אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

Phase phenomena in non-equilibrium transport through an AB ring: How to measure the transmission Phase in a two-terminal setup

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Beer Sheva, Israel

Phys. Rev. B 77, 165421 (2008)

Phys. Rev. B 80, 035416 (2009)

Phys. Rev. Lett. 104, 256801 (2010)

Outline

- **Introduction**
  - Aharonov-Bohm effect
  - Mesoscopic AB interferometers
  - Phase rigidity, phase jumps and phase lapses
- **Phase switching**
  - Experiments by Sigrist et al.
  - Phase switching out of equilibrium
- **Phase rigidity breaking out of equilibrium**
- **Measuring transmission phase in a two-terminal AB interferometer**

Mesoscopic AB interferometers

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Coherence and Phase Sensitive Measurements in a Quantum Dot

A. Yacoby, M. Heiblum, D. Mahalu, and Hadas Shtrikman

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

(Received 10 November 1994)

Via a novel interference experiment, which measures
coefficient through a quantum dot in the Coulomb regir
transport through the dot has a coherent component. V
coefficient at successive Coulomb peaks, each representin
however, as we scan through a single Coulomb peak w
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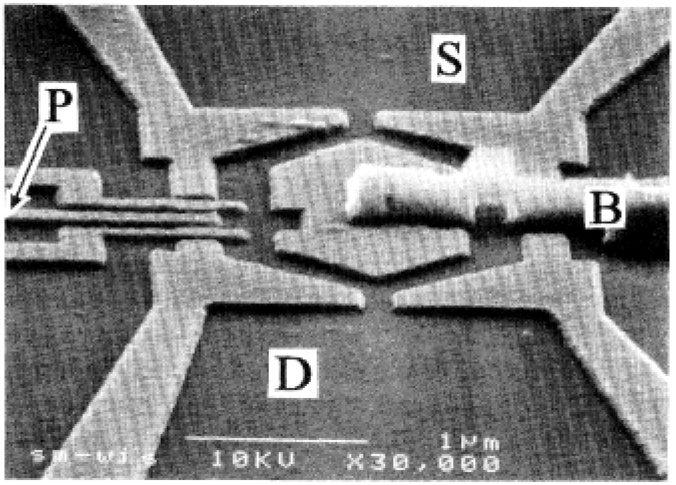
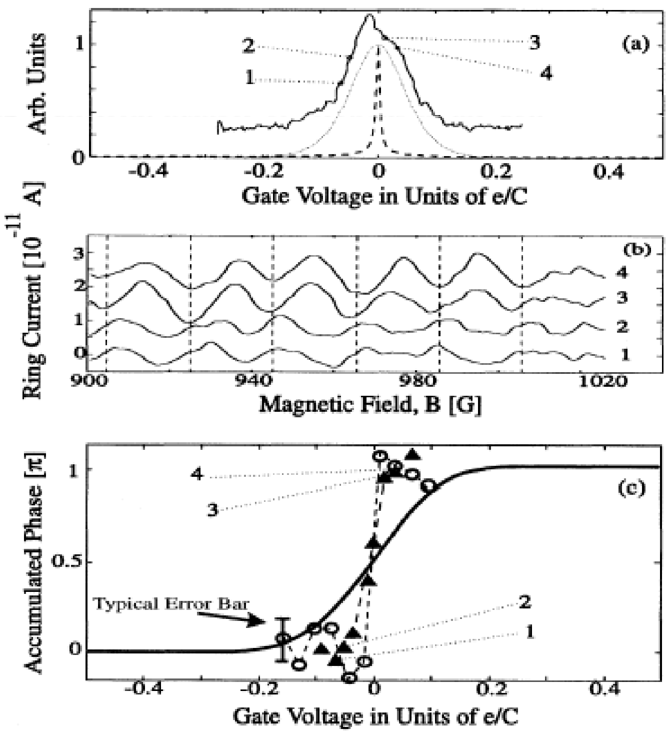
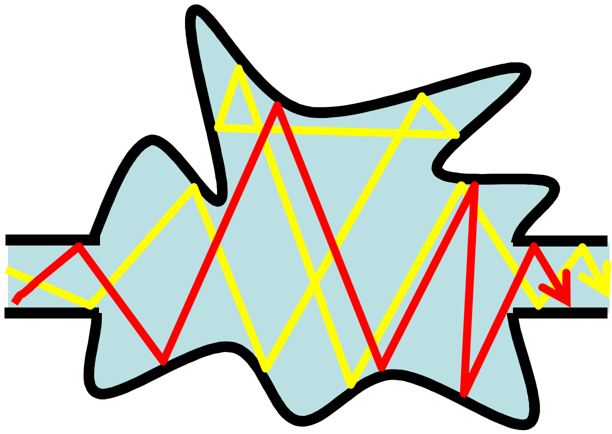
(b)





Phase symmetry
(phase rigidity, phase locking)



Two-terminal device

Onsager-Büttiker relation for a linear response conductance of a two-terminal device:

$$G(B) = G(-B)$$

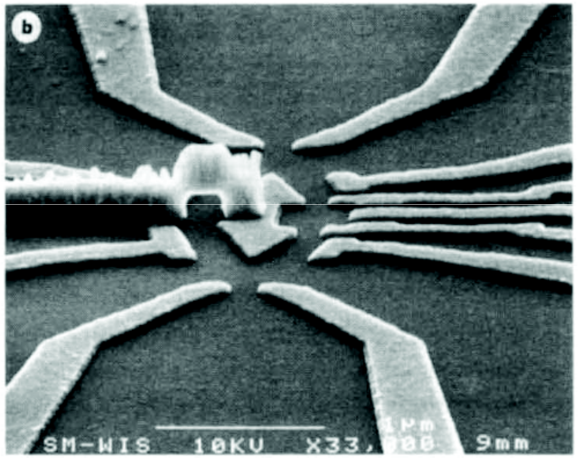
This relation is a result of the time-reversal symmetry!

M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

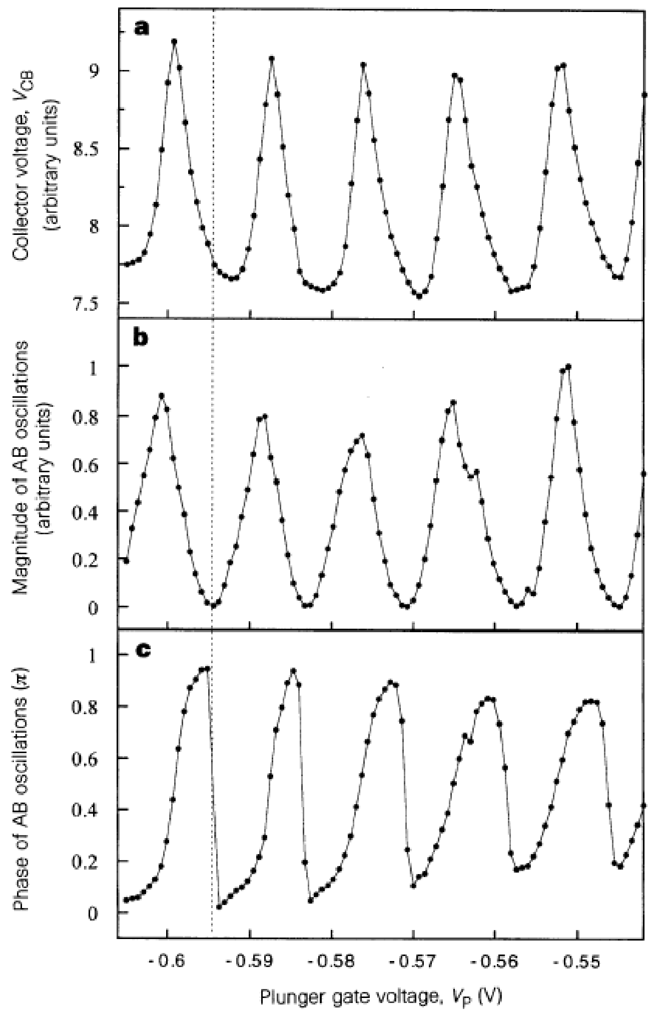
$$G(\Phi) \propto cos\left(2\pi\frac{\Phi}{\Phi_0} + \phi_0\right) \Rightarrow \phi_0 = 0, \pi$$

Bypassing phase symmetry

Phase lapse – abrupt change of the transmission phase via a quantum dot AB by π (even when no QD levels cross the Fermi level)



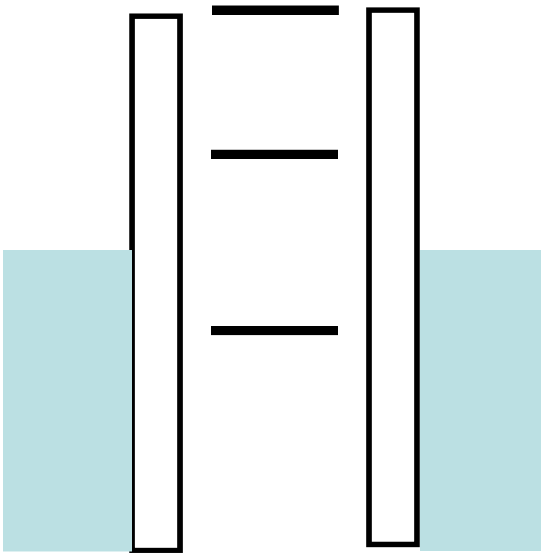
Schuster et al., Nature 385, 417 (1997)

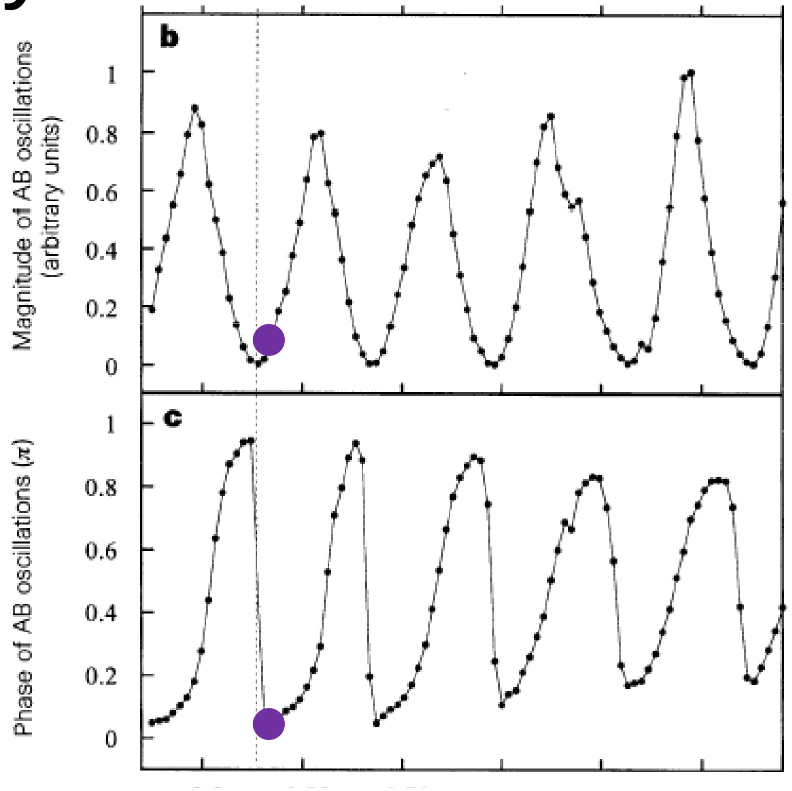


Phase slips are due to competition between different levels with opposite parity

number of phase slips =

number of levels





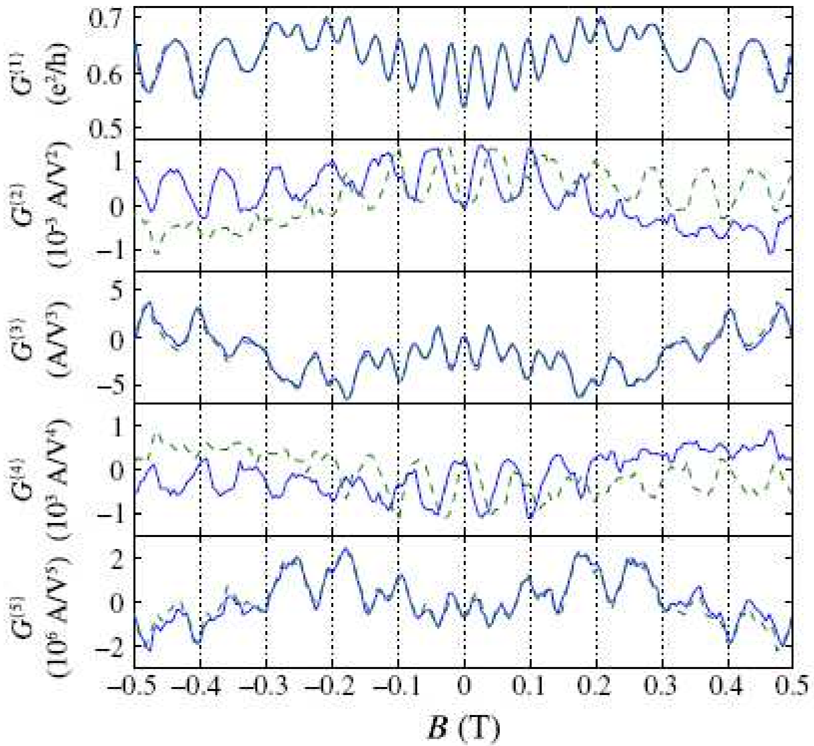
Oreg and Gefen, Phys. Rev. B 55, 13726 1997
Baltin and Gefen, Phys. Rev. Lett. 83, 5094 1999
Silvestrov and Imry, Phys. Rev. Lett. 85, 2565 2000
Golosov and Gefen, Phys. Rev. B 74, 205316 2006
Karrasch, et al., Phys. Rev. Lett. 98, 186802 2007

Breaking of phase symmetry out of equilibrium

- Breaking of phase symmetry only for even components
- Continuous breaking of symmetry with voltage

$$I = \sum_{n=1}^{5} G^{\{n\}}(V - V_0)^n.$$

Leturcq et al. (2006)

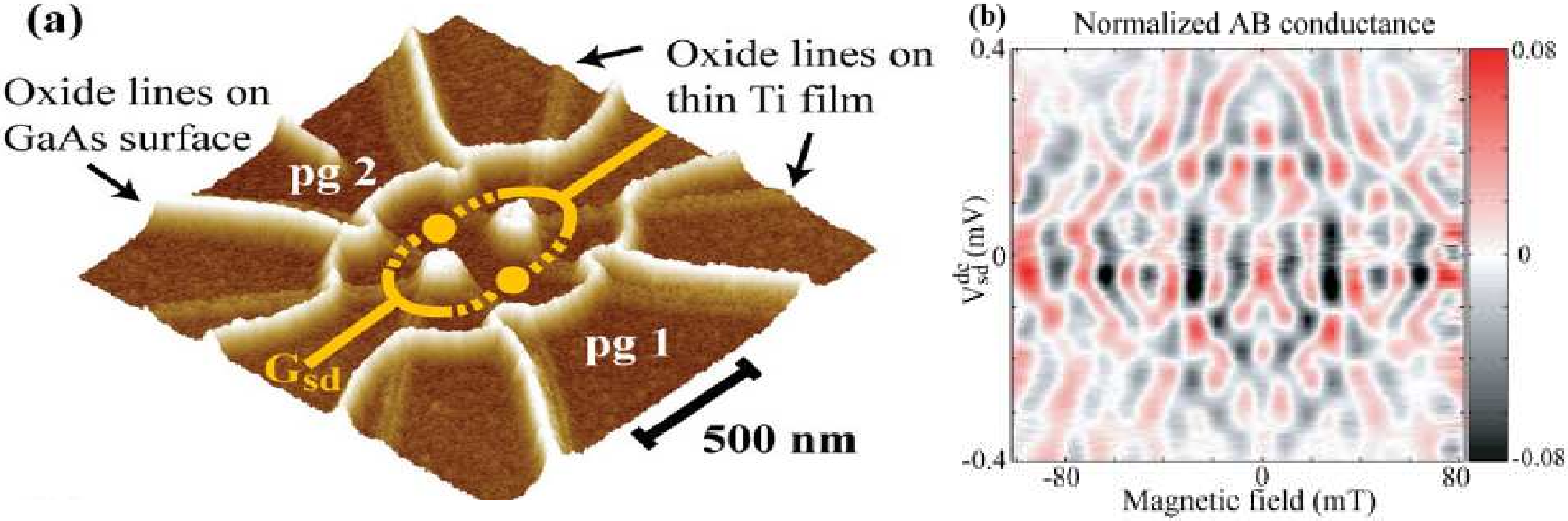


Sanchez and Buttiker (2004)

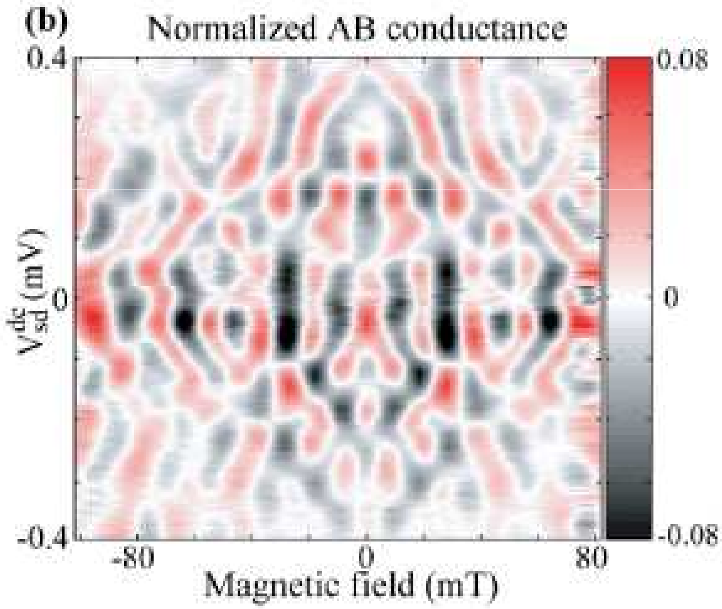
Puzzle: Experiment by Sigrist et al.

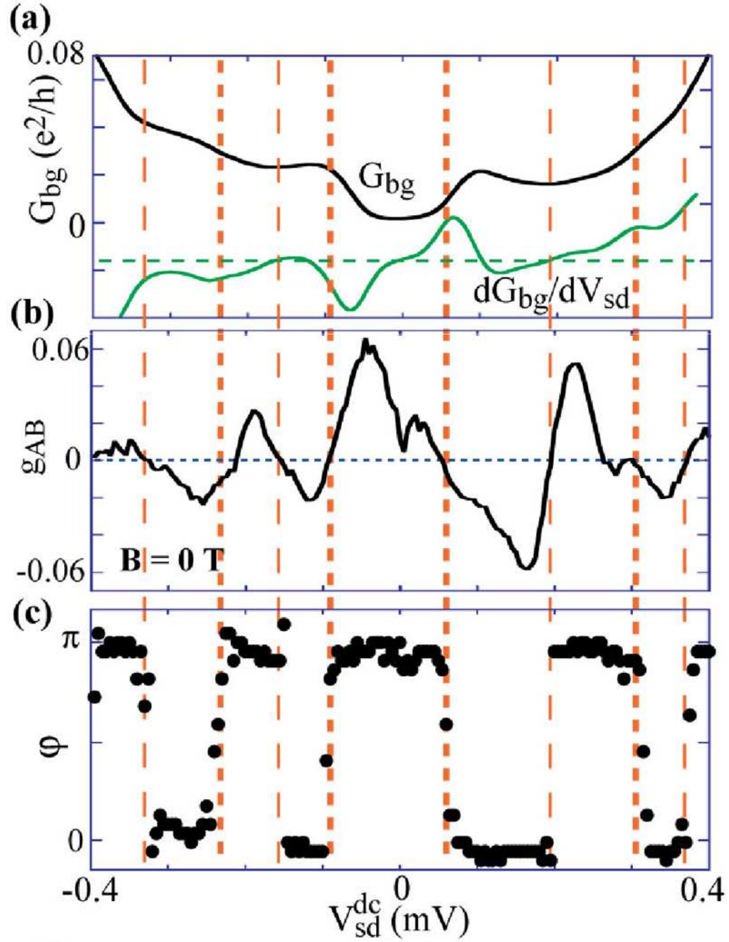
PRL 98, 036805 (2007) PHYSICAL REVIEW LETTERS week ending 19 JANUARY 2007

- *Conductance is symmetric in magnetic field even at high bias*
- *Conductance phase switches between 0 and π*
- *Switching begins for $V\sim\Delta$*
- *More Switches than levels*

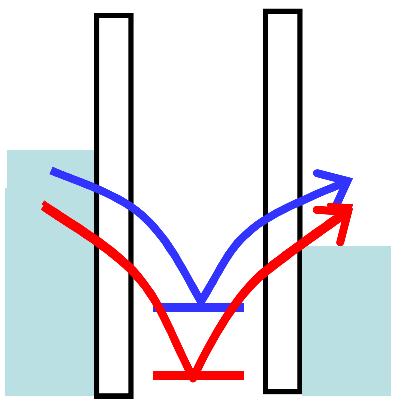


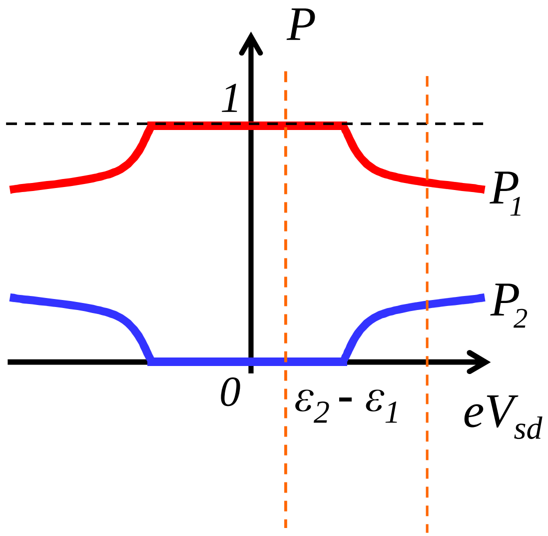
- *Conductance is symmetric in magnetic field even at high bias*
- *Conductance phase switches between 0 and π*
- *Switching begins for V~Δ*
- *More Switches than levels*



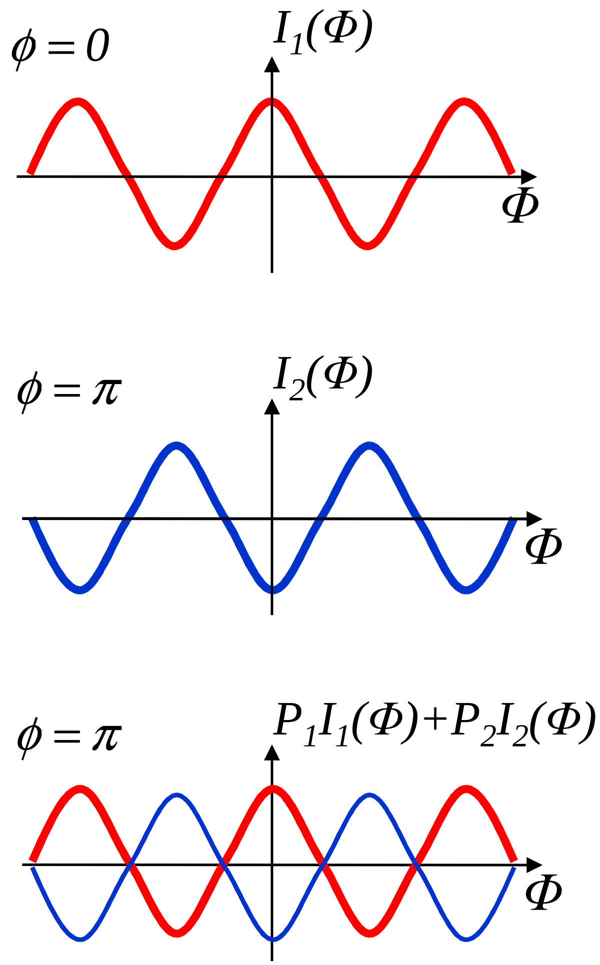


Phase switching





$V_{sd}<\varepsilon_2-\varepsilon_1$ $V_{sd}>>\varepsilon_2-\varepsilon_1$



Approximate condition for switching

$$t_1(E) = \frac{+\Gamma_1/2}{E-\varepsilon_1+i\Gamma_1/2} \qquad I_1(\Phi) \propto t_{ref}\left|t_1(E)\right| cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

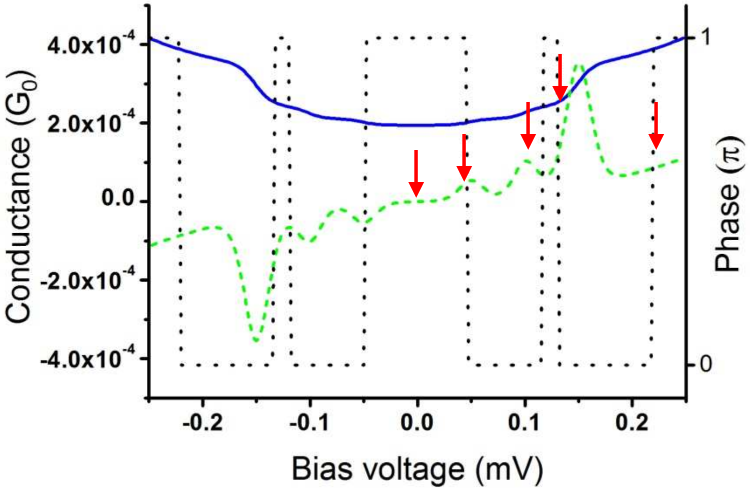
$$t_2(E) = \frac{-\Gamma_2/2}{E-\varepsilon_2+i\Gamma_2/2} \qquad I_2(\Phi) \propto -t_{ref}\left|t_2(E)\right| cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

$$I(\Phi) = P_1 I_1(\Phi) + P_2 I_2(\Phi) \propto \left(P_1\Gamma_1 - P_2\Gamma_2\right) cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

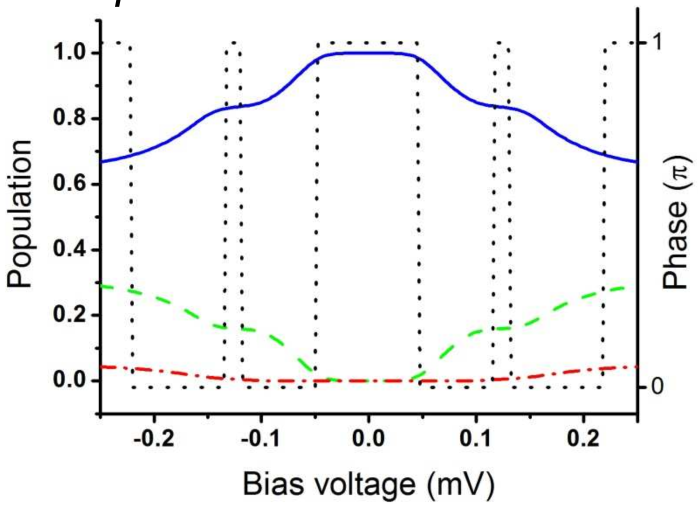
$$P_2\Gamma_2 \geq P_1\Gamma_1$$

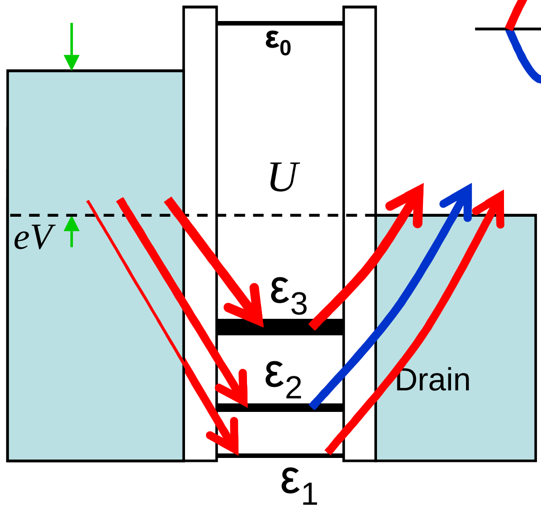
Multiple switchings

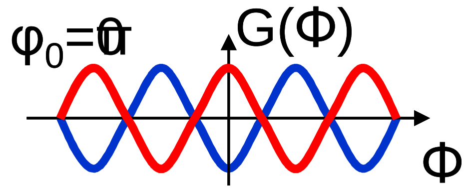
Mean Conductance



Occupation numbers







Chronology of switching events:

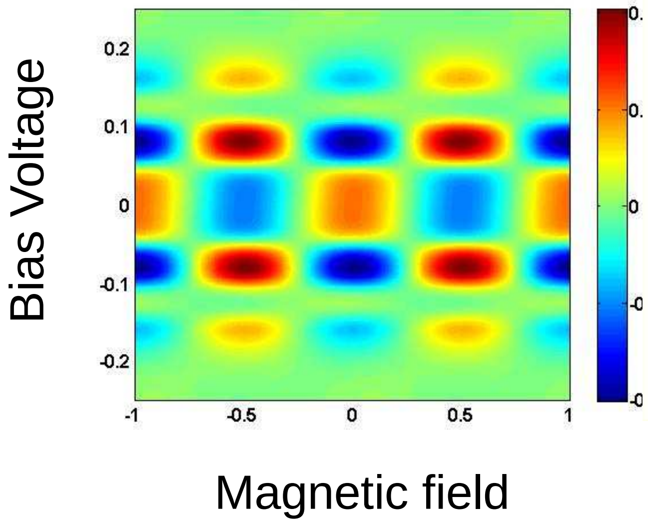
1st switching: population of level 2 starts to grow: $V>\varepsilon_2-\varepsilon_1$;

2nd switching: level 3 starts being populated from level 2: $V>\varepsilon_3-\varepsilon_2$;

3rd switching: contribution from level 2 grows faster than that due to levels 1 and 3;

4th switching: level 3 is being directly populated from level 1: $V>\varepsilon_3-\varepsilon_1$.

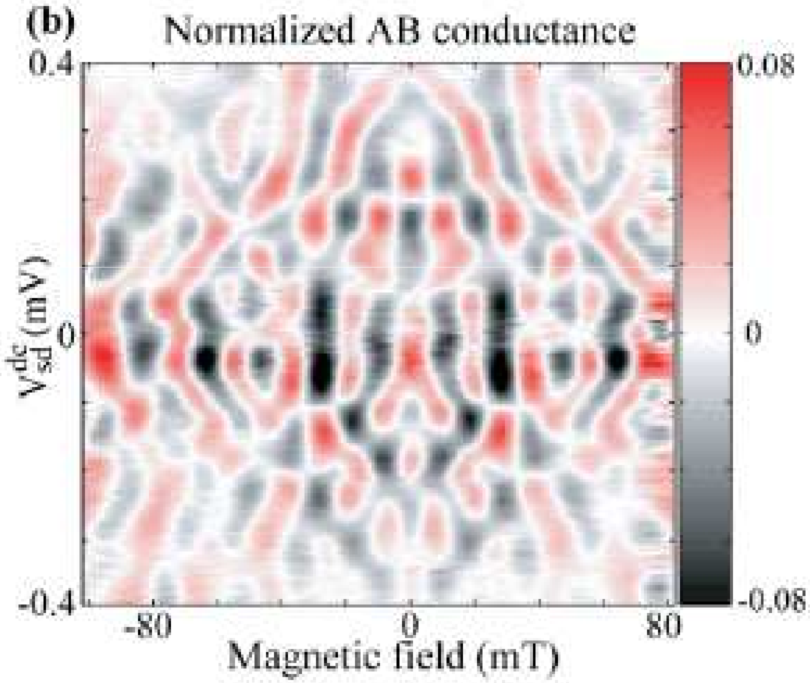
Art or applicative science?

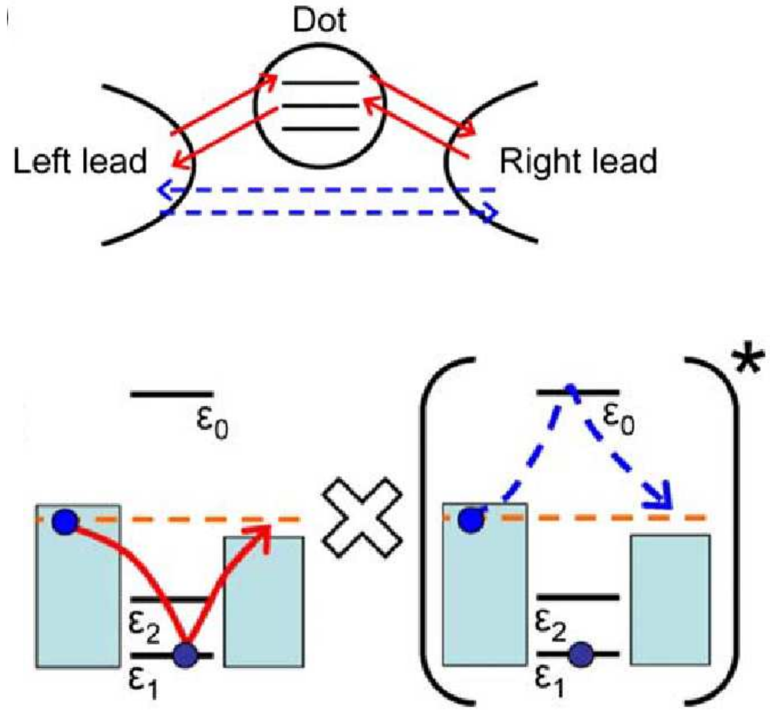


Wassily Kandinsky,
Farbstudie Quadrate (1913)

Commercial product

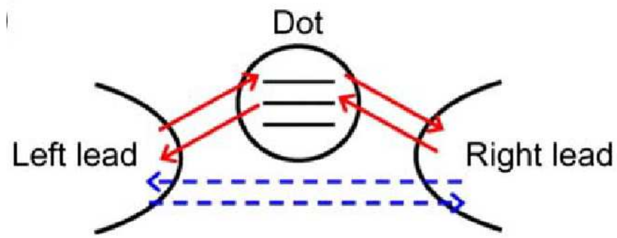
Conductance even in magnetic field even at finite bias ????





Leading contribution to the AB oscillations is even in the magnetic field!

Higher order processes

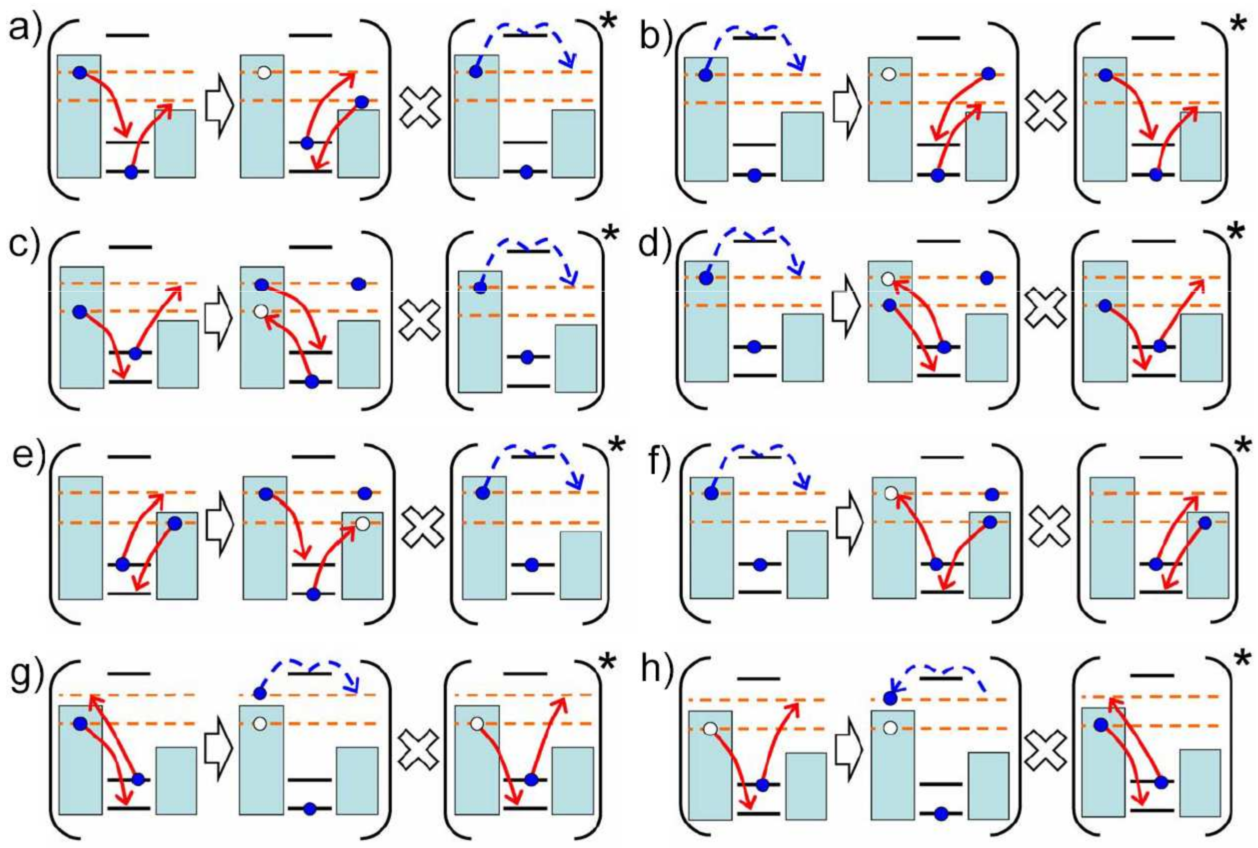


cancel out

cancel out

relevant

relevant



cancel out

cancel out

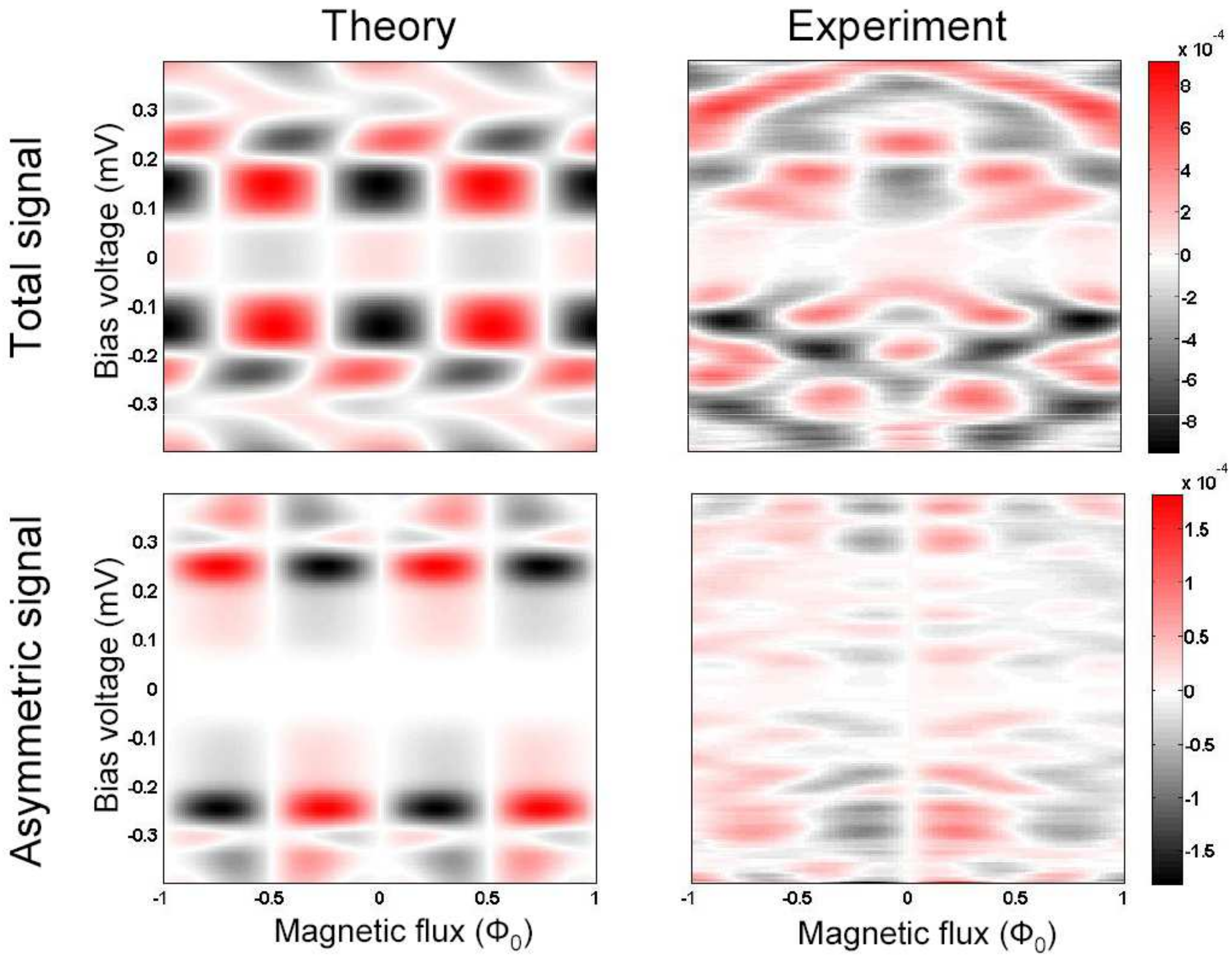
No current

No current

Theoretical analysis: summary

- Leading contribution to the AB oscillations is even in the magnetic field
- Odd contribution to the AB oscillations originates from the higher–order processes (in which the intermediate state lies on the same energy shell with the initial and the final states)
- Breaking of the phase symmetry may happen only after the onset of inelastic co-tunneling
- Breaking of the phase symmetry is best observed near phase switching when leading order vanishes.

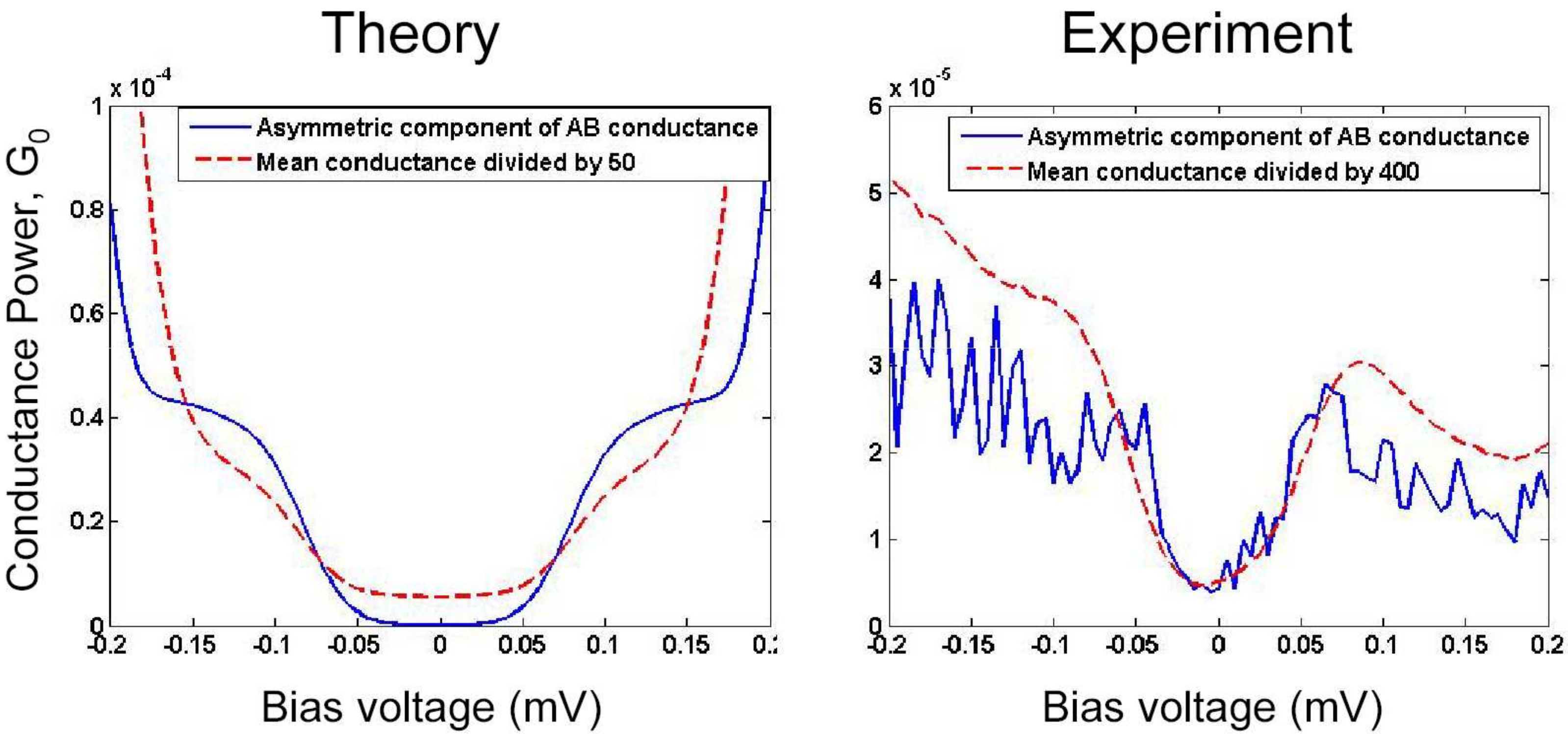
Comparing to the experiment



- The phase of the AB oscillations switches sharply between 0 and π
- Any significant asymmetry only near phase switching
- The asymmetric component of the AB oscillations is zero before the onset of inelastic co-tunneling

Comparing to the experiment

$$P(V_{sd}) = \sqrt{\int_{B_{min}}^{B_{max}} dB\, G_{asym}^2(B, V_{sd})} \,/\, (B_{max} - B_{min})$$



The asymmetric component of the AB oscillations is zero before the onset of inelastic co-tunneling

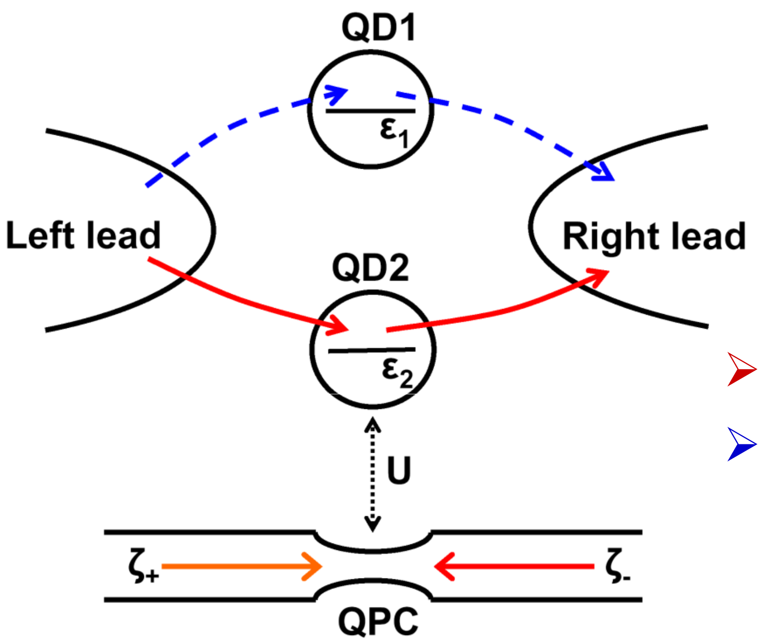
Nonequilibrium can lead to breaking of phase rigidity, can it be used to measure the transmission phase through the dot ?

Problem: we want to measure the equilibrium transmission phase !

Solution: couple other parts of the system to nonequilibrium environment

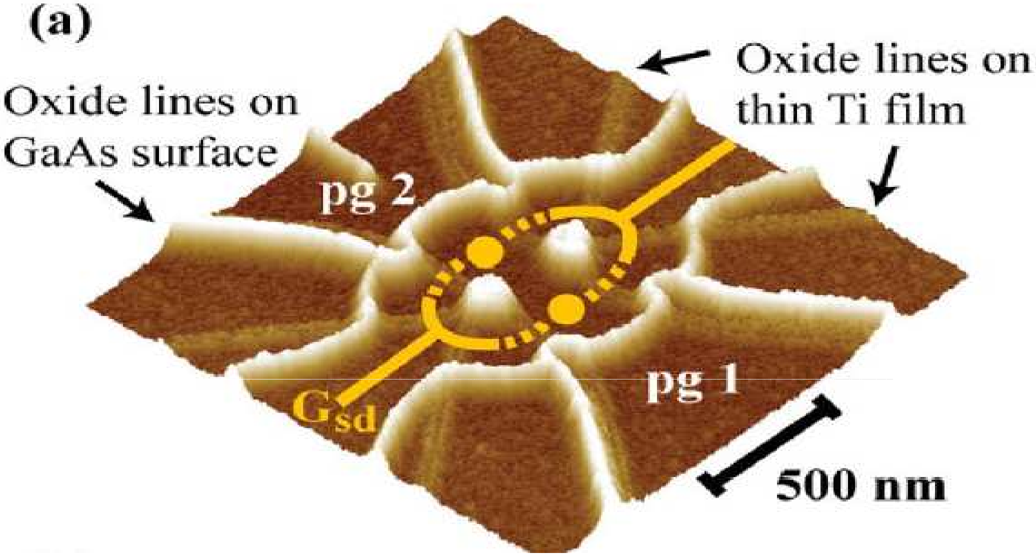
D. Sánchez and K. Kang, (2008)

Proposed Experiment



- Aim: measuring transmission phase via QD1
- Phase rigidity breaking due to coupling QD2 to a non-equilibrium environment, played here by a quantum point contact.

‘Which path?’ detector



letters to nature

Dephasing in electron interference by a ‘which-path’ detector

E. Buks, R. Schuster, M. Heiblum, D. Mahalu & V. Umansky

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Wave–particle duality, as manifest in the two-slit experiment, provides perhaps the most vivid illustration of Bohr’s complementarity principle: wave-like behaviour (interference) occurs only when the different possible paths a particle can take are indistinguishable, even in principle[1]. The introduction of a which-path (*welcher Weg*) detector for determining the actual path taken by the particle inevitably involved coupling the particle to a measuring environment, which in turn results in dephasing (suppression of interference). In other words, simultaneous observations of wave and particle behaviour is prohibited. Such

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Dephasing and the Orthogonality Catastrophe in Tunneling through a Quantum Dot: The “Which Path?” Interferometer

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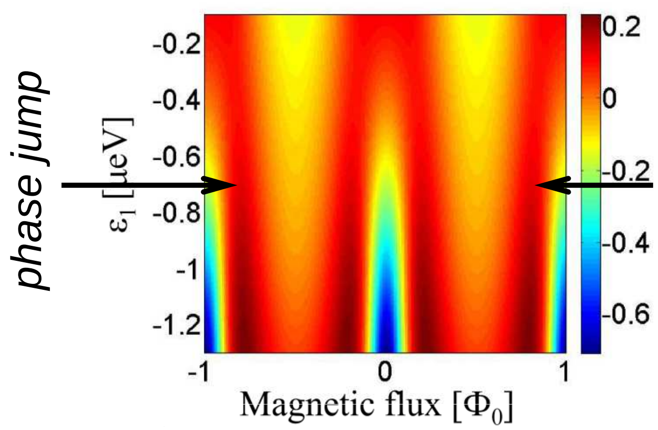
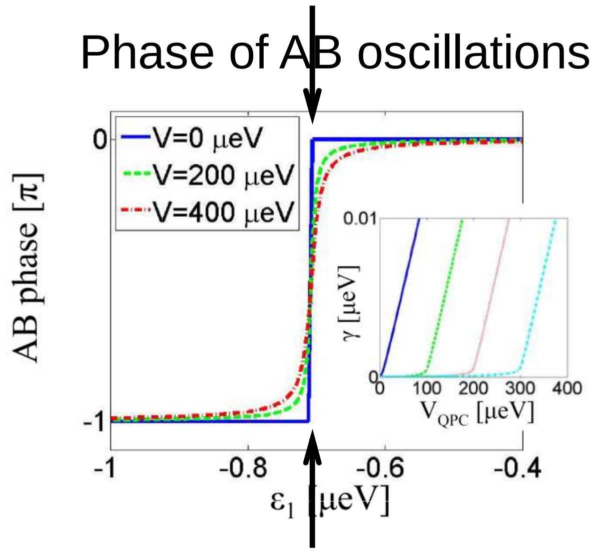
[2]*Physics Department, Ben Gurion University, Beer Sheva, 84105, Israel*

Phase rigidity breaking

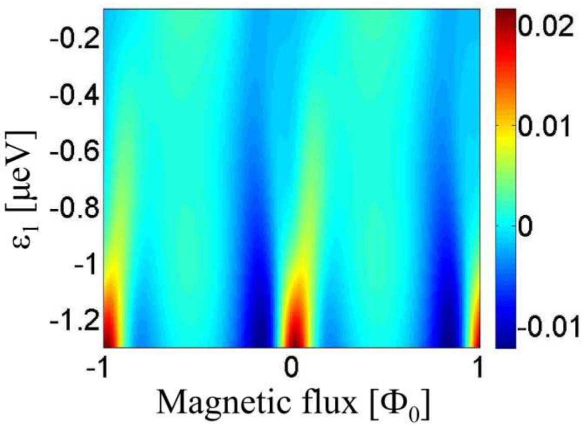
AB conductance for $V_{QPC}=0$



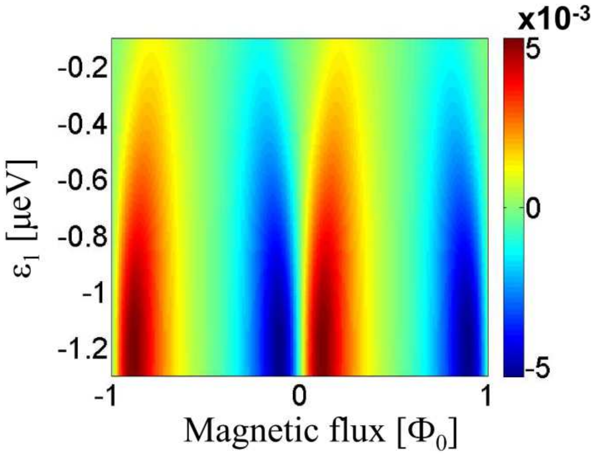
Phase of AB oscillations



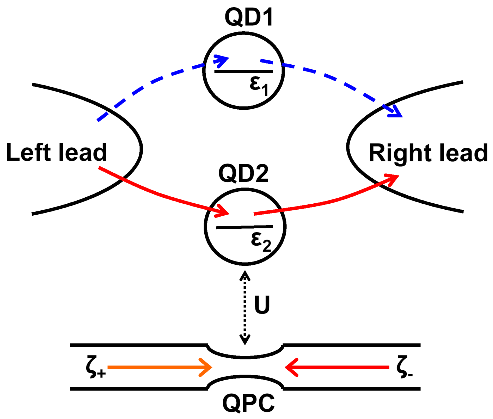
Change in AB conductance for $V_{QPC}=400\mu eV$



Odd AB conductance for $V_{QPC}=400\mu eV$



Measuring the transmission phase



Transmission coefficient through QD1:

$$t_{QD1}(\epsilon_F) = |t_{QD1}(\epsilon_F)|\, e^{i\varphi_{QD1}(\epsilon_F)}$$

Conductance of (isolated) QD1:

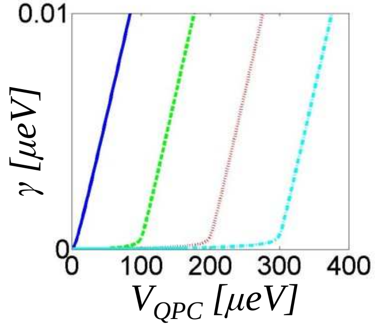
$$G_{QD1}(\epsilon_F) = \frac{G_0}{2}\, |t_{QD1}(\epsilon_F)|^2$$

Odd component of AB conductance ($\gamma(\epsilon_F) \ll \Gamma_{22} \ll |\epsilon_2 - \epsilon_F|$):

$$G^{odd}(\epsilon_F, \varphi) \simeq \pm sin\varphi \frac{G_0}{2} \frac{\gamma(\epsilon_F)\sqrt{\Gamma^L_{22}\Gamma^R_{22}}\,\left|\Gamma^L_{22} - \Gamma^R_{22}\right|}{2\left(\Gamma_{22} + \gamma(\epsilon_2)\right)\left[(\epsilon_2 - \epsilon_F)^2 + \frac{1}{4}\left(\Gamma_{22} + \gamma(\epsilon_2)\right)^2\right]} \Re\left[t_{QD1}(\epsilon_F) - t_{QD1}(\epsilon_2)\right]$$

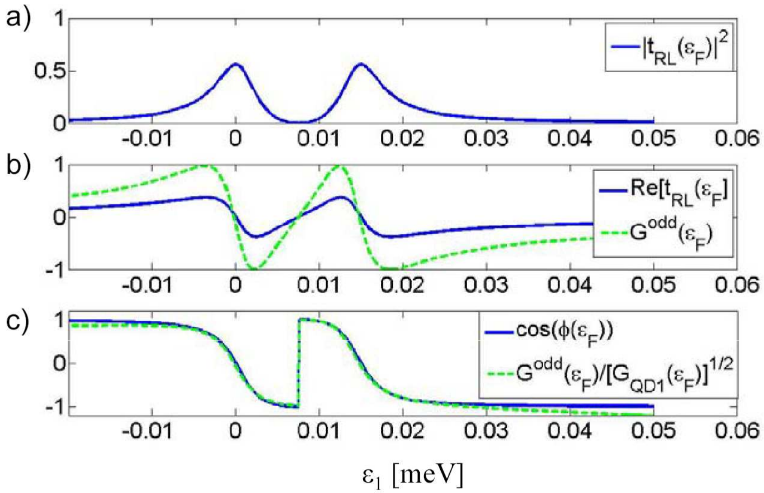
$$\cos\left[\varphi_{QD1}(\epsilon_F)\right] \propto \frac{G^{odd}}{\sqrt{G_{QD1}}}$$

Dephasing rate (for different ε_2)

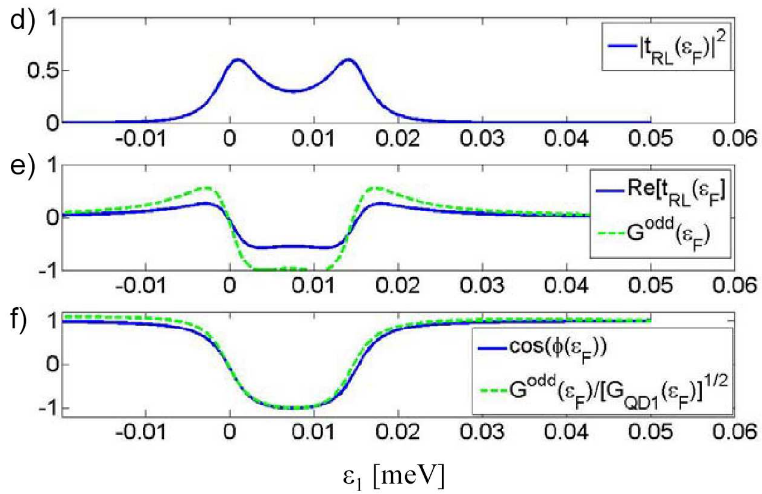


Measuring transmission phase

Two levels of the same parity



Two levels of different parities



- Zeros of G^{odd} correspond to phase $\pi/2$ or a phase jump/lapse by π
- Zeros of G^{odd} have to be matched with those of $|t_{RL}(\varepsilon_F)|$
- Normalizing cosine to $[-1,1]$: prefactor is proportional to dephasing rate

V.P.&Y.M, PRL 104, 256801 (2010).

Summary

- **Phase switching:**

  'Phase switching' in non-equilibrium co-tunneling transport via quantum dots can be explained as a result of the current via the interferometer being dominated at different bias by the QD levels of different parity.

- **Phase rigidity breaking in co-tunneling transport:**

  The processes contributing to breaking of the phase rigidity in the co-tunneling transport are of higher order than the main AB contribution which explains why the AB phase significantly deviates from 0 and π only close to the phase switching points. In addition, no breaking of the phase symmetry occurs up to the onset of the inelastic co-tunneling.

- **Measuring transmission phase via a closed AB interferometer:**

  The transmission phase through a quantum dot can be measured by inserting it into the reference arm of a 'which path?' detector.