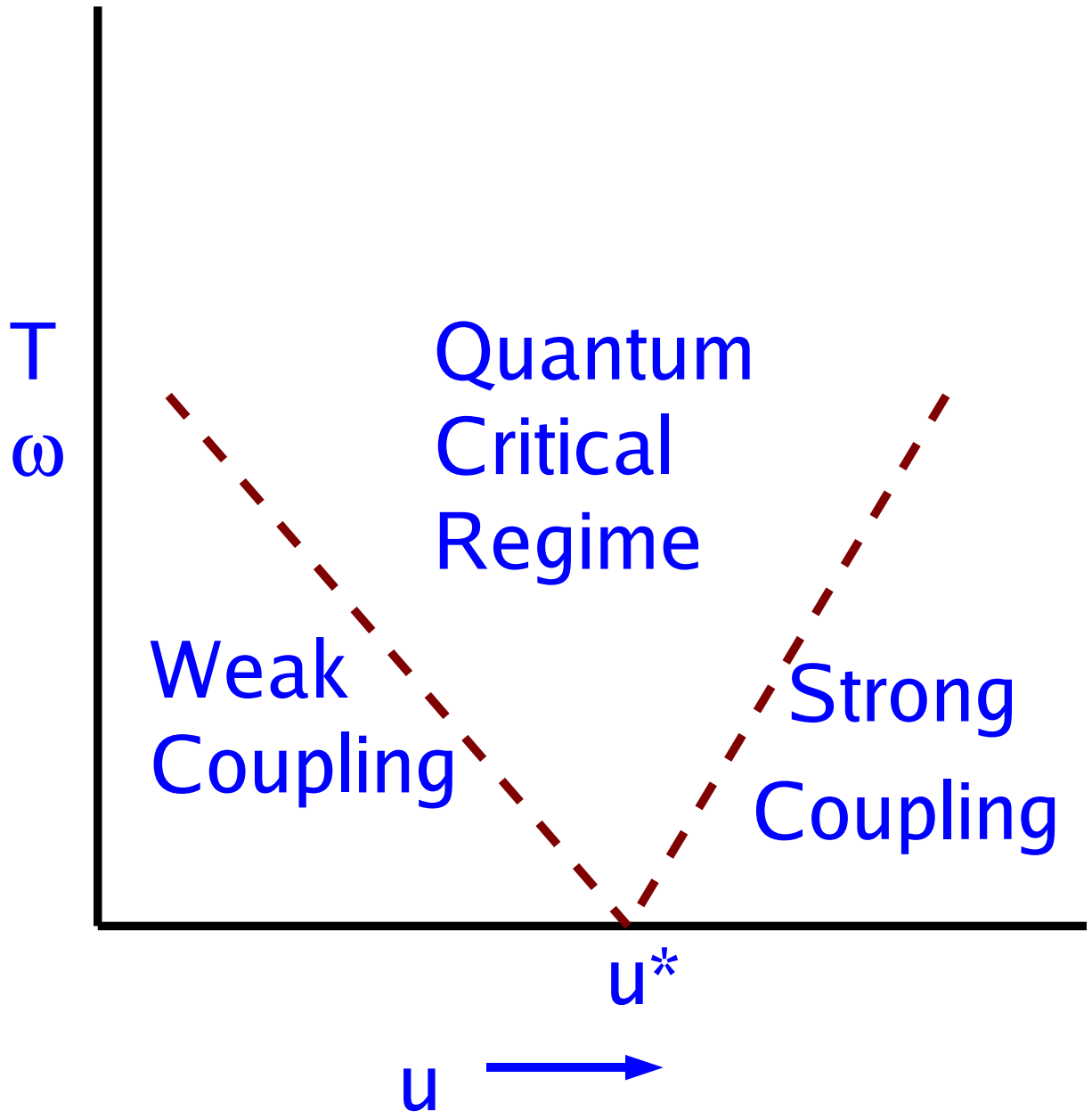
$$E_X \gg E_{Zeeman}$$

Interesting new physics in the form of quantum criticality becomes possible in this regime.

Quantum Criticality

A Quantum Phase Transition happens as a result of the change of some parameter at $T = 0$. If that transition is II-order, then

- At the $T = 0$ transition, the frequency dependences of physical quantities should be power law. Away from the transition, they are universal scaling functions of ω/E_{QCX}, where E_{QCX} is the crossover energy scale to the quantum critical regime.
- For $T, \omega \neq 0$ a whole fan-shaped quantum critical regime is controlled by the quantum critical point. All physical quantities as a function of ω are given as universal scaling functions of ω/T at the critical parameter.
- At the QPT, and therefore in the QCR, the ground and excited states are dominated by many-body quantum fluctuations. Quasiparticles may not exist.
- One needs nonperturbative methods to study the QCR. We will use large-N methods.


T
ω
Quantum
Critical
Regime
Weak
Coupling
Strong
Coupling
u*
u

Interactions

Put in all the simplest interactions which we know must be there, Charging energy, Stoner exchange, and possibly BCS.

$$H_{CJ} = \sum \epsilon_\alpha c^\dagger_{\alpha s} c_{\alpha s} + \frac{U}{2} N^2 - J\vec{S}^2 - \lambda \sum c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow} c_{\beta\downarrow} c_{\beta\uparrow}$$

This is the Universal Hamiltonian, and leads to good agreement with experiment for Coulomb Blockade peak spacings and heights.

Andreev and Kamenev, PRL 81, 3199 (98)
Brouwer, Oreg, and Halperin, PRB 60, R13977 (99)
Baranger, Ullmo, and Glazman, PRB 61, R2425 (00)
Kurland, Aleiner, and Altshuler, PRB 62, 14886 (00)

Main message of this talk

It is possible to use RMT crossovers between different symmetry classes to tune access to universal quantum critical crossover regimes in many instances. One needs (i) An order parameter undergoing a phase transition in a pure ensemble (ii) A small external perturbation which breaks the symmetry and enhances quantum fluctuations.

Murthy, PRB 70, 153304 (2004); Zelyak, Murthy, and Rozhkov, PRB 76, 125314 (2007); Murthy, PRB 77, 073309 (2008).

Setup: RMT Crossovers

For the GOE $\rightarrow$GUE crossover, we parametrize the Hamiltonian as

$$H_X = \frac{H_S + iXH_A}{1 + X^2}$$

where $X = \phi/\phi_0$ is the crossover parameter ($\phi_0 = hc/e$ is the flux quantum). As the electron wanders through the dot, every circumnavigation accumulates a phase $\pm 2\pi \frac{\phi}{\phi_0}$ and takes a time $\frac{L}{v_F}$. After N circumnavigations, the typical phase accumulated is

$$\theta \simeq 2\pi \frac{\phi}{\phi_0} \sqrt{N}$$

When this phase becomes of order 2π the electron "knows" that there is a flux through the dot. This takes a time

$$\tau_X \simeq \frac{\phi_0^2}{\phi^2}\,\frac{L}{v_F} = X^{-2}\frac{L}{v_F}$$

By the uncertainty principle, this corresponds to a crossover energy scale

$$E_X \simeq X^2 E_T$$

States separated by less than E_X are fully crossed over, while those separated by more than E_X have correlations in the original symmetry class. Extra correlations develop during the crossover. For example

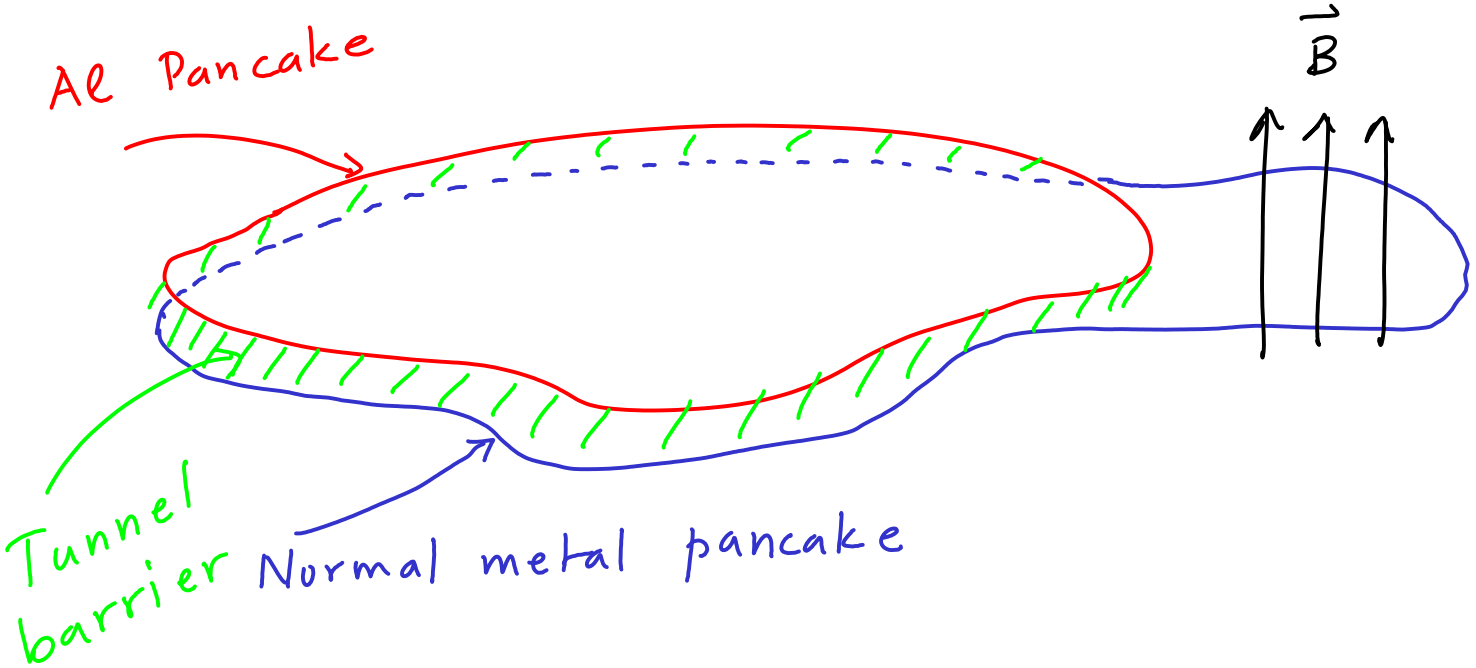
$$\langle \phi_\mu^*(i)\phi_\nu^*(j)\phi_\nu(k)\phi_\mu(l)\rangle = \frac{\delta_{il}\delta_{jk}}{g^2} + \frac{\delta_{ij}\delta_{kl}}{g^2}\,\frac{E_X\delta/\pi}{E_X^2 + (\epsilon_\mu - \epsilon_\nu)^2}$$

Adam, Brouwer, Sethna, and Waintal, PRB 66, 165310 (2002).

A Central Point

RMT crossovers produce enhanced correlations between wave functions which strongly enhance quantum fluctuations in interacting models.

Example 1: Two coupled dots above T_c



The Hamiltonian

$$H = H_1 + H_2 + H_{tunnel}$$

$$H_1 = \sum_{ijs} H_{ij}^{(O)} c_{is}^{\dagger} c_{js} - g \sum_{ij} c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} c_{j\downarrow} c_{j\uparrow}$$

$$H_2 = \sum_{\alpha\beta s} H_{\alpha\beta}^{(X)} c_{\alpha s}^{\dagger} c_{\beta s}$$

$$H_{tunnel} = \sum_{i\alpha s} V_{i\alpha} c_{is}^{\dagger} c_{\alpha s} + h.c.$$

There are actually two different RMT crossover scales here. One is the GOE$\rightarrow$GUE crossover in dot 2, with the crossover scale E_X, and the other is the crossover scale induced by the tunneling between the two dots E_U. For $|\epsilon_1 - \epsilon_2| \ll E_U$, the two wave functions have RMT correlations, while for large energy separations they are uncorrelated.

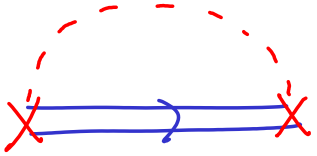
Technical Details

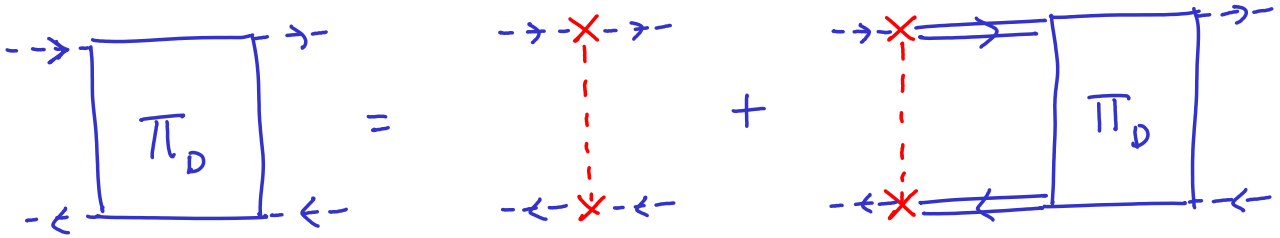
- Decompose the BCS interaction in Hubbard-Stratanovich in the imaginary time path integral to get the auxiliary fields $\Delta, \quad \Delta^*$.
- Above T_c they have no mean value, and are purely fluctuating. Integrate out the fermions to obtain the effective action for $\Delta, \quad \Delta^*$.
- For $E_U, \ E_X \gg \delta$, the effective action is the sum of many terms, and is self-averaging.
- The matrix elements that enter the effective action are products of noninteracting wavefunctions, whose averages can be found by Diffuson-Cooperon methods.

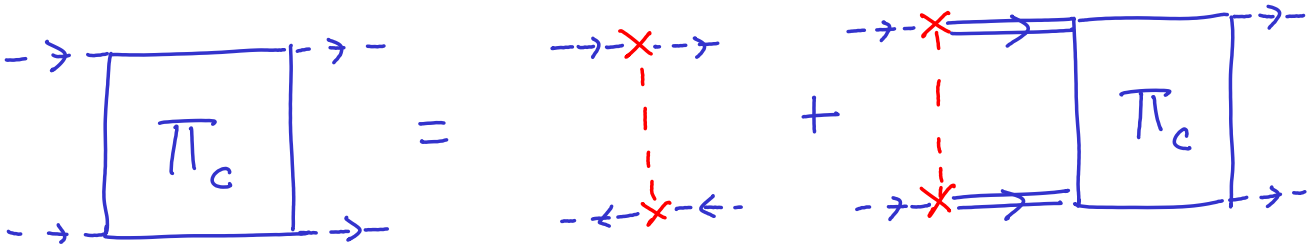
Some diagrams

$$\bar{G}_R(\omega) = \frac{1}{\omega - \Sigma_R(\omega)}$$

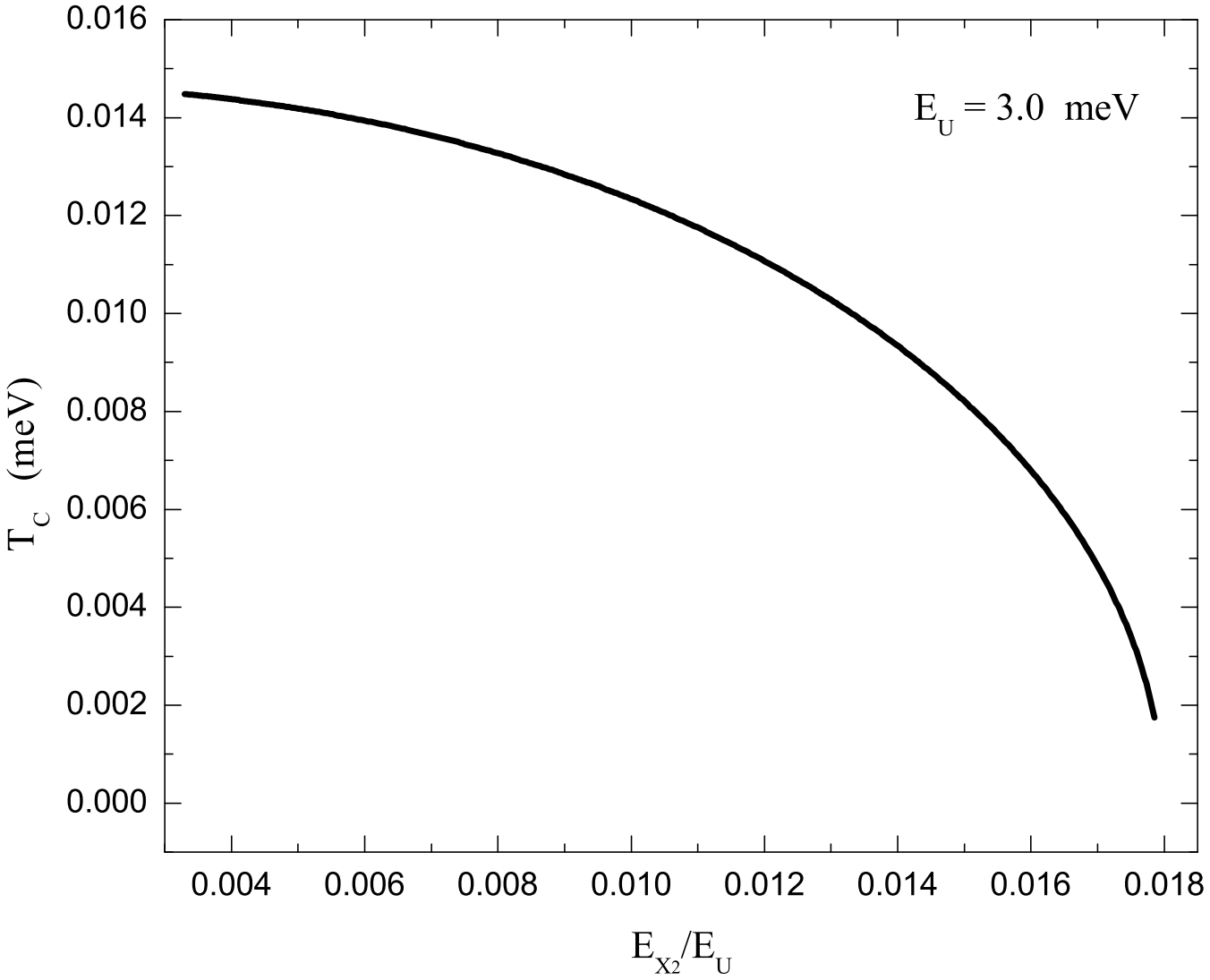
$\Sigma_R =$

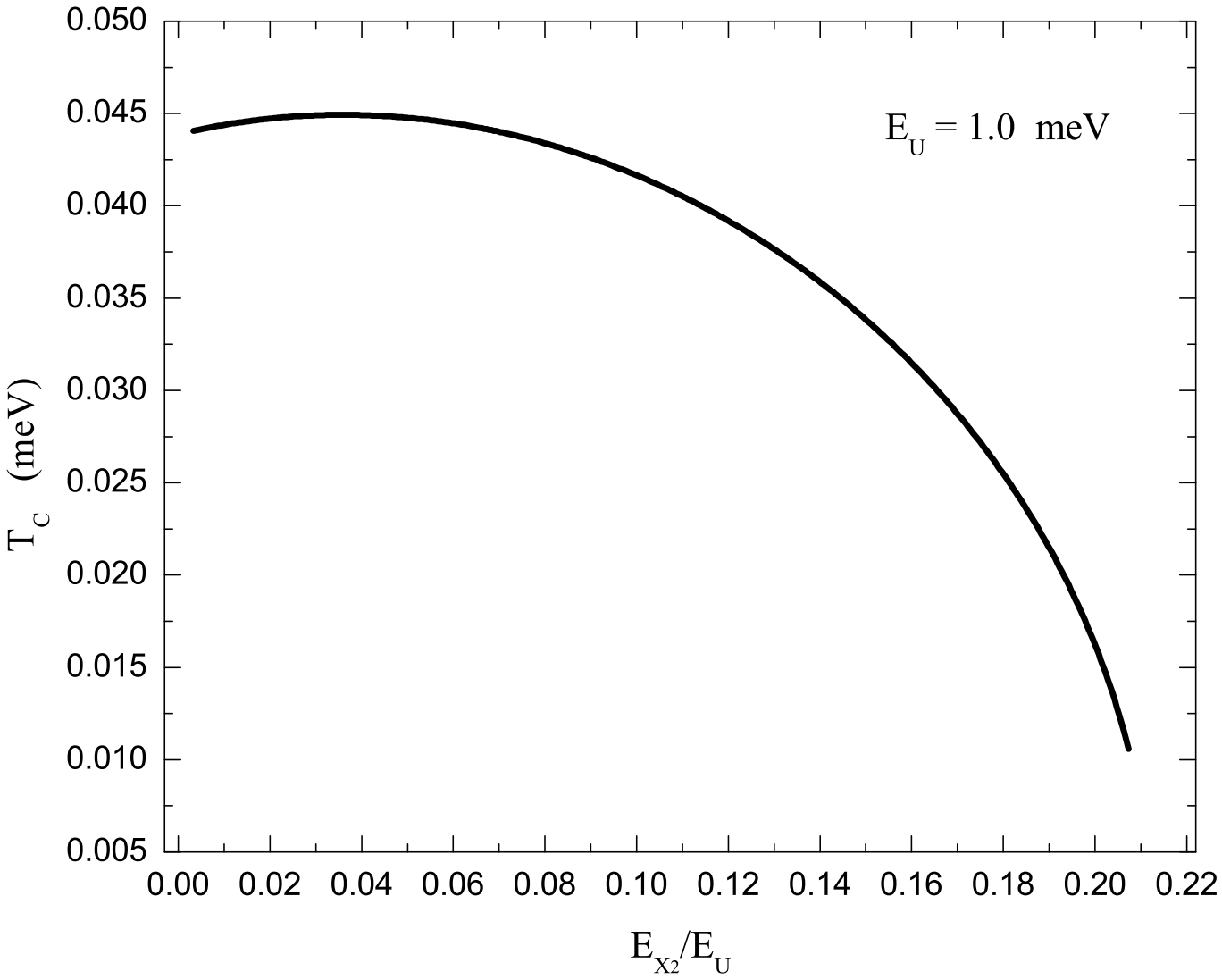


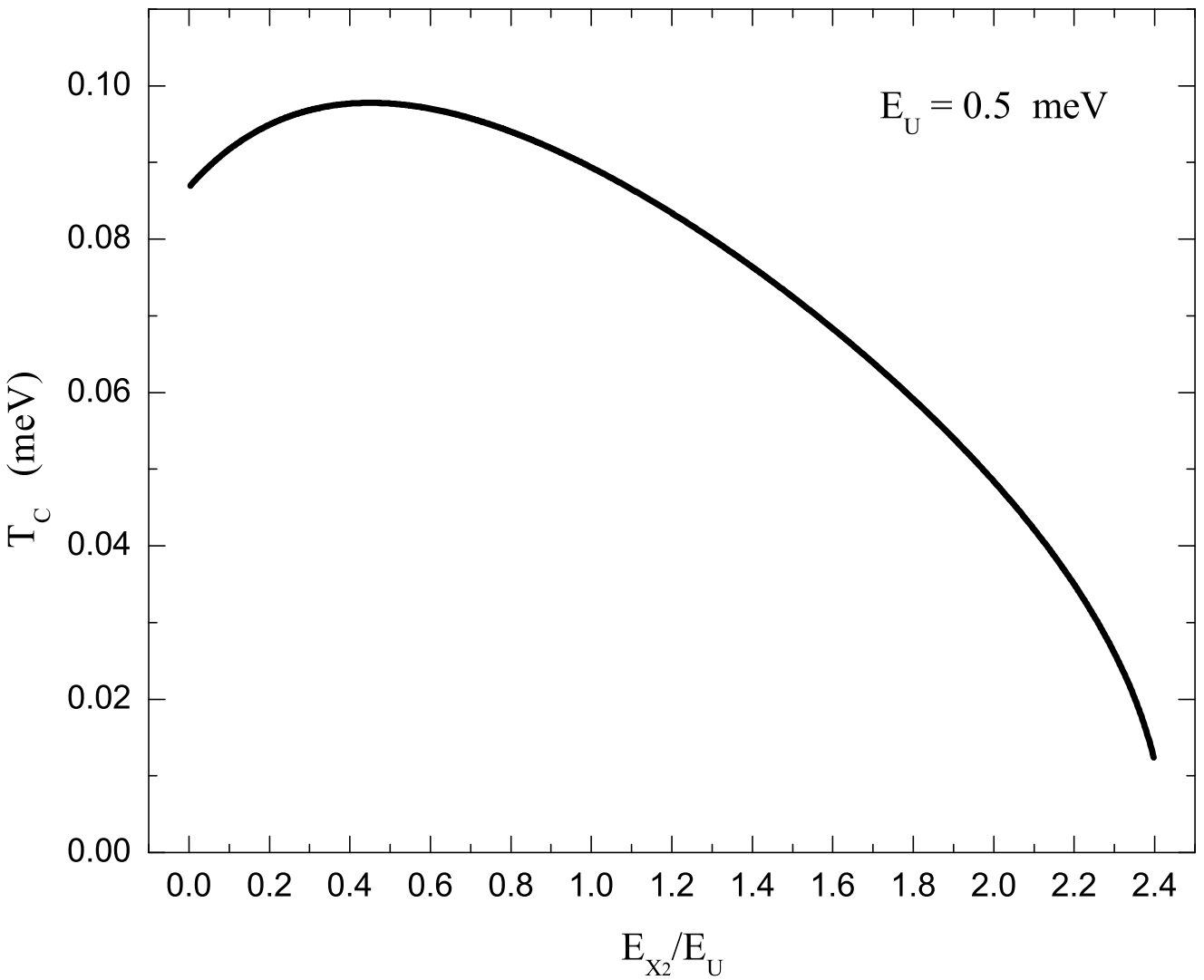




Results for T_c




0.050
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
T_C (meV)
E_U = 1.0 meV
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
E_{X2}/E_U


$E_U = 0.5$ meV
T_C (meV)
0.00
0.02
0.04
0.06
0.08
0.10
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
E_{X2}/E_U

Why is T_c nonmonotonic?

For small E_U, note that T_c increases as the orbital flux through dot 2 is increased. This can be understood as follows: When the two dots are coupled, the T_c drops from that of the Al grain (call it $T_{c0} = 0.218 meV$) because the BCS interaction has to be shared between the dots. The approximate expression of this suppression for $E_U \gg T_{c0}$ is

$$T_c \simeq T_{c0} \frac{\omega_D}{E_U} e^{-1/g}$$

Heuristically, when E_X increases the first effect is to gap the Cooperon in dot 2, thus suppressing the tunneling, which has the effect of increasing T_c.

Quasiparticle Broadening

At the quantum critical point at $T = 0$, the BCS order parameter fluctuations have the dissipative action

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |\omega| |\Delta(\omega)|^2$$

Quasiparticles scatter off these quantum critical fluctuations, and acquire a width

$$\Gamma(\omega) \simeq \delta \log\left(\frac{\omega}{\delta}\right)$$

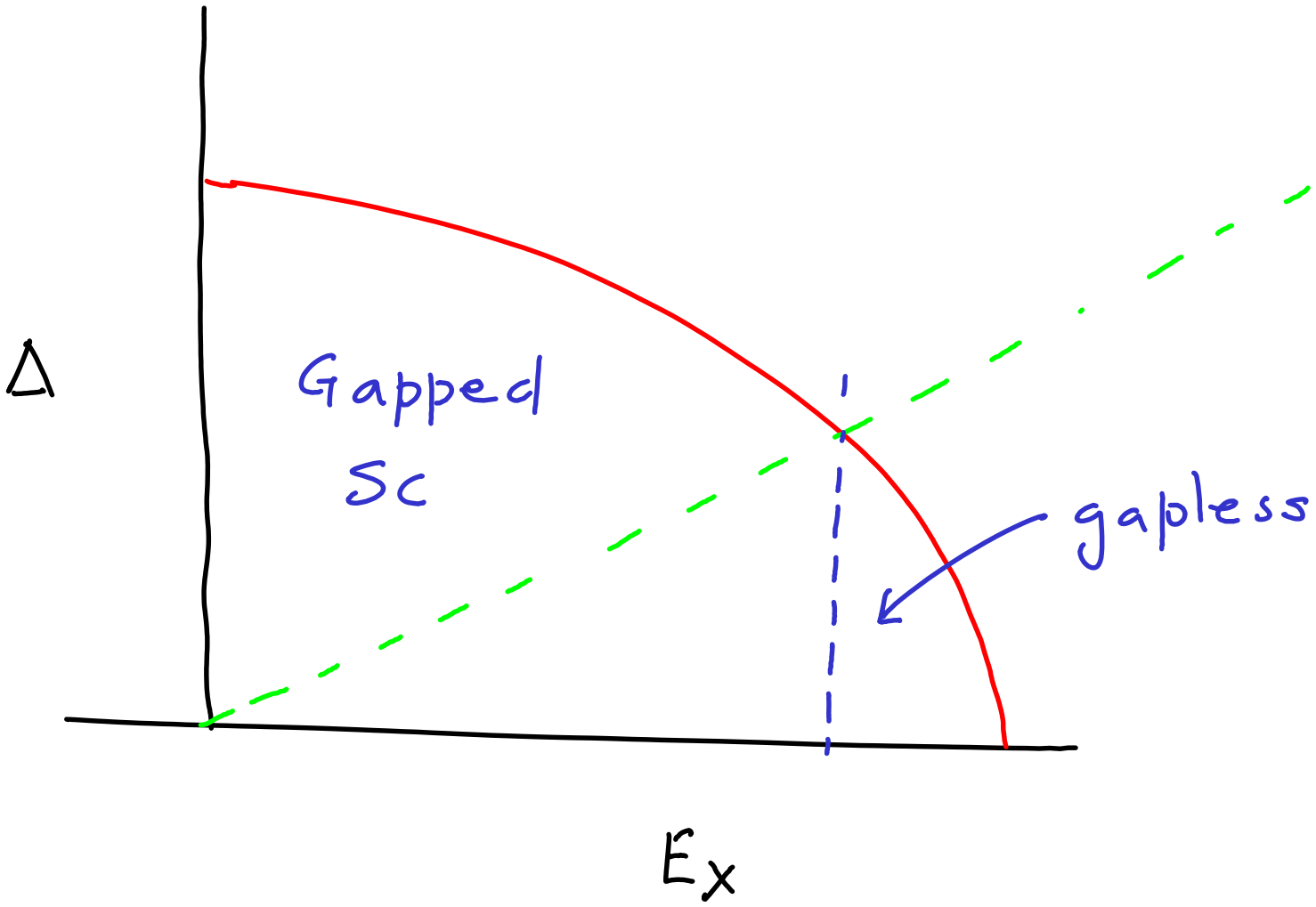
which is much broader than the usual Fermi-liquid-like form $\Gamma \simeq \omega^2/E_T$ for weakly interacting dots.

One dot below T_c

So far we have been looking at the critical point, where there is no mean value for Δ. Now let us go below T_c and see what happens when an orbital flux is turned on. Some previous work

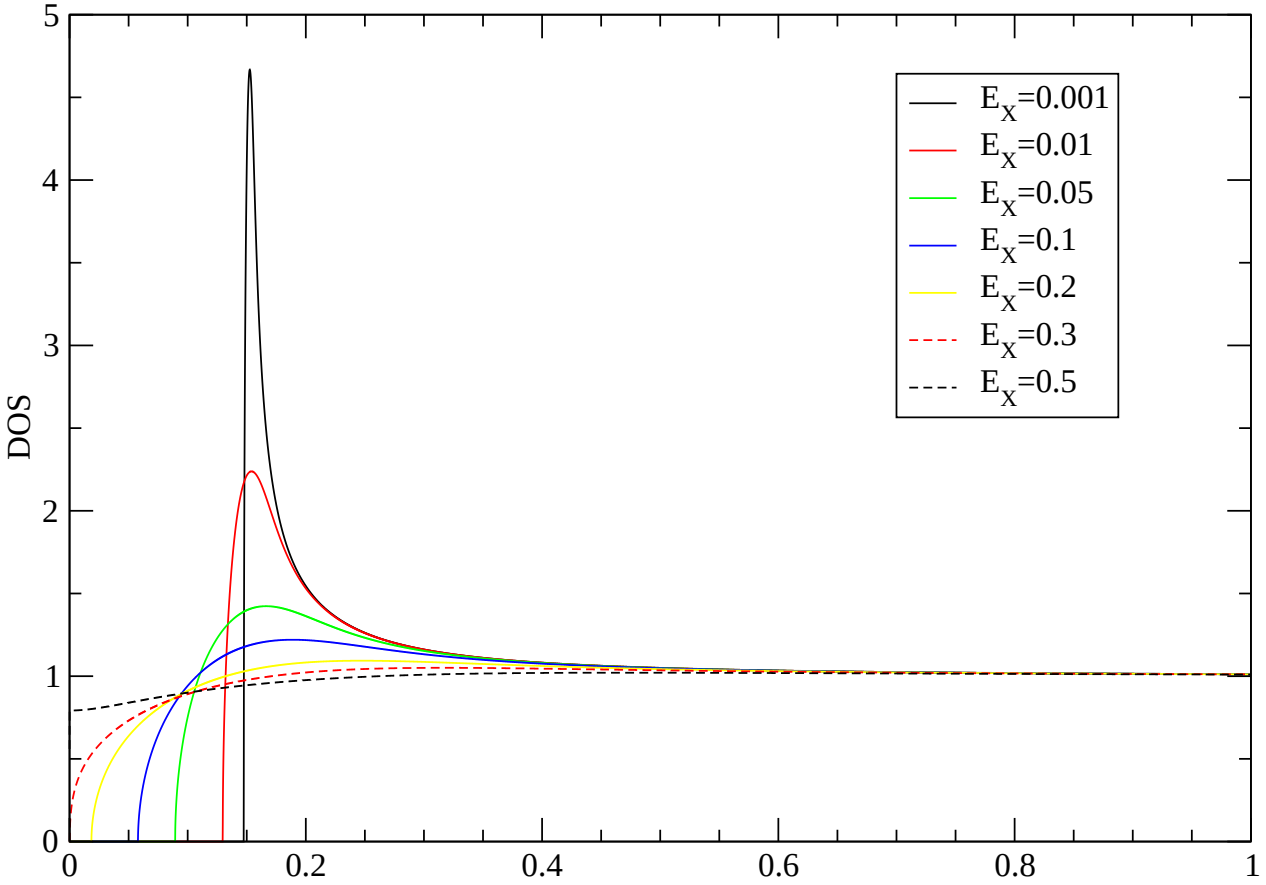
Abrikosov and Gorkov, Sov. Phys. JETP 12, 1243 (1961)$\Rightarrow$"Gapless superconductor"
Larkin, Sov. Phys. JETP 21, 153 (1965)$\Rightarrow$"Gapless SC Grain"
S. Bahcall, PRL 77, 5276 (1996)$\Rightarrow$ RMT in the bulk of high-T_c
Beloborodov, Efetov and Larkin, PRB 61, 9145 (2000)$\Rightarrow$ Phase diagram

Larkin 1965



Density of States

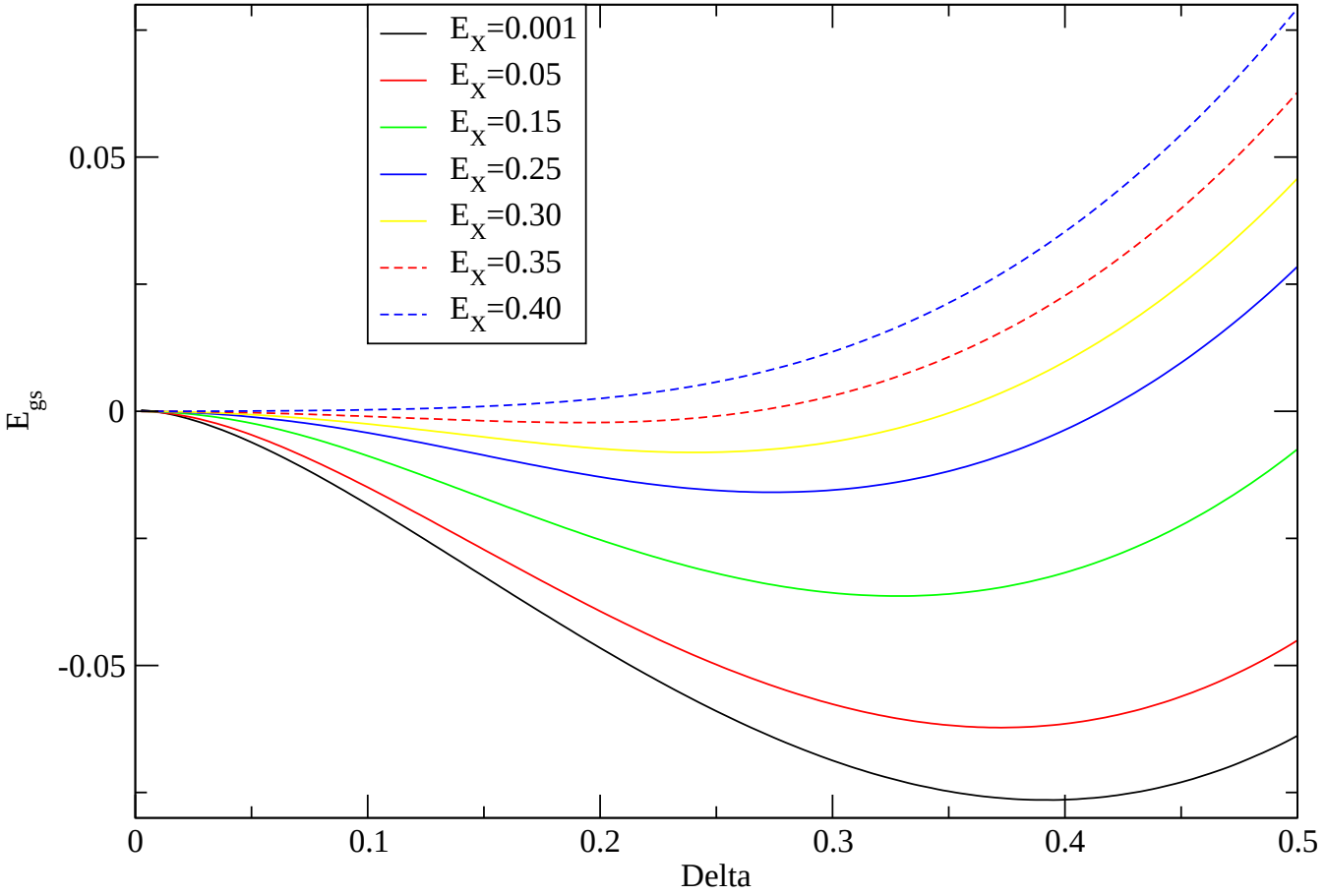
Assume for simplicity that $E_{Zeeman} = 0$, and the T-breaking is purely orbital, characterized by $E_X = X^2 E_T$. One can obtain the mean-field DOS by large-N methods.



The basic features of the DOS are:

- The sharp singularity at Δ is broadened into a peak of width $\simeq E_X^{\left(\frac{2}{3}\right)}$
- The gap gets reduced to $\Delta - \frac{3\Delta^{(1/3)} E_X^{(2/3)}}{2^{(5/3)}}$
- Beyond a critical $E_X = 2\Delta$ the DOS at $\omega = 0$ is nonzero.
- In the mean-field solution of the BCS problem, there is a region of "gapless superconductivity" beyond this critical E_x.

Ground state energy vs Δ



Wildly broad quasiparticles?

We already know that the gapless superconductor has large quantum fluctuations of Δ. The wavefunction of Δ has a width of $\simeq E_X$. Consider the exact single-particle Green's function for a particular sample

$$G_{ij}^{ss'} = \sum_{\alpha\sigma} \frac{\phi_{\alpha\sigma}(is)\phi^*_{\alpha\sigma}(js')}{\omega - \sigma E_\alpha}$$

Both $\phi_{\alpha\sigma}$ and E_α are implicit functions of Δ. Consider the static path approximation for G, which means integrating it over Δ with the proper weight, that is, the proper wavefunction. From studies of parametric correlations, we know that even tiny changes of Δ of order $\delta/\sqrt{N}$ scramble the wavefunction completely. So the integral over Δ is equivalent to a disorder average $\Rightarrow$ Broad quasiparticles with $\Gamma \simeq E_X$.

Summary

- Disorder and interactions can be treated nonperturbatively together in quantum dots described by RMT, thanks to large N or large $N_X = E_X/\delta$, $N_\Delta = \Delta/\delta$.
- Soft symmetry-breaking by RMT crossovers strongly enhances quantum fluctuations.
- The gapless superconducting grain with $E_X \gg E_{Zeeman}$ seems particularly suited to large quantum fluctuations and very broad quasiparticles.

Take-home message

A class of universal interacting crossover ensembles which are the many-body descendants of RMT single-particle crossover ensembles.

Open Questions

- Tunneling DOS of the gapless superconductor?
- Spin susceptibility$\Rightarrow$ Pseudogap?
- Quantum critical scaling for nonzero T ?