Non-equilibrium bosonization of Luttinger liquids: Tunneling spectroscopy and full counting statistics

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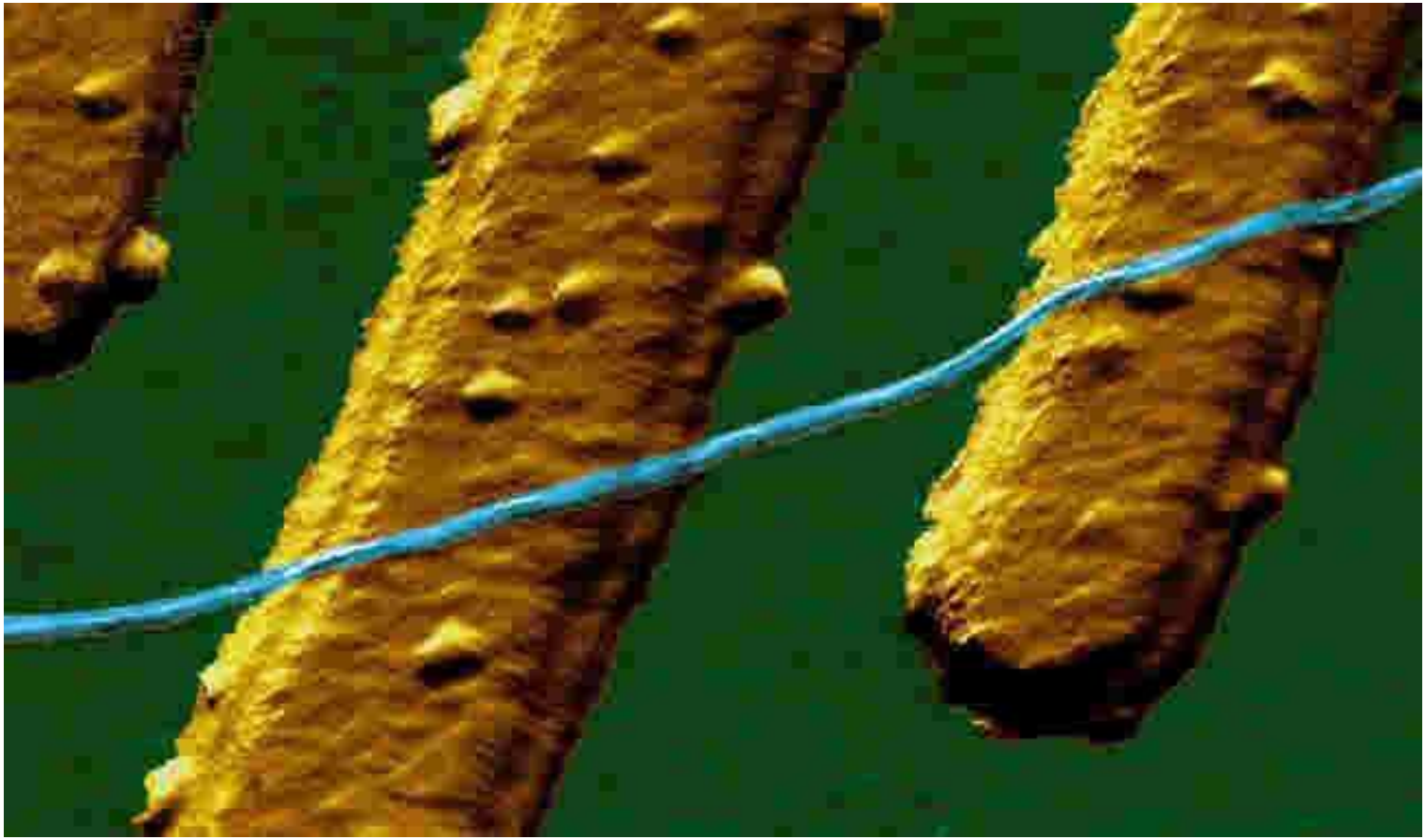
Outline

- Non-equilibrium Luttinger liquids: Setups
- Tunneling spectroscopy: Partial non-equilibrium
  - Tunneling DOS, zero-bias anomaly (ZBA), dephasing
  - Energy relaxation
- Full non-equilibrium: Non-equilibrium bosonization
  - Free electrons
  - Interacting electrons
  - Generalizations: non-equilibrium LL interferometry, spin
- Full counting statistics

Gutman, Gefen, ADM,
PRL 101, 126802 (2008); PRB 80, 045106 (2009);
EPL 90, 37003 (2010); PRB 81, 085436 (2010); arXiv:1003.5433;

Ngo Dinh, Bagrets, ADM, PRB 81, 081306(R) (2010)

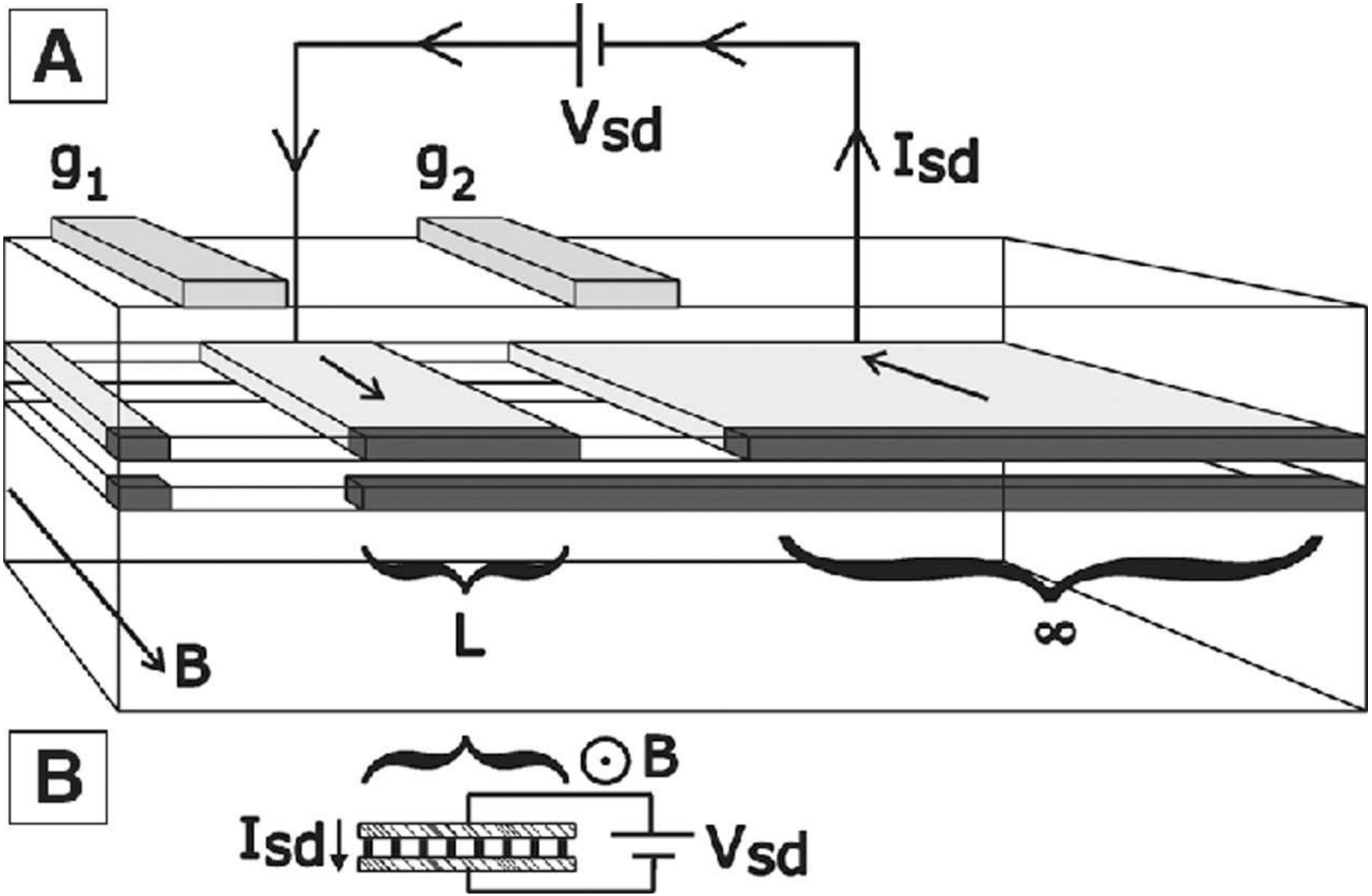
Quantum wires: Carbon nanotubes



Tans et al. (Dekker group), Nature '98

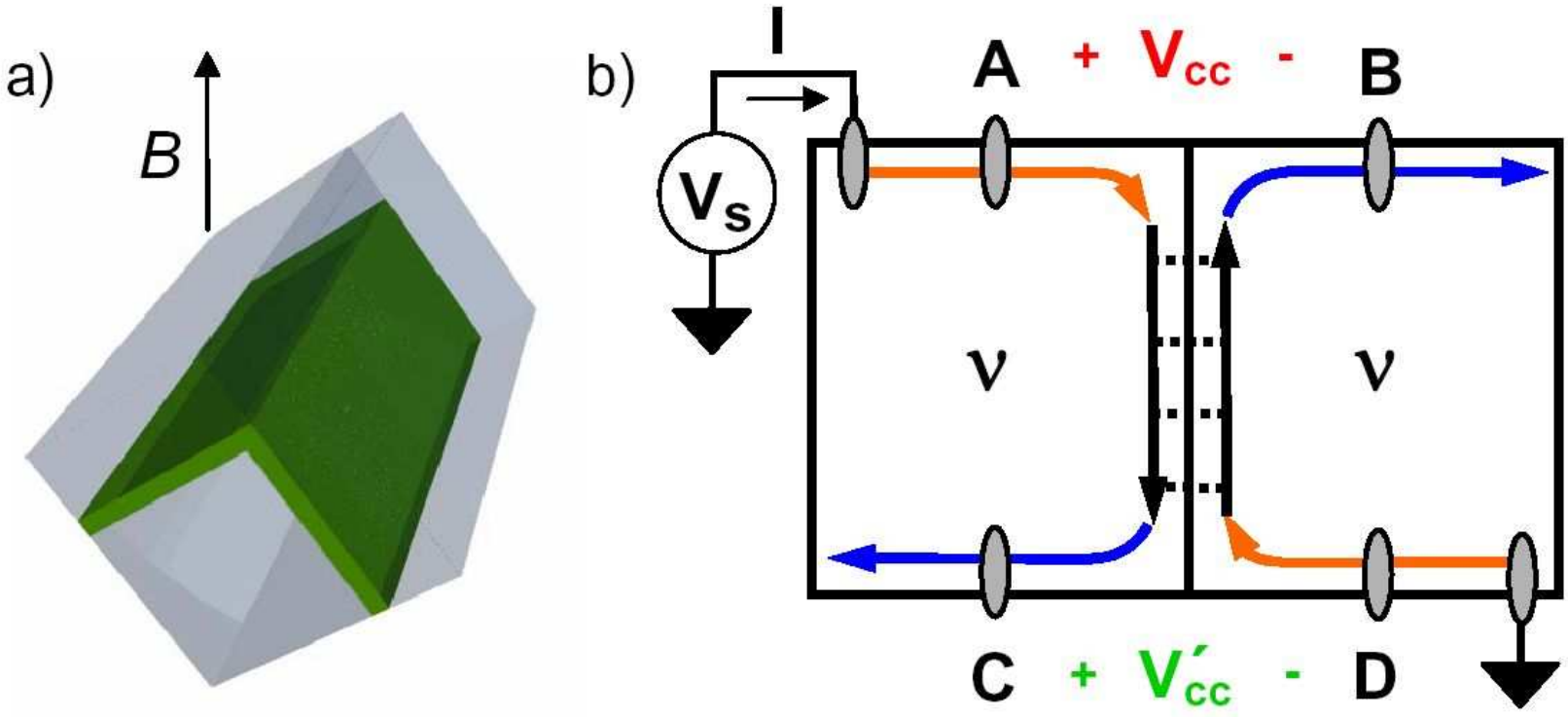
Semiconductor quantum wires:

Auslaender et al (Weizmann Inst./Bell Labs), Science 2002



Quantum wires: Quantum Hall edges

quantum wires with length of several mm



M. Grayson et al. APL'05, PRB'07

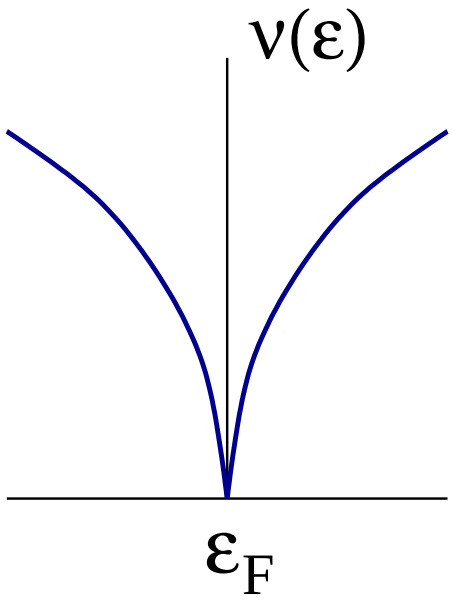
Interacting 1D fermions: What is special?

- strongly correlated state — Luttinger liquid
- Tomonaga-Luttinger model: exact solution by bosonization
- paradigmatic example of a non-Fermi-liquid state
- described in terms of bosonic modes (plasmons)
- no Fermi-liquid quasipartcle pole (residue $Z = 0$)

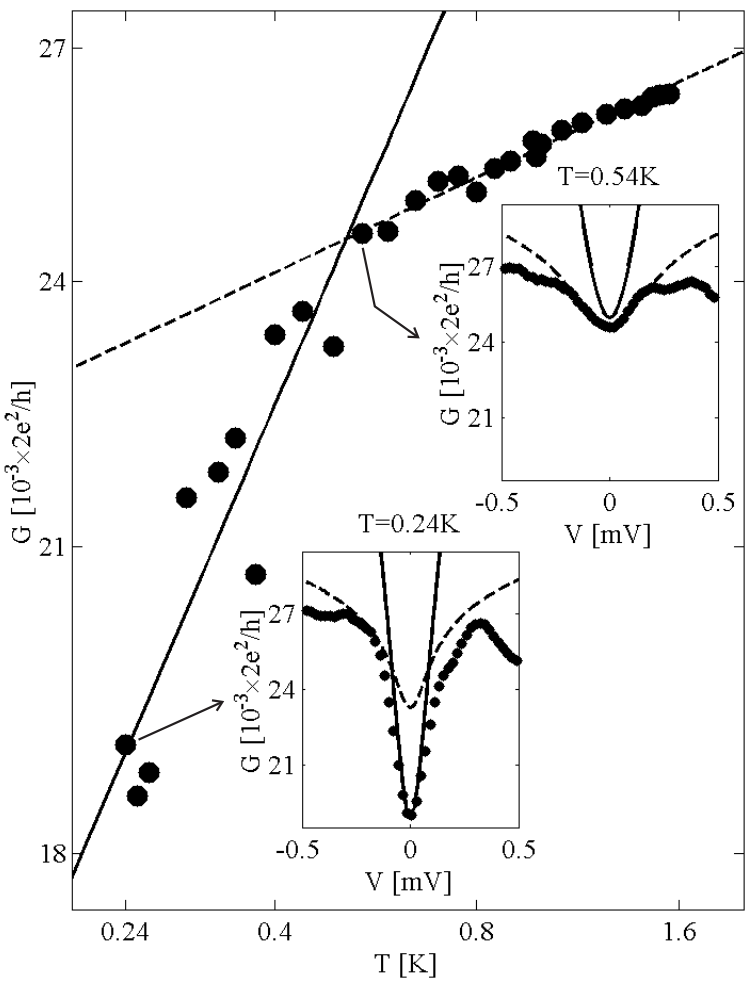
$\longrightarrow$ common wisdom: no fermionic excitations

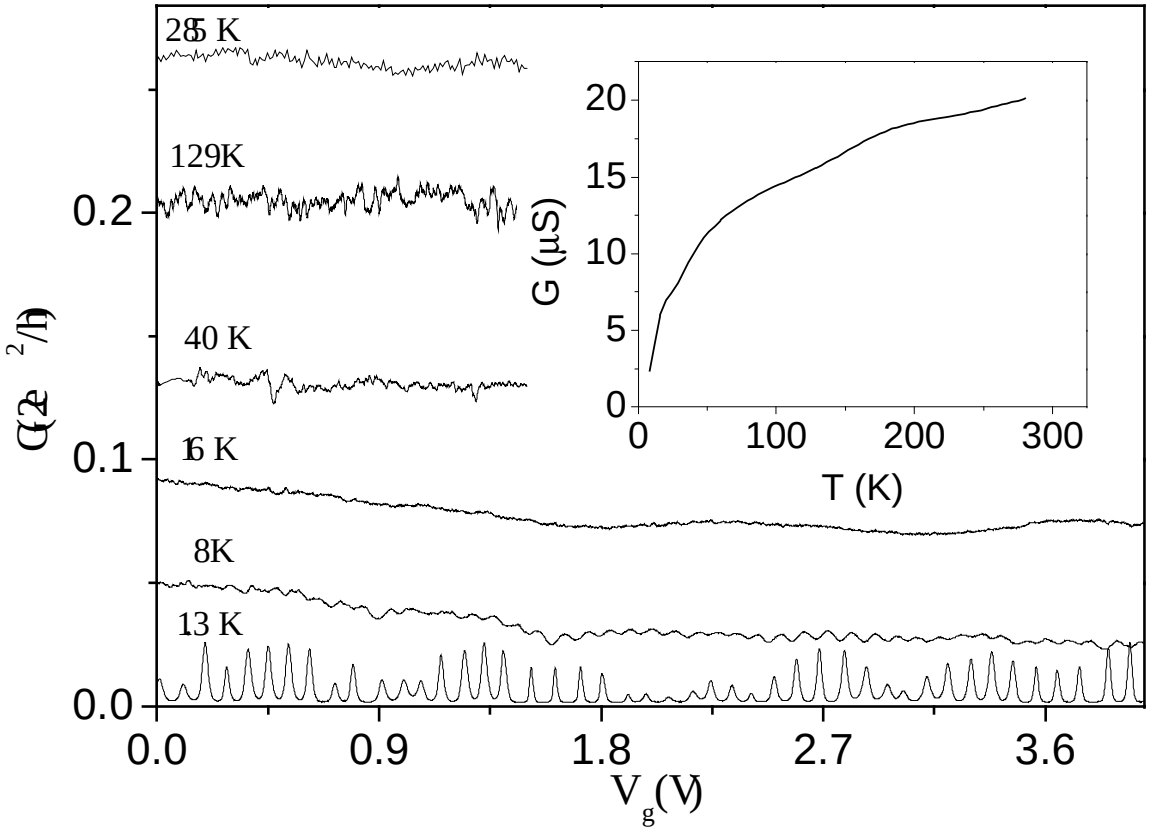
- zero-bias anomaly:

tunneling DoS vanishes at the Fermi level: $\nu(\epsilon) \propto |\epsilon|^{\alpha}$



1D: Luttinger liquid behavior in experiments





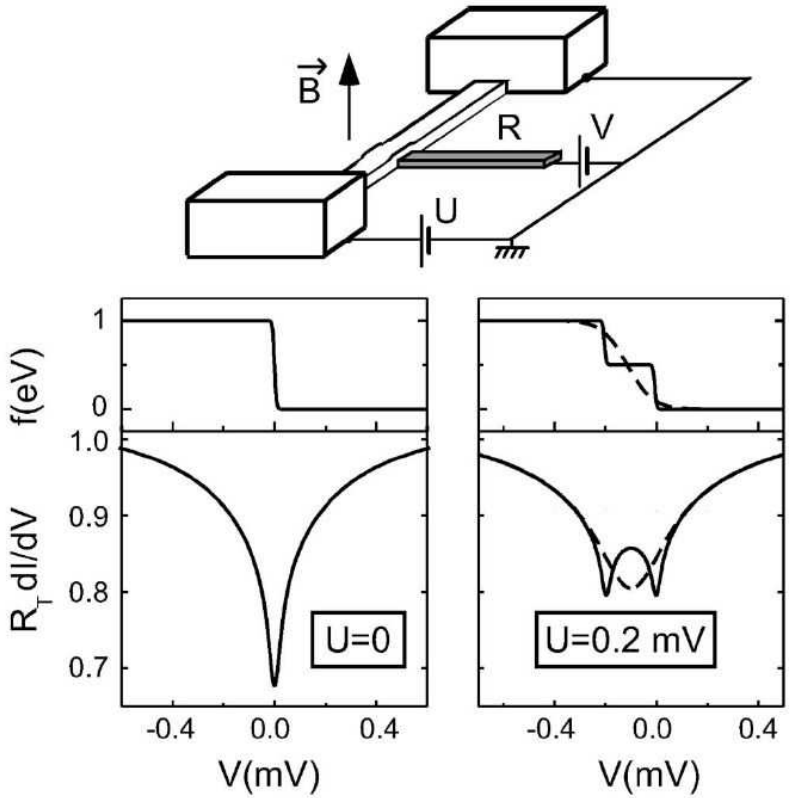
Tserkovnyak, Halperin, Auslaender, Yacoby, PRB'03

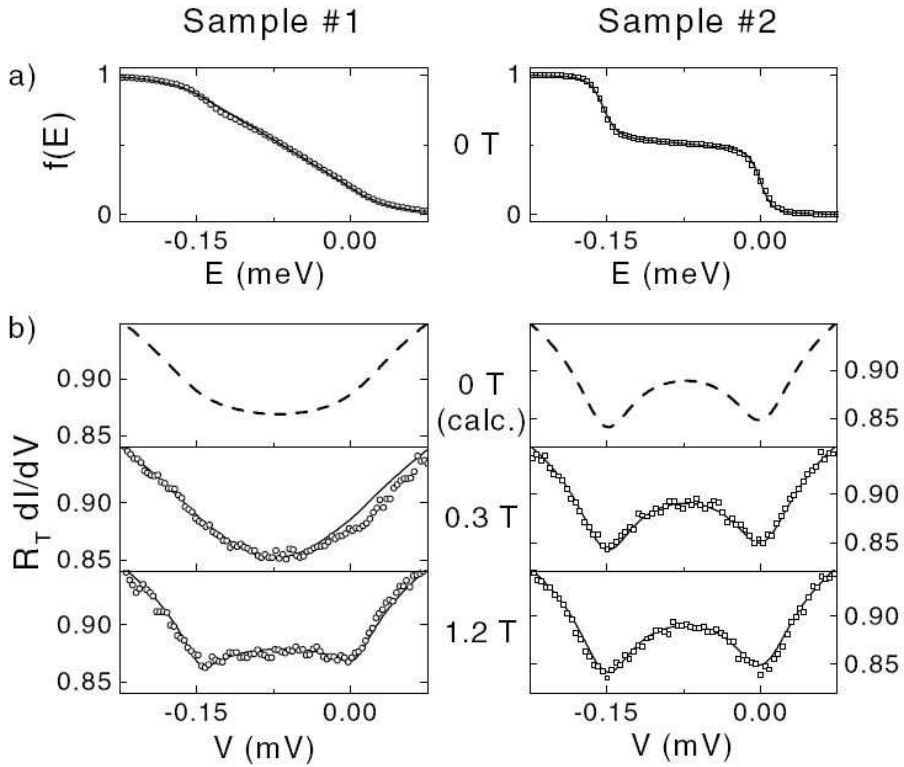
zero-bias anomaly in tunneling into GaAs LL wire

Bockrath et al, Nature '99

lowering T $\longrightarrow$ power-law suppression of conductance of a nanotube with impurities

Tunneling spectroscopy of thick metallic wires





tunneling current $\longrightarrow$

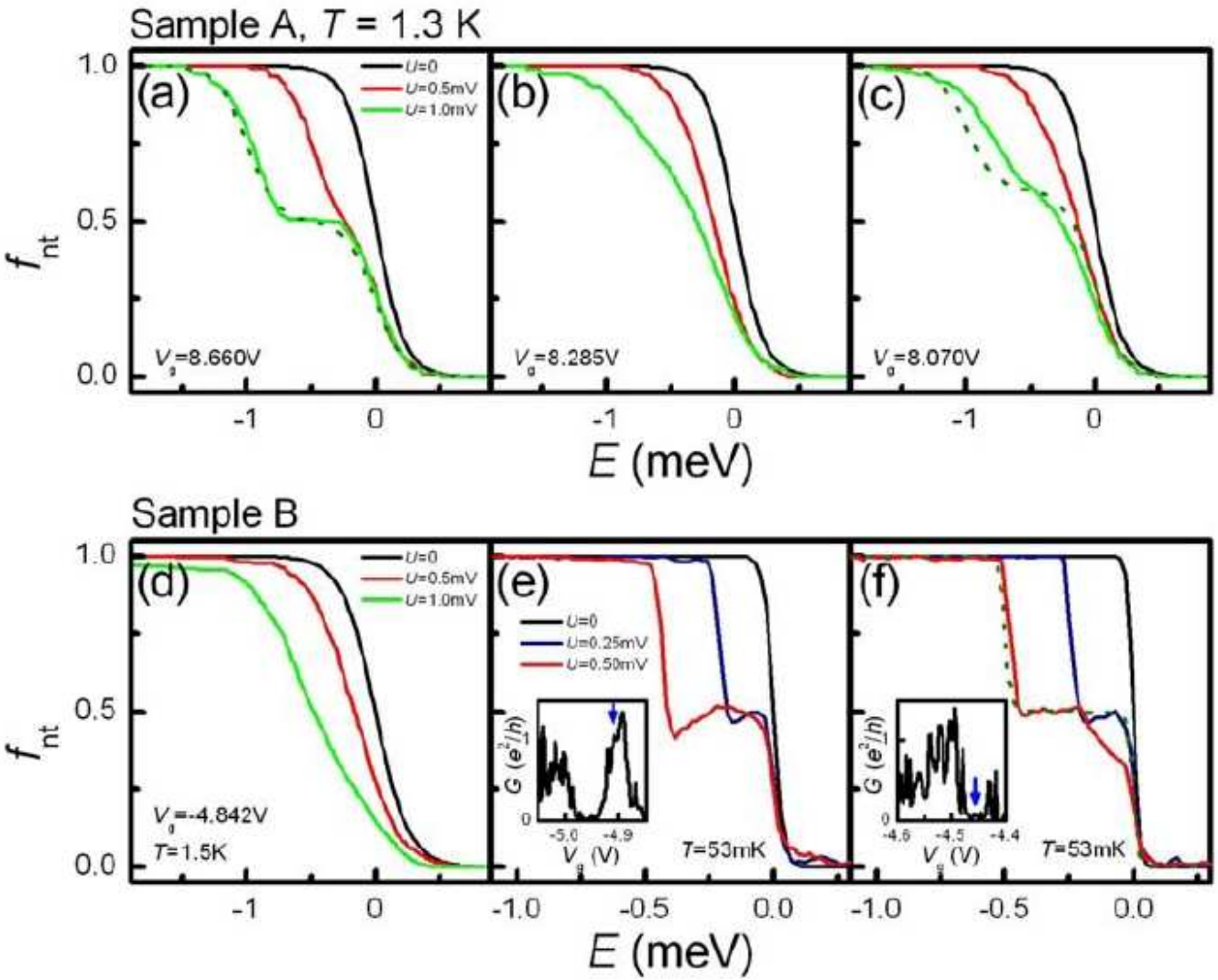
information about local Green functions $G^{\gtrless}(x, x; t)$ $\longrightarrow$

- tunneling DOS
- distribution function

Pothier, Gueron, Birge, Esteve, Devoret, PRL'97

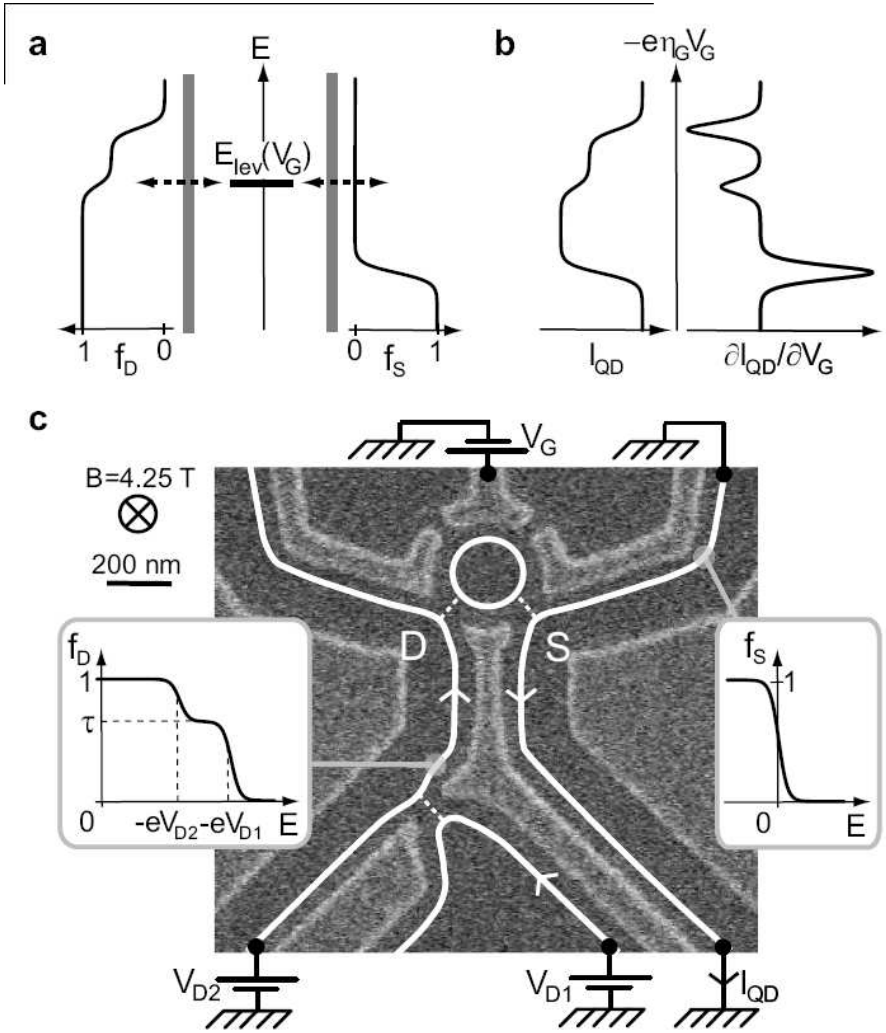
Anthore, Pierre, Pothier, Esteve PRL'03

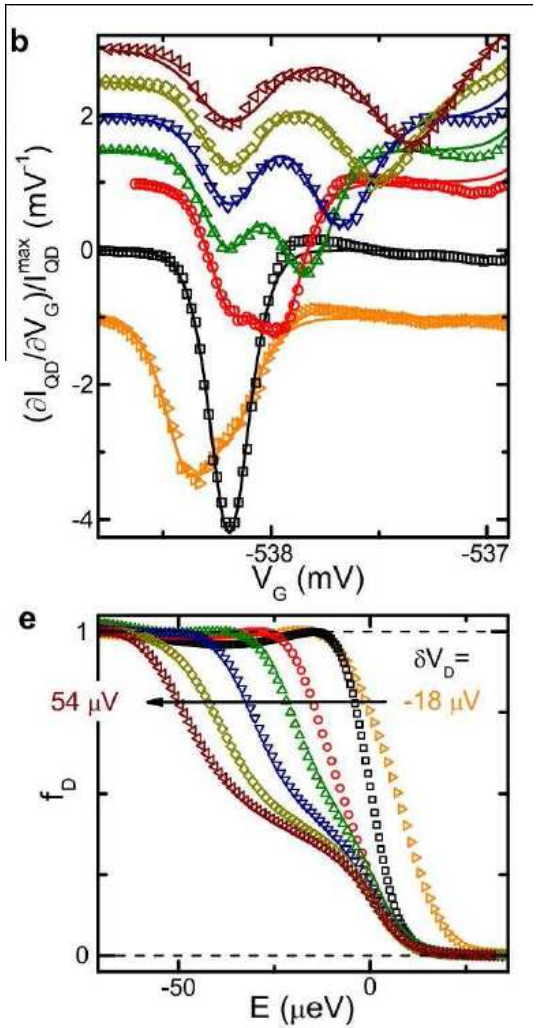
Tunneling spectroscopy of carbon nanotubes



Experiment: Y. Chen, T. Dirks, G. Al-Zoubi, N. Birge & N. Mason, PRL'08

Tunneling spectroscopy of quantum Hall edge states





Experiment: Altimiras, le Sueur, Gennser, Cavanna, Mailly, Pierre, Nature Physics 6, 34 (2010)

Non-equilibrium 1D: What is special?

- **Strongly correlated state** (Luttinger liquid) **out of equilibrium**

- **No energy relaxation in LL**

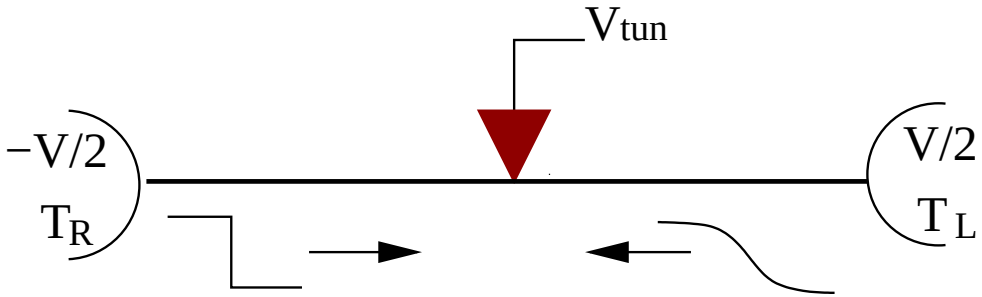
  (in the absence of inhomogeneities, neglecting non-linearity of spectrum and momentum dependence of interaction)

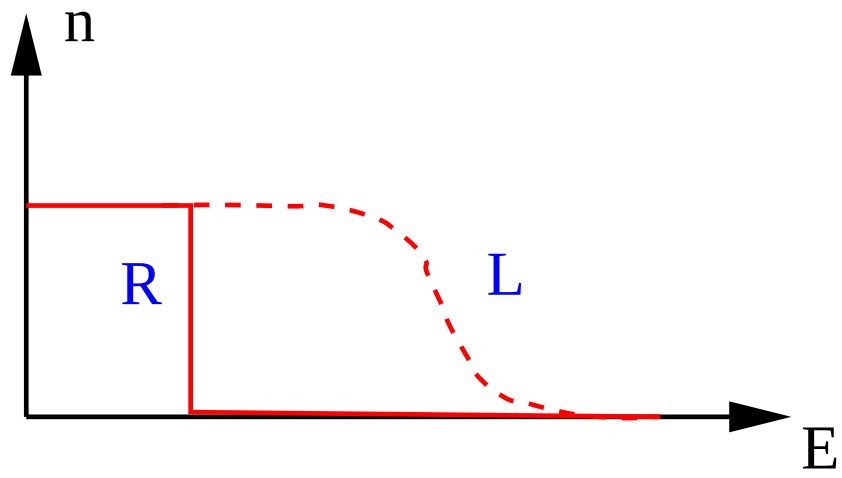
- Equilibrium: exact solution via **bosonization**

  Non-equilibrium – ?

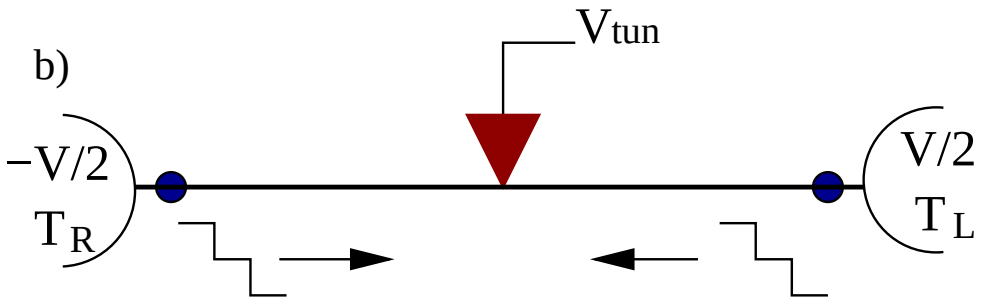
  Fermionic distribution within the **bosonization** formalism – ?

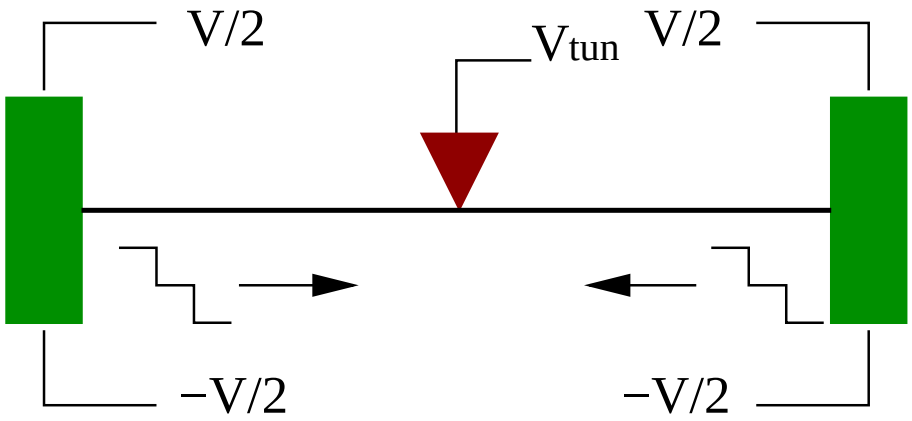
Tunneling spectroscopy of non-equilibrium LL: Setups





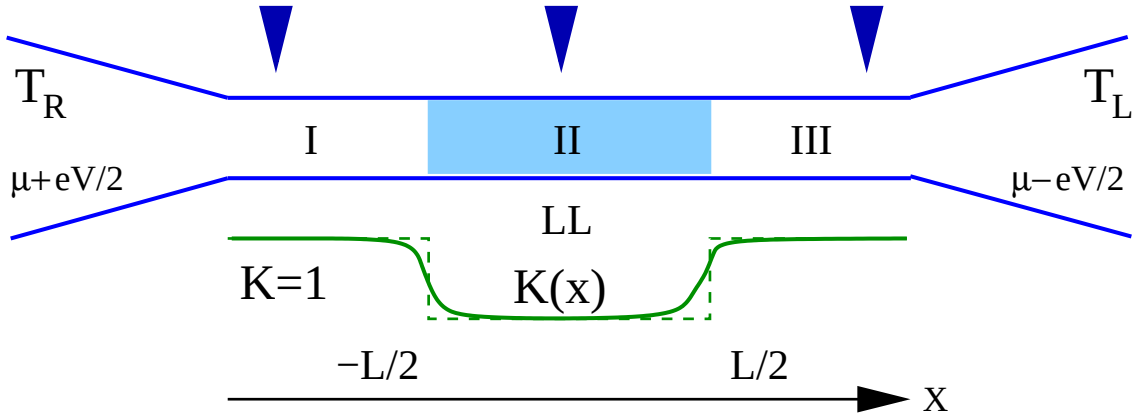
Partial non-equilibrium: **Left- and right-movers separately in equilibrium but** $V \neq 0$, $T_R \neq T_L$





Fully non-equilibrium LL setups

Luttinger Liquid in partial non-equilibrium



Hamiltonian $$H_0 = \sum_{k,\eta} v(\eta k - k_F)\psi^\dagger_\eta(k)\psi_\eta(k)$$

$$\eta = R/L \longrightarrow \pm 1$$

$$H_{\mathrm{e-e}} = \int dx \frac{g(x)}{2}(\rho_R + \rho_L)^2 \qquad K(x) = [1 + g(x)/\pi v]^{-1/2}$$

Bosonization: Fermionic operators $$\psi_\eta \simeq \left(\frac{\Lambda}{2\pi v}\right)^{1/2} \exp(i\phi_\eta)$$

electron density $$\rho_\eta = \frac{\eta}{2\pi}\partial_x \phi_\eta$$

Tunneling spectroscopy: General results

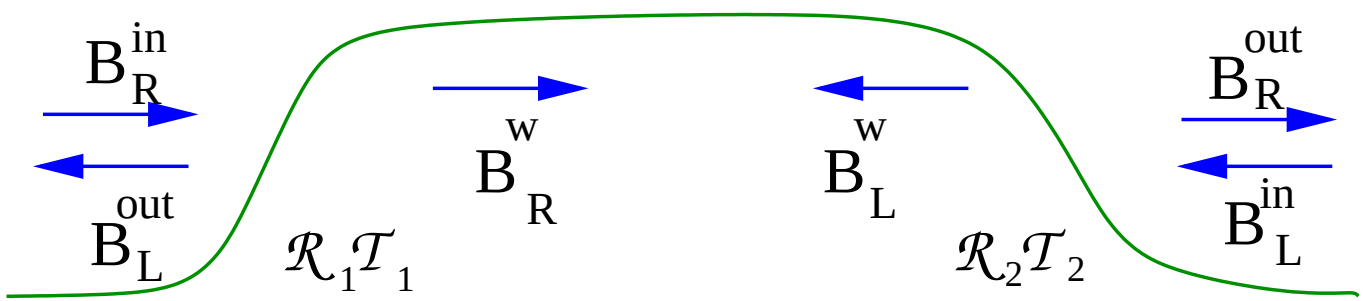
Keldysh Green functions $\quad G^{\gtrless}_{\eta}(x,t;x,0) = G^{\gtrless}_{\eta,0}(t)e^{-i\eta eVt/2}e^{-\mathcal{F}^{\gtrless}_{\eta}}$

$$G^{\gtrless}_{\eta,0}(t) = -\frac{T}{2v}\frac{1}{\sinh \pi T(t \mp i/\Lambda)}$$ — free fermions

$$\mathcal{F}^{\gtrless}_{R} = \int_0^\infty \frac{d\omega}{\omega}\left[(B^{\rm w}_R - B^{(0)}_R)(1-\cos\omega t) + \gamma\Big((B^{\rm w}_R + B^{\rm w}_L)(1-\cos\omega t) \pm i\sin\omega t\Big)\right]$$

$\gamma = (K-1)^2/2K$ — interaction strength

$B_\eta(\omega)$ — plasmon distribution function



$$B^{\rm w}_R = \mathcal{T}_1 B^{\rm in}_R + \mathcal{R}_1 B^{\rm w}_L \qquad B^{\rm out}_L = \mathcal{R}_1 B^{\rm in}_R + \mathcal{T}_1 B^{\rm w}_L \qquad B^{\rm in}_\eta \equiv B^{(0)}_\eta$$

Non-interacting parts of the wire

- no effect on tunneling DOS
- fermionic distr. function: energy redistribution due to plasmon scattering

$$n_R(t) = n_{R,0}(t)e^{-\mathcal{F}_R(t)} = \frac{i}{2}e^{-ieVt/2}\left(\frac{T_R}{\sinh \pi T_R t + i0}\right)^{\mathcal{T}}\left(\frac{T_L}{\sinh \pi T_L t + i0}\right)^{\mathcal{R}}$$

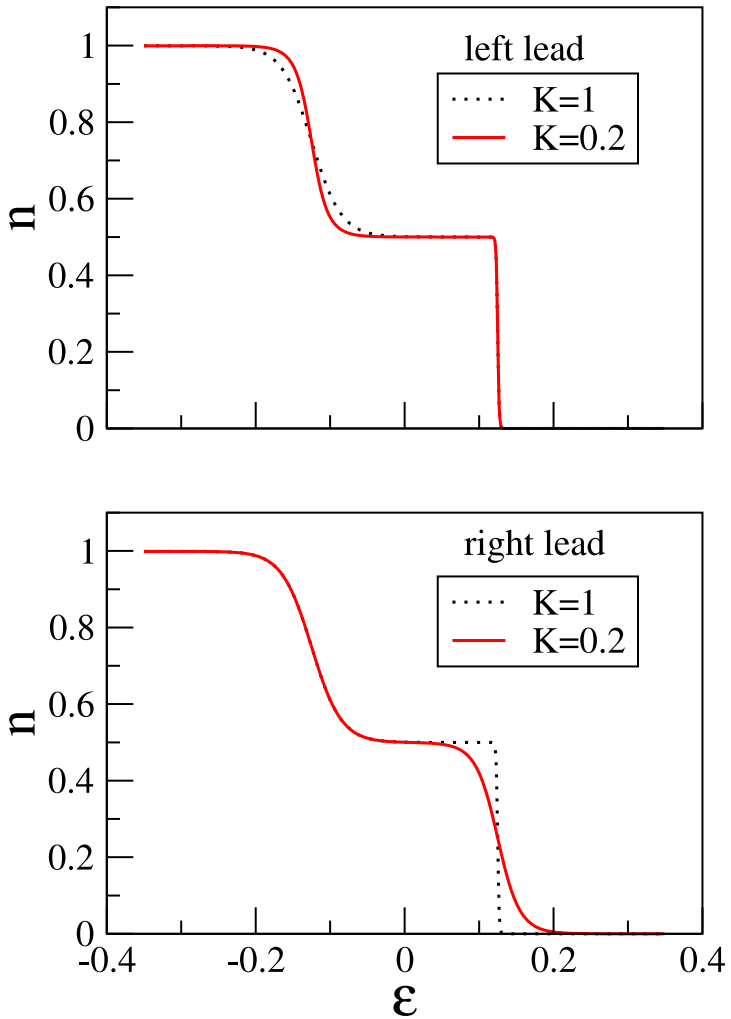


Figure:

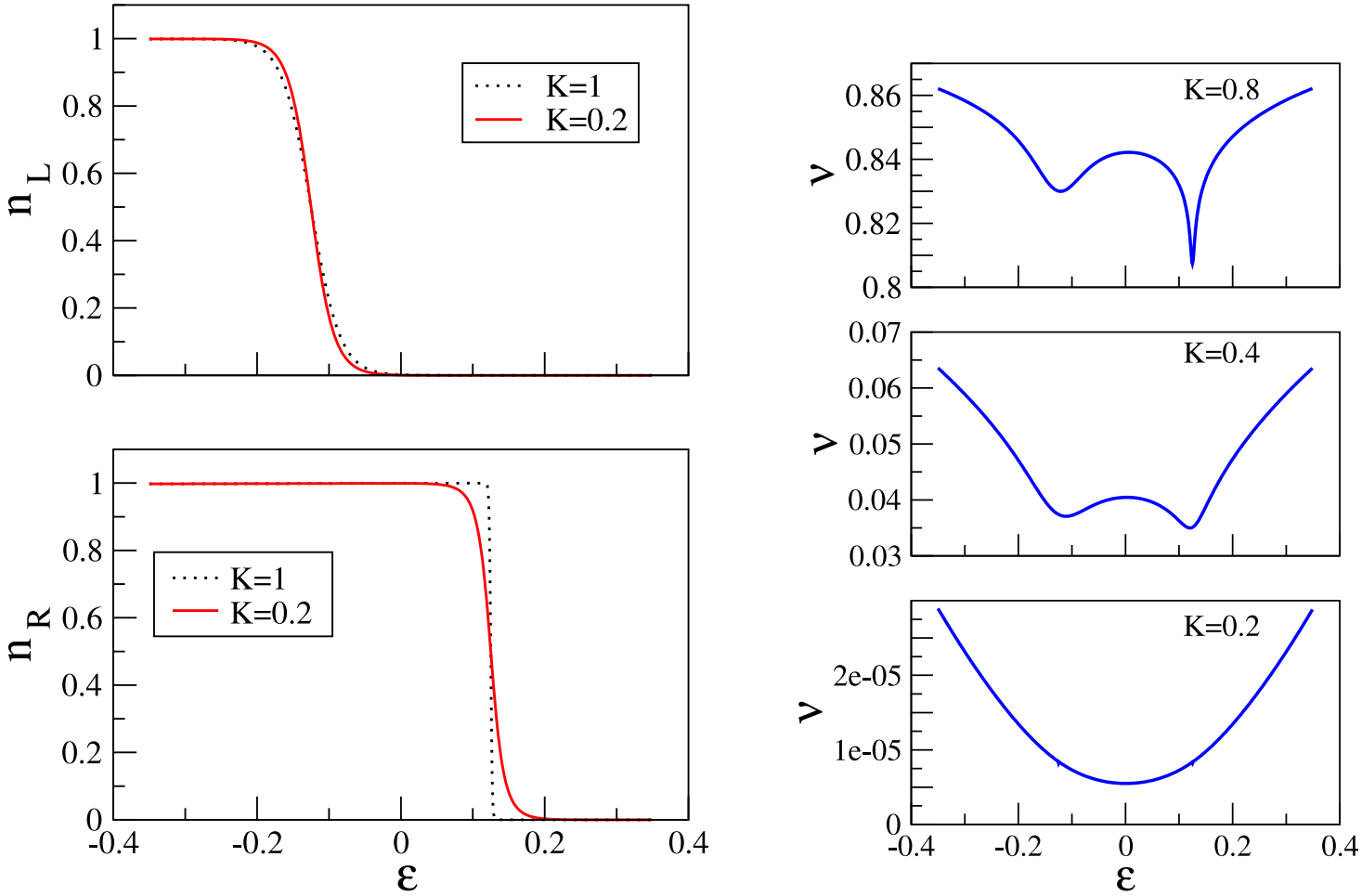
$n = n_R + n_L$

sharp boundaries

$T_R = 0.001, \quad T_L = 0.2, \quad eV = 0.25$

Interacting part of the wire

- distr. function $n_\eta(t) = n_{\eta,0}(t) \exp\left\{-\int_0^\infty \frac{d\omega}{\omega}[B_\eta^{\rm w}(\omega) - B_\eta^{(0)}(\omega)](1-\cos\omega t)\right\}$
- broadening of $G^{\gtrless}(\epsilon)$, TDOS $\nu(\epsilon)$ by distribution function + dephasing



- Dephasing despite absence of energy relaxation in the wire; contributes crucially to broadening of singularities

Relation to electric and thermal conductance

Electric current $$I = ev(N_R - N_L) = \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi}[n_R(\epsilon) - n_L(\epsilon)] = \frac{e^2}{h}V$$

Maslov, Stone '95; Ponomarenko '95; Safi, Schulz '95; Oreg, Finkelstein '96

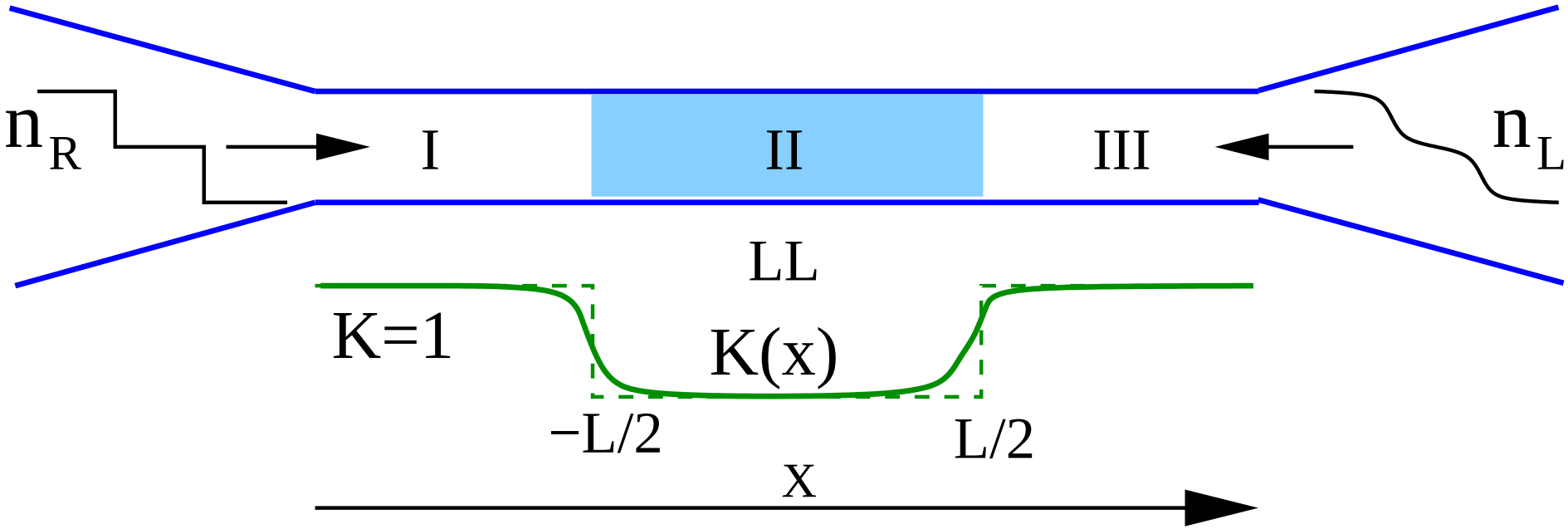
Thermal current

$$\begin{aligned} I_E &= v\partial_t \left[G_R^<(t,t') - G_L^<(t,t')\right]\big|_{t=t'} \qquad \text{in non-interacting part} \\ &= \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi}\epsilon[n_R(\epsilon) - n_L(\epsilon)] \\ &= \frac{1}{4\pi}\int_0^{\infty} d\omega\omega\mathcal{T}(\omega)[B_R^{(0)}(\omega) - B_L^{(0)}(\omega)] \\ &= \frac{\pi}{12}\mathcal{T}(T_R^2 - T_L^2) \qquad \text{for } \omega\text{-independent transmission} \end{aligned}$$

Fazio, Hekking, Khmelnitskii '98

Full non-equilibrium:

Non-equilibrium bosonization of Luttinger liquid



Non-equilibrium bosonization: Free fermions

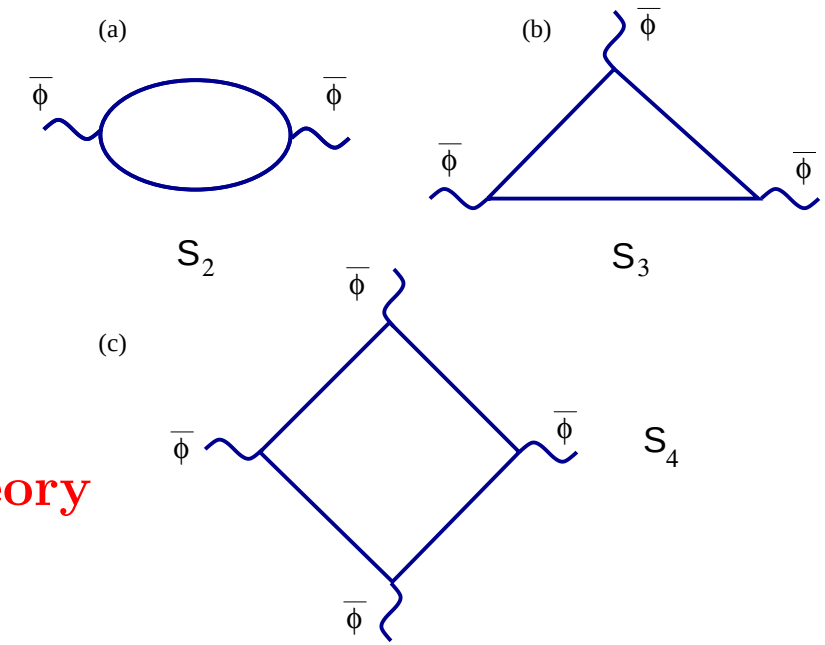
bosonized Keldysh action $$S_0 = \sum_\eta (\rho_\eta \Pi_\eta^{a^{-1}} \bar\rho_\eta - i \ln Z_\eta[\bar\chi_\eta])$$

$\rho, \bar\rho = (\rho_+ \pm \rho_-)/\sqrt{2}$ — classical and quantum components of density

$i \ln Z_\eta[\bar\chi_\eta] = \sum_n (-1)^{n+1} \bar\chi_\eta^n \mathcal{S}_{n,\eta}/n$

— partition function of free chiral fermions in the field $\bar\chi_\eta = \Pi_\eta^{a^{-1}} \bar\rho_\eta$

$\mathcal{S}_{n,\eta} = \langle \rho_{1,\eta} \rho_{2,\eta} \ldots \rho_{n,\eta} \rangle$ — density cumulants



Equilibrium: $S_n = 0$ for all $n > 2$

$\longrightarrow$ gaussian theory

Out of equilibrium all $S_n \neq 0$

$\longrightarrow$ looks as interacting field theory

But crucial simplifications:

- Z_η depends only on $\bar\rho_\eta$ $\longrightarrow$ action linear in ρ_η

  $\longrightarrow$ integral over ρ_η can be performed, yields an equation fixing $\bar\rho_\eta$
- $Z_\eta[\bar\chi_\eta]$ is restricted to mass-shell ($\omega = \eta v q$)

Non-equilibrium bosonization: Green function of free Fermions

$$G^{\gtrless}_{0,\eta}(\tau) = -\frac{1}{2\pi v}\frac{1}{\tau \mp i/\Lambda}\Delta_{\eta,\tau}(2\pi)$$

expressed in terms of Fredholm-Toeplitz functional determinant:

$$\Delta_\eta[\delta(t)] = \det\left[1 + \left(e^{-i\hat{\delta}} - 1\right)\hat{n}_\eta\right]$$

$\Delta_\eta[\delta(t)] = \Delta_{\eta,\tau}(\lambda)$ for rectangular pulse $\delta(t) = \lambda[\theta(t+\tau) - \theta(t)]$

$\hat{\delta}$ diagonal in t space, $\hat{n}_\eta$ diagonal in ϵ space

Free particles: $\lambda = 2\pi$

Electron is a 2π soliton of the bosonic problem

Relation to the counting statistics problem

Counting Statistics

generating function

$$\kappa(\lambda) = \sum_N e^{iN\lambda} p_N \,, \qquad \ln \kappa(\lambda) = \sum_{k=1}^{\infty} m_k \frac{(i\lambda)^k}{k!}, \qquad m_k = \langle\langle \delta N^k \rangle\rangle$$

N — number of particles passing in time τ

non-interacting fermions: $\kappa(\lambda) = \det[\hat{1} - \hat{n} + \hat{S} \exp(i\lambda)\hat{n}]$

$\hat{S}$ — scattering matrix

Levitov, Lesovik '93

$\Delta_{\eta,\tau}(\lambda)$ has the same form as $\kappa(\lambda)$

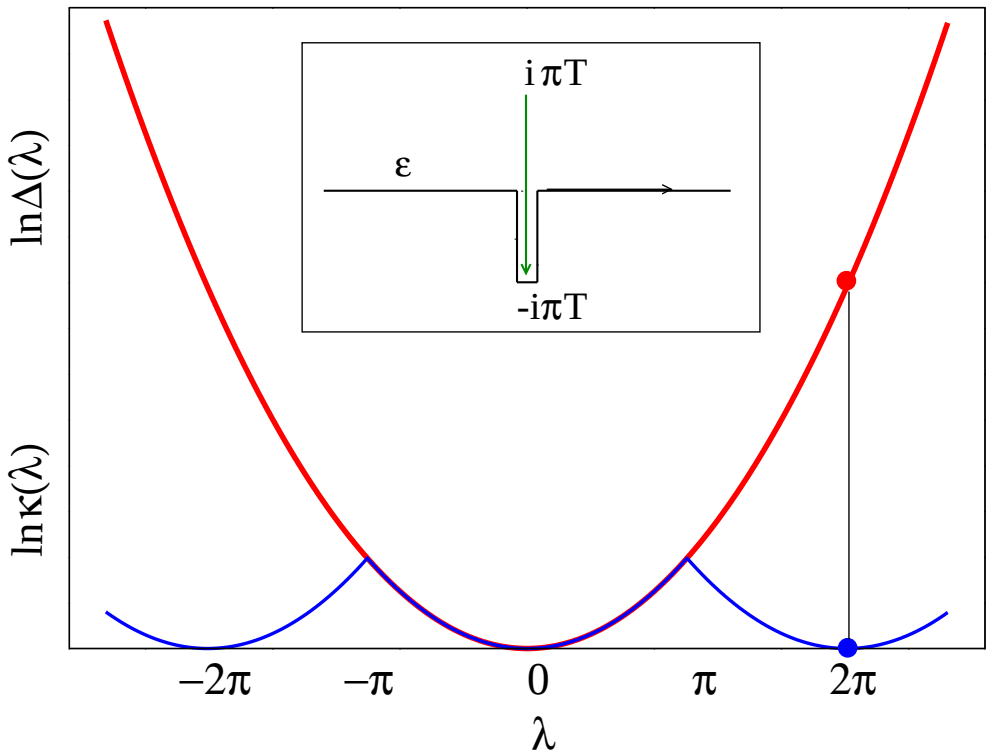
But $\kappa(2\pi) \equiv 1$ due to charge quantization – ?!

Analytic properties of $\Delta_{\eta,\tau}(\lambda)$ and $\kappa(\lambda)$

semiclassical limit

$$\ln \Delta_{\eta,\tau}(\lambda) = \frac{\tau}{2\pi\hbar} \int d\epsilon \ln[1 + (\exp i\lambda - 1) n_\eta(\epsilon)]$$

Equilibrium: $\ln \Delta_{\eta,\tau}(\lambda) = -\tau T \lambda^2 / 4\pi$



Non-equilibrium bosonization of Luttinger liquid: Results

Green function $$G_R^{\gtrless}(\tau) = -\frac{\Delta_L[\delta_L]\Delta_R[\delta_R]}{2\pi v(\pm i\Lambda)^\gamma(\tau \mp i/\Lambda)^{1+\gamma}}$$

$$\delta_\eta(t) = \sum_{n=0}^{\infty} \delta_{\eta,n} w_\tau(t, t_n)$$ sequence of pulses

$w_\tau(t, \tilde{t}) = \theta(t - \tilde{t} + \tau) - \theta(t - \tilde{t})$ rectangular pulse of duration τ

$t_n = (n + 1/2 - 1/2K)L/u$ positions of pulses

amplitudes:

$$\delta_{\eta,2m} = \pi t_\eta \frac{1 + \eta K}{\sqrt{K}} r_L^m r_R^m \qquad \delta_{\eta,2m+1} = -\pi t_\eta \frac{1 - \eta K}{\sqrt{K}} r_\eta^m r_{-\eta}^{m+1}$$

$r_{R/L}$, $t_{R/L}$ — plasmon reflection/transmission coefficients

Fractionalization of 2π pulse at the tunneling and at boundaries

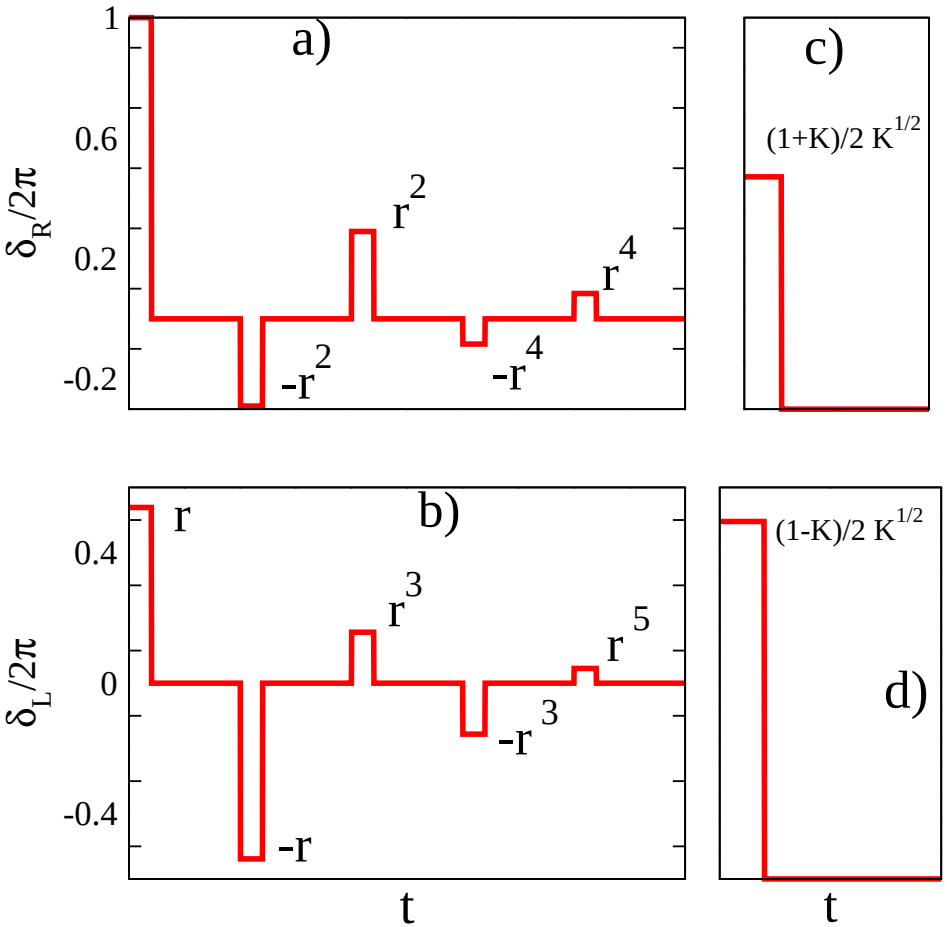
cf. Safi, Schulz '95; Le Hur '02; Steinberg et al '08; Berg et al '09

Non-equilibrium bosonization of Luttinger liquid: Results

Green function

$$G_R^{\gtrless}(\tau) = -\frac{\Delta_L[\delta_L]\Delta_R[\delta_R]}{2\pi v(\pm i\Lambda)^{\gamma}(\tau \mp i/\Lambda)^{1+\gamma}}$$

sharp **smooth** **boundary**



$$r = \frac{1-K}{1+K}$$

K – LL interaction parameter

for long wires $\tau \ll L/u$

$$\Delta_\eta[\delta_\eta(t)] \simeq \prod_{n=0}^{\infty} \Delta_{\eta,\tau}[\delta_{\eta,n}]$$

Fractionalization of 2π pulse at the tunneling and at boundaries

Toeplitz determinants

$$\Delta_\tau(\delta) = \det\left[1 + \hat{P}\left(e^{-i\delta} - 1\right)\hat{n}_\eta\right] \qquad \hat{P} - \text{projector on } [0, \tau]$$

UV-regularization: $-\Lambda < \epsilon < \Lambda \longrightarrow t_j = \pi j/\Lambda$

$\longrightarrow$ $N \times N$ **matrix** $\quad N = \tau\Lambda/\pi$

$$\Delta_N[f] = \det[f(t_j - t_k)] \qquad 0 \le j, k \le N - 1$$

$f(t_j)$ – **Fourier transform of** $\quad f(\epsilon) = [1 + n(\epsilon)(e^{-i\delta} - 1)]e^{-i\frac{\delta}{2}\frac{\epsilon}{\Lambda}}$

$\longrightarrow$ **Toeplitz matrix**

Toeplitz determinants: Asymptotics

Large–N asymptotics of Toeplitz determinants:

- smooth $f(\epsilon)$: Szegő theorem
- $f(\epsilon)$ with jumps / power-law singularities:

Fisher-Hartwig conjecture

Deift, Its, and Krasovsky, arXiv:0905.0443 and references therein

$$\Delta_N \sim \sum_k c_k N^{-\gamma_k} e^{-N(iE_k+\Gamma)}$$

Large N $\longrightarrow$ long time $\tau = N\pi/\Lambda$

- exponential decay (dephasing) $e^{-\Gamma N}$ $\longrightarrow$ $e^{-\tau/2\tau_\phi}$
- modified power laws at multiple Fermi edges $(\epsilon - \epsilon_k)^{\gamma_k - 1}$

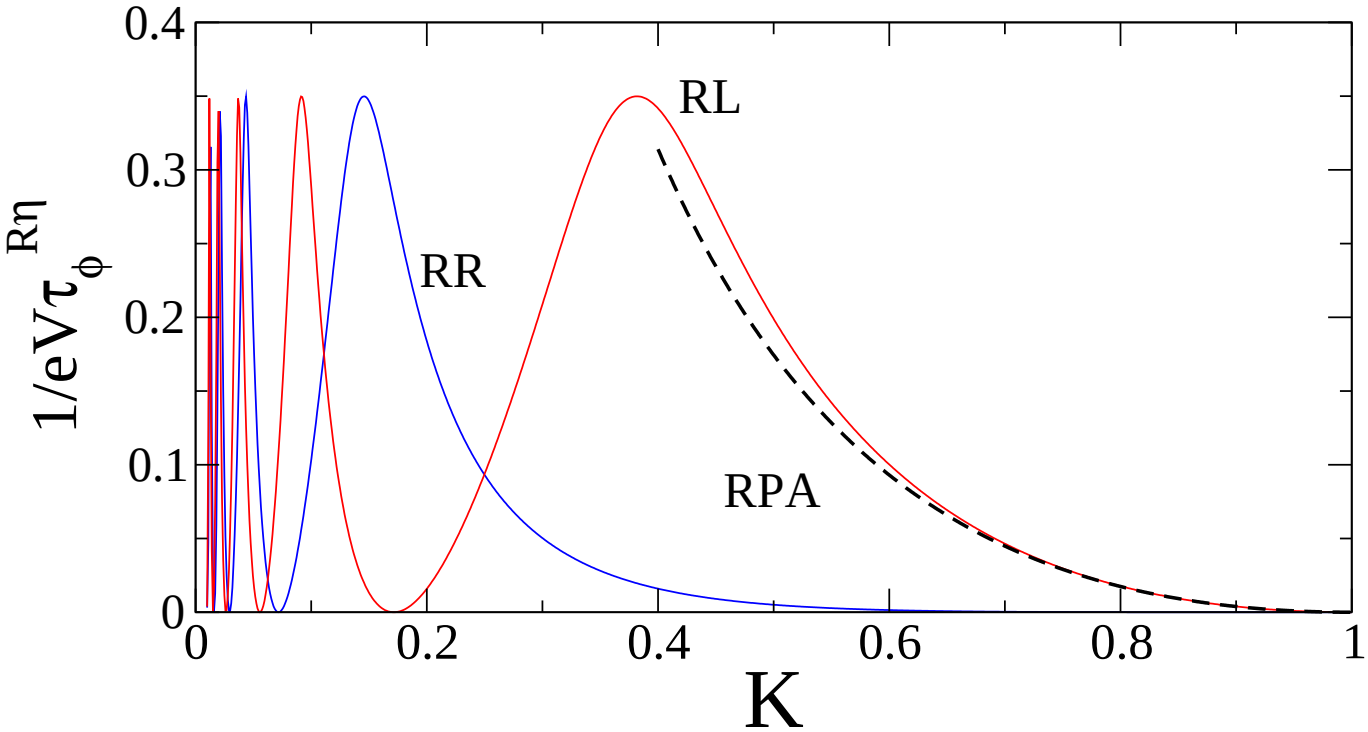
related work on Fermi edge singularity:
Abanin, Levitov, PRL 2005

Non-equilibrium dephasing
(double-step distribution, smooth boundaries)

$$1/\tau_\phi^R = 1/\tau_\phi^{RR} + 1/\tau_\phi^{RL}$$

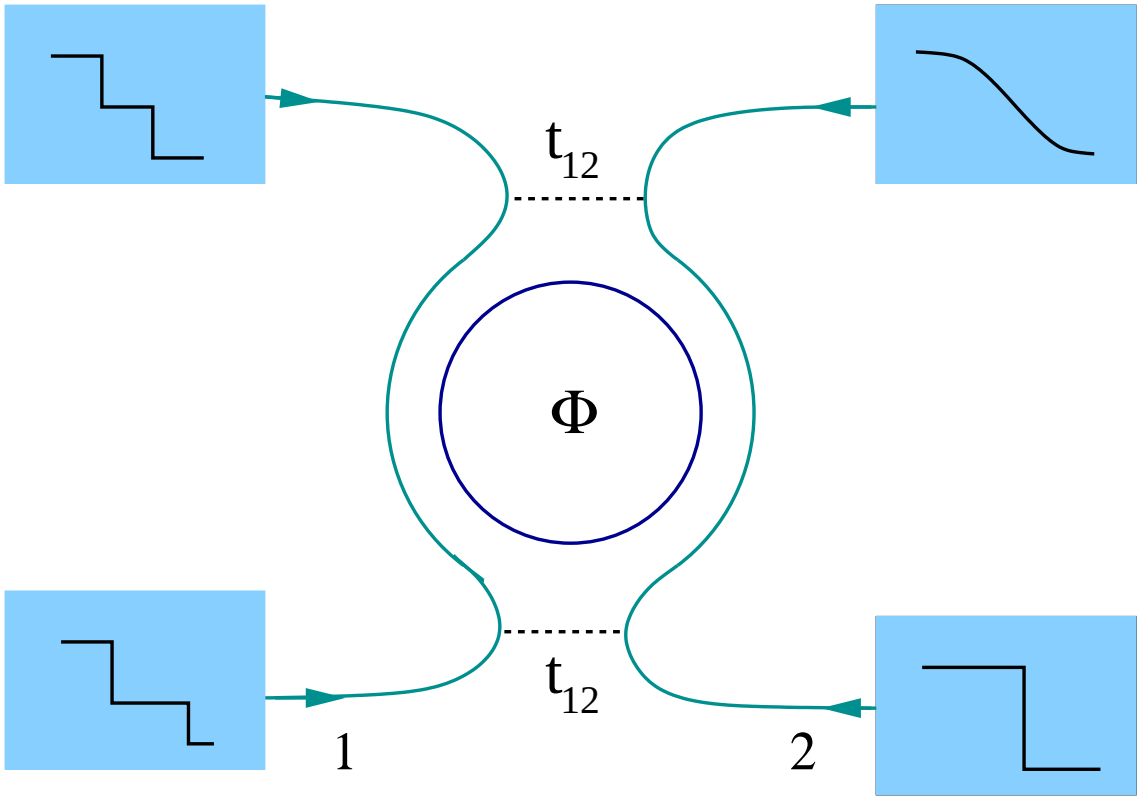
dephasing broadening of ZBA

$$1/\tau_\phi^{R\eta} = \frac{eV}{2\pi} \ln\left[1 - 4a_\eta(1-a_\eta)\sin^2\frac{\pi(1+\eta K}{2\sqrt{K}}\right]$$



- $1/\tau_\phi^{RR}$: RPA is violated even for weak interaction
- dephasing rate oscillates as a function of interaction strength !

Non-equilibrium Luttinger liquid Aharonov-Bohm interferometer



dominant contribution:

dephasing rate $1/\tau_\phi^{RL}$, **oscillatory function of** K

$\longrightarrow$ **AB oscillation amplitude strongly oscillates as a function of interaction strength**

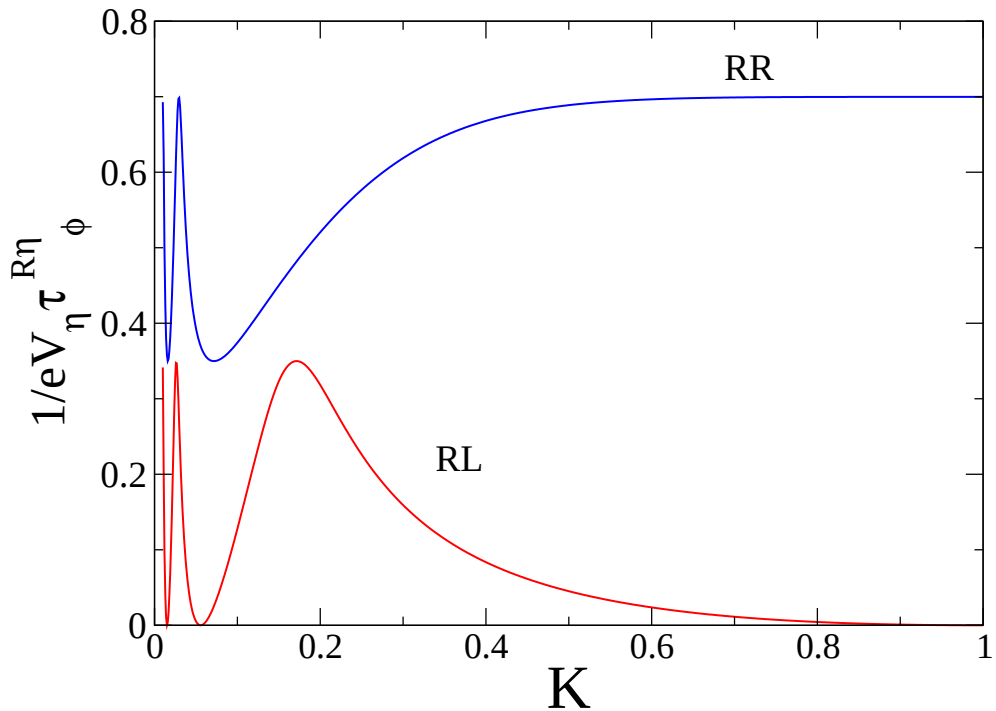
Spinful Luttinger liquid

$$G^{\gtrless}_{R,\uparrow}(\tau) = \mp \frac{i\Lambda}{2\pi\sqrt{uv}} \frac{\prod_{\eta,\sigma} \overline{\Delta}_{\eta,\sigma}[\delta_{\eta,\sigma}(t)]}{(1 \pm i\Lambda\tau)^{1+\gamma/2}}$$

$$\delta_{L,\uparrow}(t) = \delta_{L,\downarrow}(t) = \tfrac{1}{2}\delta_L(t)$$

$$\delta_{R,\uparrow}(t) = \tfrac{1}{2}\left(\delta_R(t) + \delta^0_R(t)\right)$$

$$\delta_{R,\downarrow}(t) = \tfrac{1}{2}\left(\delta_R(t) - \delta^0_R(t)\right)$$



long wire $L(v^{-1} - u^{-1})/\tau \gg 1$

$\longrightarrow$ spin-charge separation

$$\overline{\Delta}_{R,\uparrow}[\delta_{R,\uparrow}] = \overline{\Delta}_{R,\tau,\uparrow}(\pi) \prod_{n=0}^{\infty} \overline{\Delta}_{R,\tau,\uparrow}\left(\frac{\delta_{R,n}}{2}\right)$$

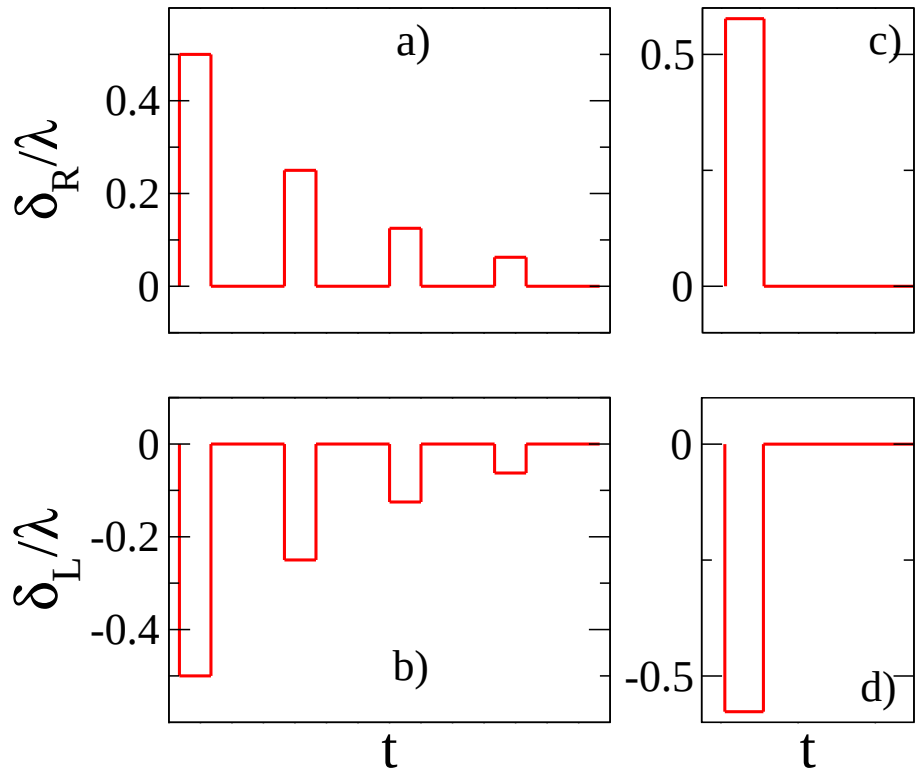
$$\overline{\Delta}_{R,\downarrow}[\delta_{R,\downarrow}] = \overline{\Delta}_{R,\tau,\downarrow}(-\pi) \prod_{n=0}^{\infty} \overline{\Delta}_{R,\tau,\downarrow}\left(\frac{\delta_{R,n}}{2}\right)$$

$$\overline{\Delta}_{L,\sigma}[\delta_{L,\sigma}] = \prod_{n=0}^{\infty} \overline{\Delta}_{L,\tau,\sigma}\left(\frac{\delta_{L,n}}{2}\right)$$

Full counting statistics of LL conductor

$$\kappa(\lambda) = \Delta_R[\delta_R(t)]\Delta_L[\delta_L(t)]$$

$$\delta_\eta(t) = \sum_{n=0}^{\infty} \delta_{\eta,n} w_\tau(t, t_n)$$



measurement point in interacting region:

$$\delta_{\eta,2n} = \eta\lambda t_{-\eta}\sqrt{K} r_\eta^n r_{-\eta}^n \equiv \eta\lambda e^*_{\eta,2n}$$

$$\delta_{\eta,2n+1} = \eta\lambda t_{-\eta}\sqrt{K} r_\eta^{n+1} r_{-\eta}^n \equiv \eta\lambda e^*_{\eta,2n+1}$$

- $\tau \gg L/u$ $\longrightarrow$ all pulses overlap $\longrightarrow$ FCS of free fermions

- $\tau \ll L/u$ $\longrightarrow$ determinant splits into product

$\longrightarrow$ superposition of FCS of non-interacting electrons

with fractional charges $e^*_{\eta,n}$

Full counting statistics and charge fractionalization

smooth boundaries $\longrightarrow$ one fractional charge $e^*_{\eta,0} = \sqrt{K}$

sharp boundaries $\longrightarrow$ sequence of fractional charges

$$e^*_{\eta,n} = 2K(1-K)^n/(1+K)^{n+1}$$

Comments:

- to be distinguished from charge fractionalization in FQHE (spectral gap, quantized fractional charge)
- how is the result for charge fractionalization compatible with electron charge quantization?

  above analysis: bosonization, slow density ρ_η
  corresponds to measurement smooth on the scale k_F^{-1}

  for spatial resolution of measurement sharp on the scale k_F^{-1}:
  fast oscillatory contributions to ρ_η (Haldane)
  $\longrightarrow$ $\kappa(\lambda)$ periodically continued beyond $[-\pi, \pi]$ $\longleftrightarrow$ charge quantization.
  This will not affect the moments of FCS (derivatives of $\kappa(\lambda)$ at $\lambda = 0$)

Non-equilibrium correlation functions of many-body problems

$G = \langle e^{-i\mathcal{O}_-(\tau)} e^{i\mathcal{O}_+(0)} \rangle \qquad \mathcal{O} = \sum_{\eta=R,L} c_\eta \phi_\eta$

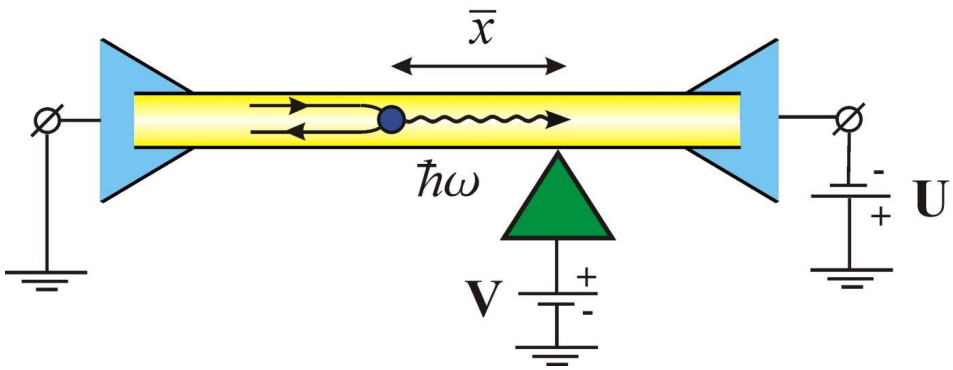
$\longrightarrow \quad G = \Delta_R[\delta_R]\Delta_L[\delta_L]$ **Toeplitz determinants**

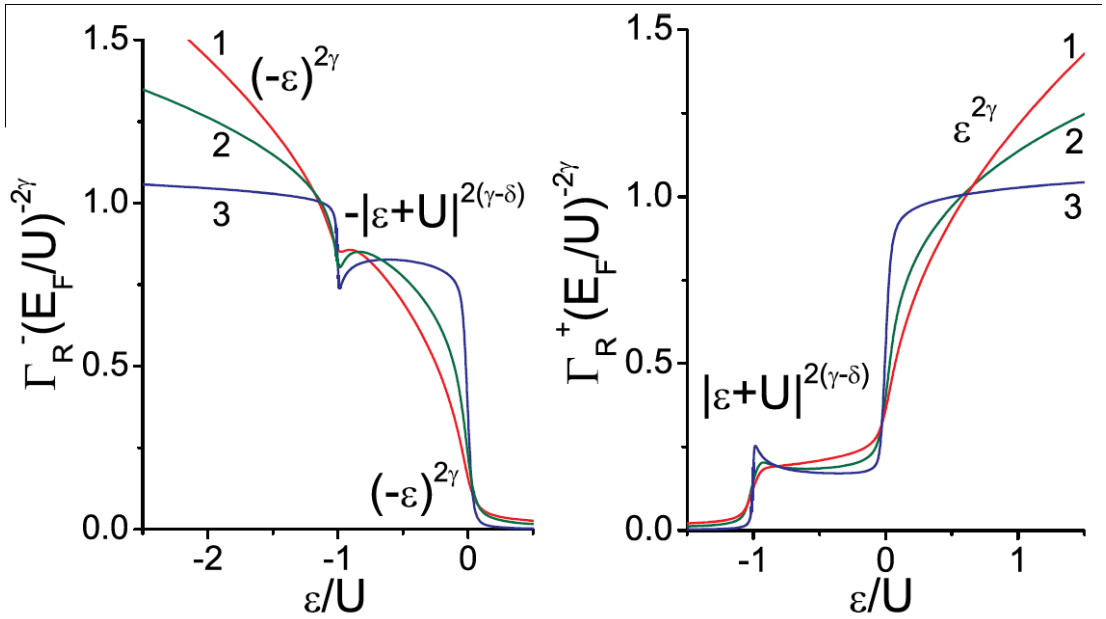
	c_R	c_L	δ_R	δ_L
G_{FES}	$-1+\frac{\delta_0}{\pi}$	0	$2(\pi-\delta_0)$	0
G_{FR}	-1	0	$2\pi\frac{1+K}{2\sqrt{K}}$	$2\pi\frac{1-K}{2\sqrt{K}}$
G_{FL}	0	-1	$2\pi\frac{1-K}{2\sqrt{K}}$	$2\pi\frac{1+K}{2\sqrt{K}}$
$\chi_\tau(\lambda)$	$-\frac{\lambda}{2\pi}$	$\frac{\lambda}{2\pi}$	$\lambda\sqrt{K}$	$-\lambda\sqrt{K}$
G_B	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\pi}{\sqrt{K}}$	$\frac{\pi}{\sqrt{K}}$

smooth boundaries

Related activity

tunneling spectroscopy of LL driven out of equilibrium by bias and an impurity inside the LL





no exact solution; saddle-point approach

similar (but different) results:

- modified tunneling exponents
- oscillatory dependence of dephasing rate on K (interaction)

Ngo Dinh, Bagrets, ADM, PRB 81, 081306(R) (2010)

Summary

- Bosonization technique out of equilibrium
- Tunneling Spectroscopy of LL: exact solution via bosonization
  - $G^{\gtrless}$ in terms of Toeplitz determinants $\Delta[\delta_\eta(t)]$
  - Energy distribution:

    plasmon scattering on the boundaries affects $n(\epsilon)$
  - Non-equilibrium dephasing:

    broadening of ZBA, LL interferometry

    double-step distrib.: oscillatory dephasing, RPA breakdown
- Spinful LL: spin-charge separation out of equilibrium
- Full counting statistics: superposition of FCS of non-interacting particles with fractional charges
- Outlook: bosonic 1D systems (cold atoms), FQHE edges,...