

# **Nonequilibrium current and relaxation dynamics of a charge-fluctuating quantum dot**

Volker Meden

Institut für Theorie der Statistischen Physik

**RWTHAACHEN  
UNIVERSITY**

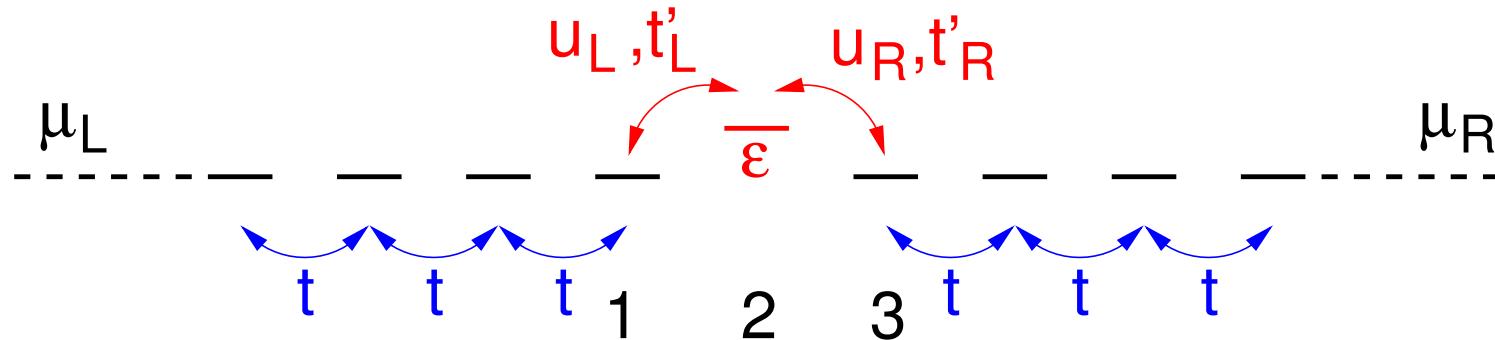
**JARA** | Jülich Aachen  
Research  
Alliance

with Christoph Karrasch, Sabine Andergassen, Dirk Schuricht,  
Mikhail Pletyukhov, László Borda, Herbert Schoeller

# Finite-bias current through quantum dots—limiting cases

spin fluctuations: nonequilibrium physics of the Kondo model

charge fluctuations: the interacting resonant level model (lattice version)



$$\begin{aligned}
 H = & \epsilon n_2 + u_L \left( n_1 - \frac{1}{2} \right) \left( n_2 - \frac{1}{2} \right) + u_R \left( n_2 - \frac{1}{2} \right) \left( n_3 - \frac{1}{2} \right) - t'_L (d_1^\dagger d_2 + \text{H.c.}) \\
 & - t'_R (d_2^\dagger d_3 + \text{H.c.}) - t \sum_{\alpha=L,R} \sum_{j=1}^{\infty} (c_{j,\alpha}^\dagger c_{j+1,\alpha} + \text{H.c.}) - t (d_1^\dagger c_{1,L} + d_3^\dagger c_{1,R} + \text{H.c.})
 \end{aligned}$$

model parameters:

- level position:  $\epsilon$  ( $\epsilon = 0$ : p-h symmetry)
- local Coulomb interaction:  $u_\alpha$  ( $= U_\alpha / \rho_\alpha$ )
- local tunneling:  $t'_\alpha$  ( $= \Gamma_\alpha^\infty / (2\pi\rho_\alpha)$ )
- band width:  $B = 4t$
- bias voltage:  $\mu_{L/R} = \pm V/2$

## In equilibrium with $L$ - $R$ symmetry: scaling limit

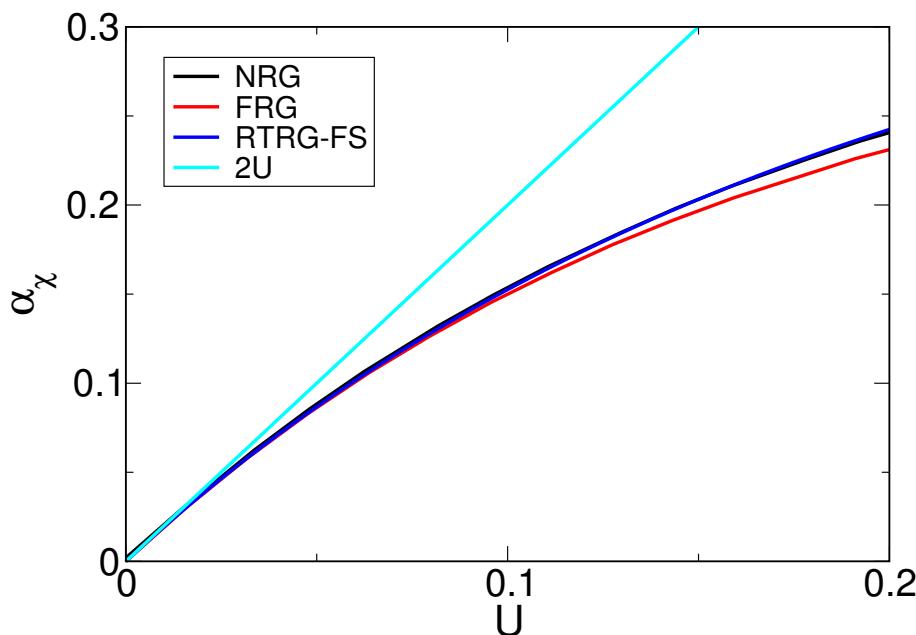
consider field theoretical limit with  $|\epsilon|, |u|, t' \ll B$ :  $\rho(\omega) \rightarrow \rho(0)$

- relation to x-ray edge problem (Nozière & de Dominicis '69)
- single lead: mapping to Kondo model (Wiegmann & Finkel'shtein '78)
- bosonization and RG methods (Schlottmann '80-82, . . . )
- numerical RG (Borda et al. '07; Kashcheyevs et al. '09)

here: functional RG and real-time RG in frequency space, small  $U$

(Karrasch, Enss & VM '06; Schoeller '09)

example: susceptibility  $\chi = -\left. \frac{d\langle n_2 \rangle}{d\epsilon} \right|_{\epsilon=0}$   $\rightarrow$  power law



$$\begin{aligned}\chi^{-1} &\sim (\Gamma^\infty)^{1-\alpha_\chi} \\ \alpha_\chi &= 2U + \mathcal{O}(U^2)\end{aligned}$$

RG:  $\Gamma^\infty$  acts as cutoff

define scale:  $T_K = \frac{2}{\pi\chi}$

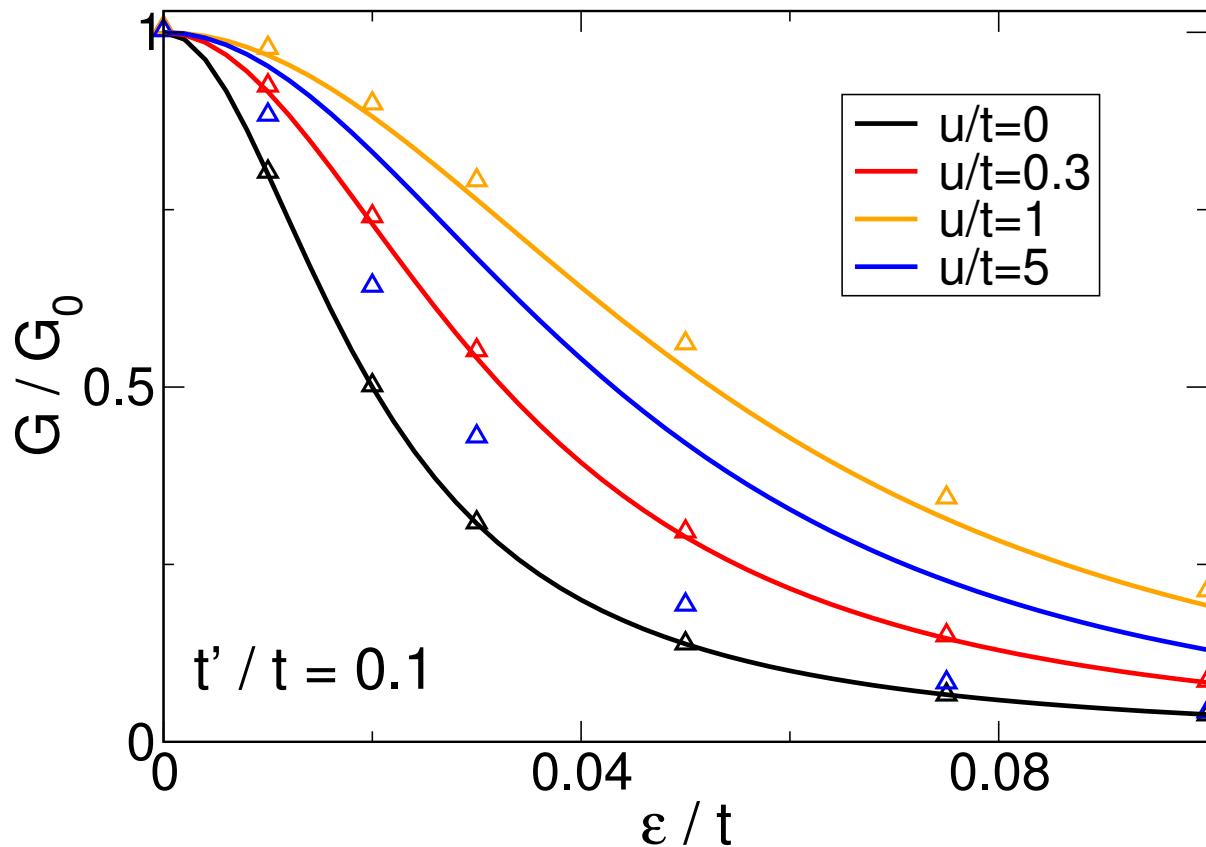
# In equilibrium with $L$ - $R$ symmetry: lattice model

use numerical DMRG and Kubo formula

(Bohr & Schmitteckert '07)

here: functional RG for small  $u$

(Karrasch, Pletyukhov, Borda & VM '10)



symbols: DMRG

lines: FRG

## Finite bias: steady state at particle-hole and $L$ - $R$ symmetry

scaling limit and  $V \gg T_K$ :  $I \sim V^{-\alpha_V}$ ,  $\alpha_V = 2U + \mathcal{O}(U^2)$

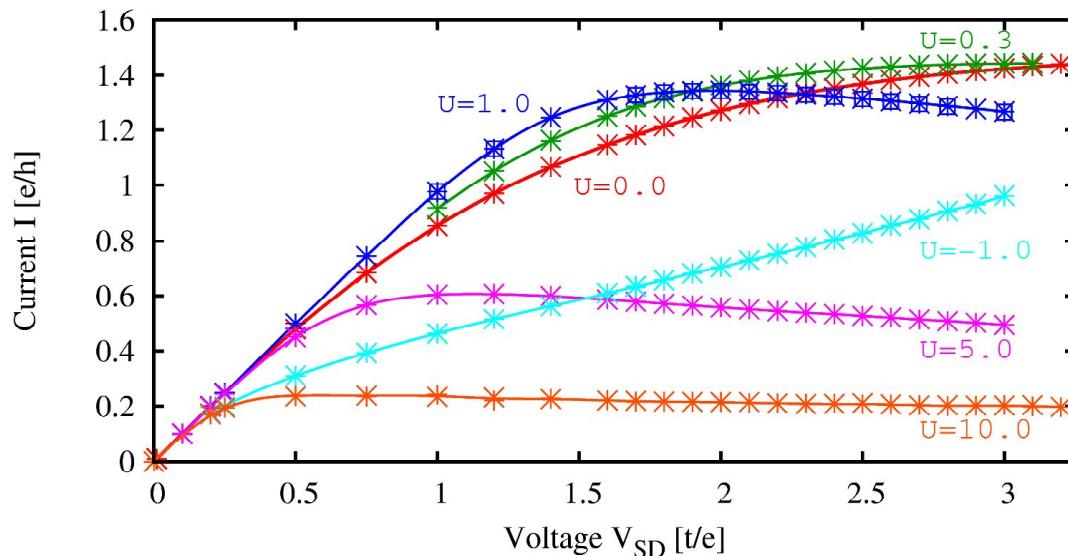
(Doyon '07; Boulat, Saleur & Schmitteckert '08)

does  $V$  only act as (trivial) IR cutoff?

(Borda & Zawadowski '10)

lattice model, td-DMRG:

(Boulat, Saleur & Schmitteckert '08)



hopping parameter:  $t'/t = 0.5$

⇒ not in scaling limit

⇒ no power law!

questions:

- compare nonequ. FRG to td-DMRG
- confirm power law in scaling limit
- go away from p-h and  $L$ - $R$  symmetry

# Steady-state FRG on the Keldysh contour: $\epsilon = 0$ , L-R symmetric

(Gezzi, Pruschke & VM '07; Jakobs, VM & Schoeller '07; Jakobs, Pletyukhov & Schoeller '10)

cutoff: auxiliary leads to sites 1,2,3; coupling  $\Lambda$  from  $\infty \rightarrow 0$

flow equation (first order in  $u$ ):

$$\partial_\Lambda t'^\Lambda = i \frac{u}{4\pi} \int d\omega \left\{ G^\Lambda \partial_\Lambda [G_0^\Lambda]^{-1} G^\Lambda \right\}_{\text{Keldysh}}^{\text{off-diag.}}, \quad t'^{\Lambda=\infty} = t'$$

Green functions: ( $\Gamma_{\text{lead}}(\omega) = \pi\rho(\omega)t^2$ )

$$[G_{\text{ret}}^\Lambda]^{-1} = \begin{pmatrix} \omega + i\Gamma_{\text{lead}} + i\Lambda & -t'^\Lambda & 0 \\ -t'^\Lambda & \omega + i\Lambda & -t'^\Lambda \\ 0 & -t'^\Lambda & \omega + i\Gamma_{\text{lead}} + i\Lambda \end{pmatrix}, \quad G_{\text{adv}}^\Lambda = [G_{\text{ret}}^\Lambda]^\dagger$$

$$G_{\text{Keldysh}}^\Lambda = -2iG_{\text{ret}}^\Lambda \left\{ i\Lambda[1 - 2f_C] + \Gamma_{\text{lead}} \begin{pmatrix} [1 - 2f_L] & & \\ & 0 & \\ & & [1 - 2f_R] \end{pmatrix} \right\} G_{\text{adv}}^\Lambda$$

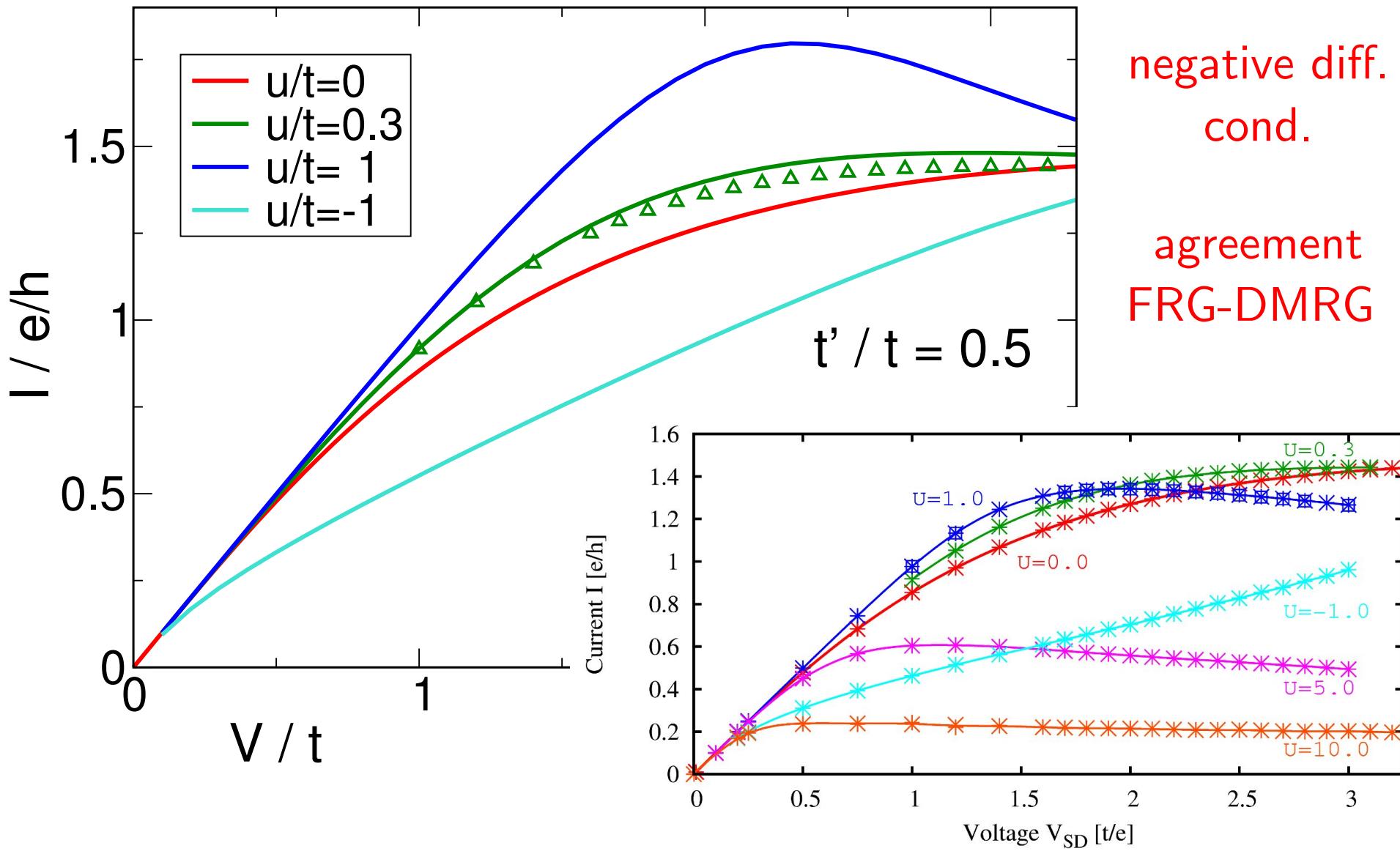
occupation functions:

$$f_{L/R}(\omega) = f(\omega \mp V/2) \quad \text{physical leads}$$

$$f_C(\omega) = f(\omega) \quad \text{auxiliary leads}$$

current (at  $\Lambda = 0$ ):  $I = 4 \int \Gamma_{\text{lead}}^2(\omega) [f_L(\omega) - f_R(\omega)] |G_{\text{ret}}^{13}(\omega)|^2 d\omega$

## Comparison to td-DMRG



(Boulat, Saleur & Schmitteckert '08; Karrasch, Borda, Pletyukhov & VM '10)

## FRG and RTRG-FS in scaling limit: $\epsilon = 0$ , L-R symmetric

flow equation for renormalized rate  $\Gamma^\Lambda$ :

$$\begin{aligned} \partial_\Lambda \Gamma^\Lambda &= -2U\Gamma^\Lambda \frac{\Lambda + \Gamma^\Lambda}{(V/2)^2 + (\Lambda + \Gamma^\Lambda)^2}, \quad \Gamma^{\Lambda=\infty} = \Gamma^\infty \\ \Rightarrow \quad \Gamma &\approx \Gamma^\infty \left( \frac{\Lambda_0}{\max\{V/2, \Gamma\}} \right)^{2U}, \quad \Lambda_0 \sim B \\ \Rightarrow \quad V \text{ and } \Gamma &\text{ act as IR cutoffs} \end{aligned}$$

observables:

- susceptibility at  $V = 0$ :

$$\chi^{-1} = \pi\Gamma/2 = \pi T_K/2 \sim (\Gamma^\infty)^{1-\alpha_\chi}, \quad \alpha_\chi = 2U + \mathcal{O}(U^2)$$

- current for  $V \gg \Gamma = T_K$ : (agrees with Doyon '07)

$$I = \frac{2\Gamma}{\pi} \arctan \frac{V}{\Gamma} \sim \Gamma \sim V^{-\alpha_V}, \quad \alpha_V = 2U + \mathcal{O}(U^2)$$

no interesting nonequilibrium physics??

## FRG and RTRG-FS in scaling limit: general case

flow of level positions is of order  $U^2$  and can be neglected

flow equation for renormalized rates  $\Gamma_\alpha^\Lambda$ : ( $\Gamma^\Lambda = \sum_\alpha \Gamma_\alpha^\Lambda$ )

$$\partial_\Lambda \Gamma_\alpha^\Lambda = -2U_\alpha \Gamma_\alpha^\Lambda \frac{\Lambda + \Gamma^\Lambda/2}{(\mu_\alpha - \epsilon)^2 + (\Lambda + \Gamma^\Lambda/2)} , \quad \Gamma_\alpha^{\Lambda=\infty} = \Gamma_\alpha^\infty$$

$$\Rightarrow \quad \Gamma_\alpha \approx \Gamma_\alpha^\infty \left( \frac{\Lambda_0}{\max\{|\mu_\alpha - \epsilon|, \Gamma/2\}} \right)^{2U_\alpha} , \quad \Lambda_0 \sim B$$

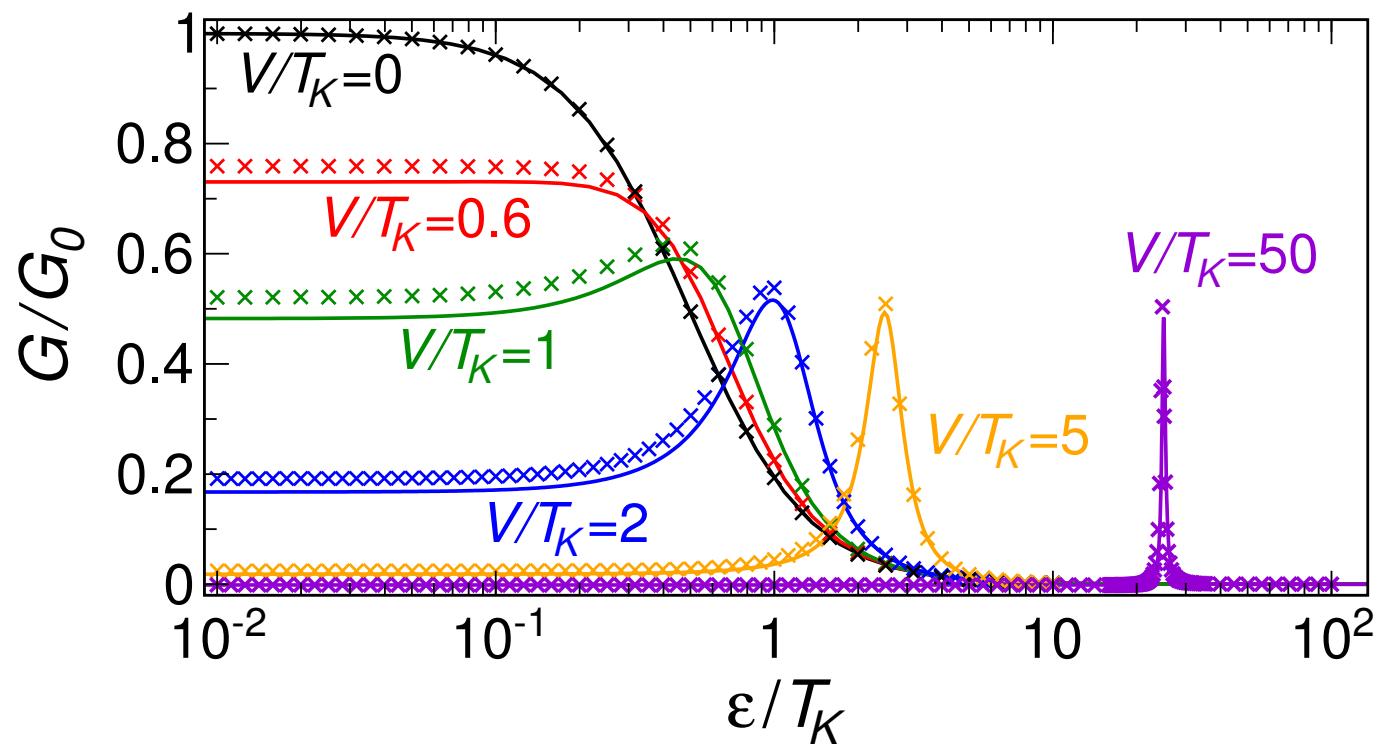
current (at  $\Lambda = 0$ ):

$$I = \frac{1}{\pi} \frac{\Gamma_L \Gamma_R}{\Gamma} \left[ \arctan \frac{V - 2\epsilon}{\Gamma} + \arctan \frac{V + 2\epsilon}{\Gamma} \right]$$

introduce  $T_K^\alpha$ :  $T_K^\alpha = \Gamma_\alpha^\infty (2\Lambda_0/T_K)^{2U_\alpha}$  ,  $T_K = \sum_\alpha T_K^\alpha$  ( $\neq \Gamma$ )

introduce asymmetry parameter:  $c^2 = T_K^L/T_K^R$

## Away from p-h symmetry, still $L$ - $R$ symmetric



$L$ - $R$  symmetric

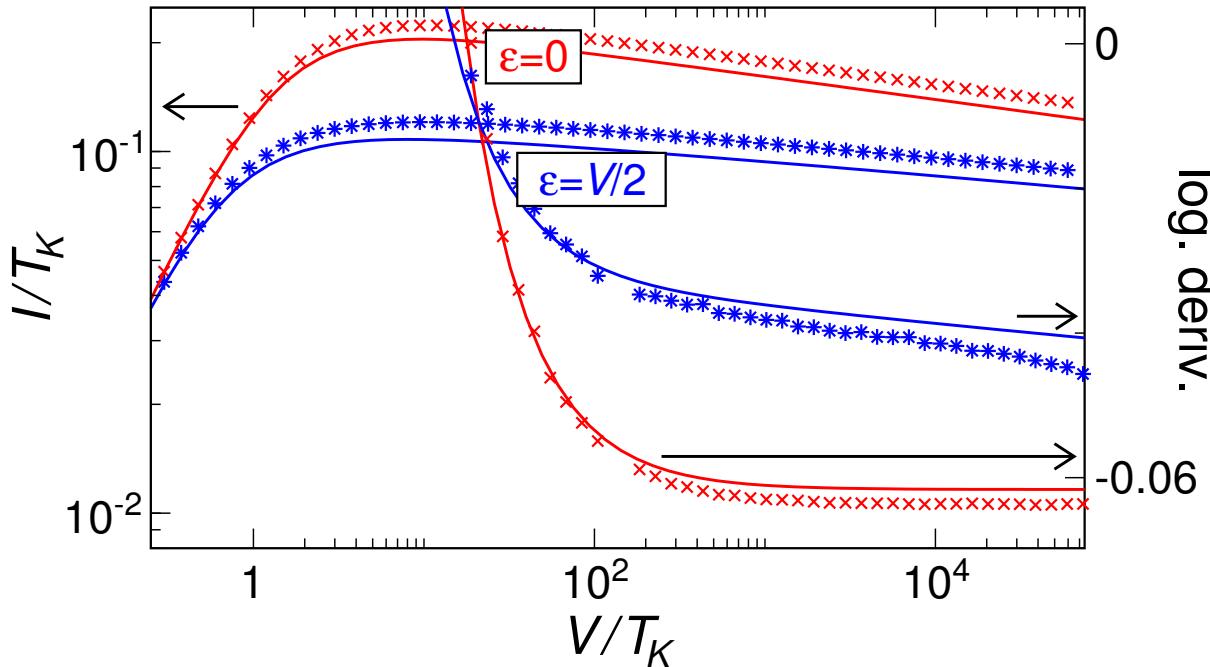
$U = 0.1/\pi$

lines: RTRG-FS

symbols: FRG

$\Rightarrow$  maximum of  $G = dI/dV$  at  $\epsilon = V/2$ : resonance!

## On-resonance current



$L-R$  symmetric

$$U = 0.1/\pi$$

lines: RTRG-FS

symbols: FRG

current for  $V \gg T_K$ :  $I(V) \approx \frac{T_K}{2} \frac{\left(\frac{T_K}{\Gamma}\right)^{2U_L} \left(\frac{T_K}{2V}\right)^{2U_R}}{c \left(\frac{T_K}{\Gamma}\right)^{2U_L} + \frac{1}{c} \left(\frac{T_K}{2V}\right)^{2U_R}} \frac{c}{1+c^2}.$

$\Rightarrow V$  is not a simple IR cutoff

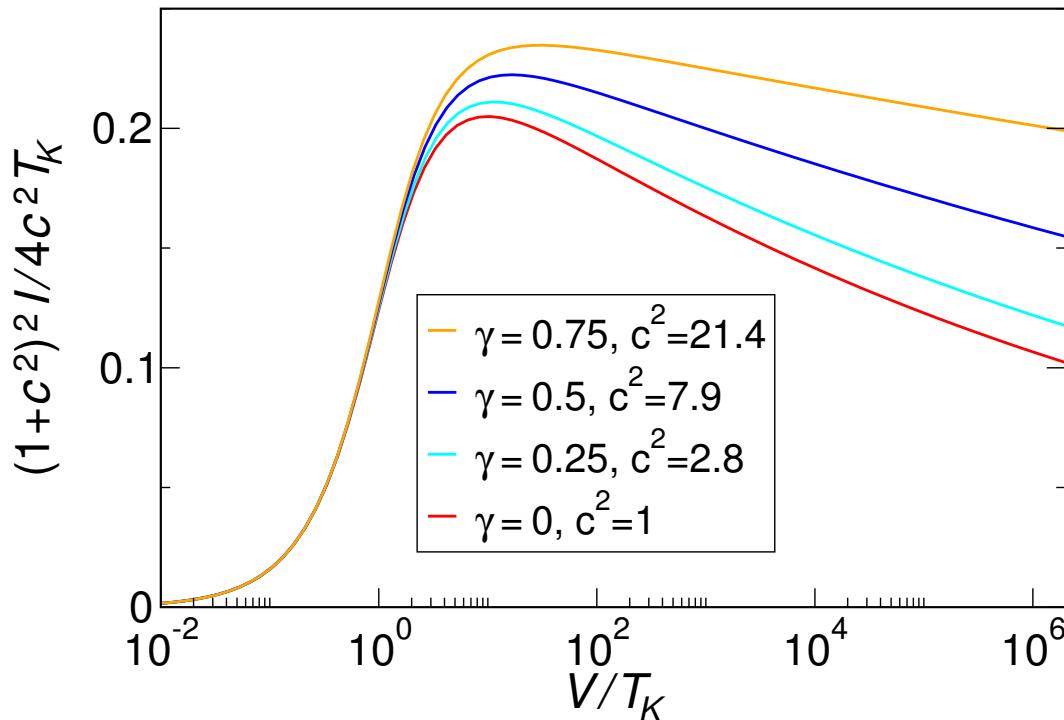
power law only for

- $V \gg T_K$
- $c \gg 1$

$$\Rightarrow I \sim V^{-2U_R}$$

(does **not** agree with Doyon '07)

## Left-right asymmetry, off-resonance



*L-R* asymmetric

$$U_{L/R} = (1 \pm \gamma) 0.1/\pi$$

$$\epsilon = 0$$

lines: RTRG-FS

$$\frac{\left(\frac{T_K}{V}\right)^{2U_L} \left(\frac{T_K}{V}\right)^{2U_R}}{c \left(\frac{T_K}{V}\right)^{2U_L} + \frac{1}{c} \left(\frac{T_K}{V}\right)^{2U_R}} \cdot \frac{c}{1+c^2}.$$

current for  $V \gg T_K, V \gg \epsilon$ :  $I(V) \approx T_K$

$\Rightarrow V$  is not a simple IR cutoff for generic cases

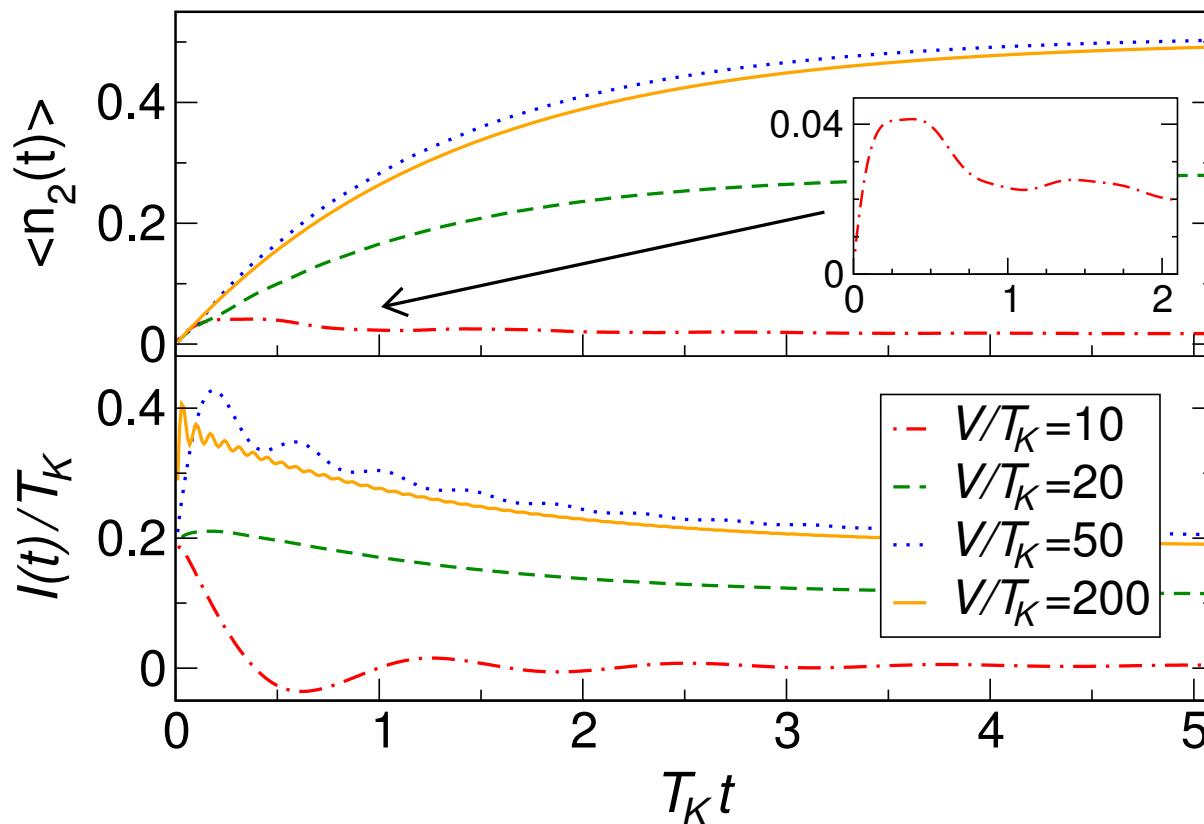
power law only for

- $U_L = U_R \quad \Rightarrow \quad I \sim V^{-2U}$
- $U_L \neq U_R, c \ll 1 \quad \Rightarrow \quad I \sim V^{-2U_L}$
- $U_L \neq U_R, c \gg 1 \quad \Rightarrow \quad I \sim V^{-2U_R}$

# Relaxation dynamics into the steady state: RTRG-FS

long-time behavior, off-resonance ( $\epsilon, V, |\epsilon - V/2| \gg T_K, 1/t$ )

$$\langle n_2(t) \rangle \approx (1 - e^{-\Gamma_1 t}) \langle n_2 \rangle - \frac{1}{2\pi} e^{-\Gamma_2 t} (T_K t)^{1+2U} \\ \times \left[ \frac{\sin((\epsilon + \frac{V}{2})t)}{(\epsilon + \frac{V}{2})^2 t^2} - \frac{\pi U}{4} \frac{\cos((\epsilon + \frac{V}{2})t)}{(\epsilon + \frac{V}{2})^2 t^2} + (V \rightarrow -V) \right]$$



*L-R symmetric*

$$\langle n_2(t = 0) \rangle = 0$$

$$U = 0.1/\pi, \epsilon = 10T_K$$

$\Gamma_1 \approx \Gamma$  (charge relaxation)

$\Gamma_2 \approx \Gamma/2$  (level broadening)

oscillation freqs.  $\epsilon \pm V/2$

⇒ exponential and power-law decay, two decay rates

## Summary

- finite bias IRLM contains interesting nonequilibrium physics . . .
- which doesn't show up for left-right symmetry and off-resonance
- interesting relaxation dynamics
- FRG and RTRG-FS are suitable tools to study the nonequ. IRLM
- td-DMRG results (not in scaling limit) confirmed

Refs.:

- Karrasch, Andergassen, Pletyukhov, Schuricht, Borda, VM & Schoeller, EPL **90**, 30003 (2010)
- Karrasch, Pletyukhov, Borda & VM, PRB **81**, 125122 (2010)