Superconducting Proximity Effect in Interacting Quantum Dots:

Equilibrium and Nonequilibrium

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SFB 491

Collaborators & Publications

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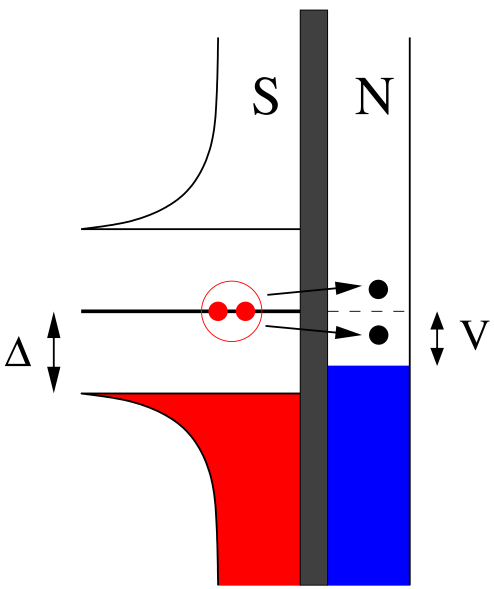
New J. Phys. **9**, *278 (2007); Phys. Rev. B* **77**, *134513 (2008);*
Phys. Rev. B **79**, *054505 (2009); arXiv:1002.4629;*
Europhys. Lett. **91**, *47004 (2010); arXiv:1006.1790;*
Phys. Rev. B **82**, *094514 (2010)*

Transport through S-N and S-I-S Interfaces

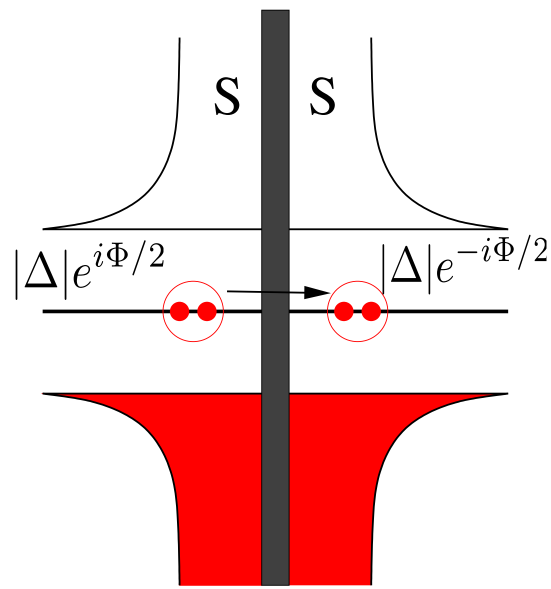
- BCS superconductivity: pair potential $\Delta = g\langle c_{-k\downarrow} c_{k\uparrow}\rangle$
  - quasiparticles with gap $|\Delta|$ in density of states
  - Cooper-pair condensate
- subgap transport

Andreev reflection



Cooper pair breaks/forms

Josephson current



transfer of Cooper pairs

$J_{\text{jos}}(\Phi) = J_c \sin\Phi$

QDs Coupled to Superconductors - Exps

- carbon nanotube quantum dot
  - *Buitelaar, Nussbaumer, Schönenberger, PRL ’02*
  - *Cleziou, Wernsdorfer, Bouchiat, Ondarcuhu, Monthioux, Nature Nanotech. ’06*
  - *Jarillo-Herrero, van Dam, Kouwenhoven, Nature ’06*
  - *Jorgensen, Grove-Rasmussen, Novotny, Flensberg, Lindelof, PRL ’06*
  - *Hermann, Portier, Roche, Levy Yeyati, Kontos, Strunk, PRL ’10*
- quantum dot in InAs nanowire
  - *van Dam, Nazarov, Bakkers, De Franceschi, Kouwenhoven, Nature ’06*
  - *Hofstetter, Csonka, Nygard, Schönenberger, Nature ’10*
- InAs quantum dot with Al electrodes
  - *Buizert, Oiwa, Shibata, Hirakawa, Tarucha, PRL ’07*
  - *Deacon, Tanaka, Oiwa, Sakano, Yoshida, Shibata, Hirakawa, Tarucha, PRL ’10*

QDs Coupled to Superconductors - Thy

- Andreev and multiple Andreev reflections through QDs

  Levy Yeyati, Cuevas, Lopez-Davalos, Martin-Rodero, PRB '97; Fazio, Raimondi, PRL '98, PRL '99; Kang, PRB '98; Schwab, Raimondi, PRB '99; Johansson, Bratus, Shumeiko, Wendin, PRB '99; Clerk, Ambegaokar, Hershfield, PRB '00; Cuevas, Levy Yeyati, Martin-Rodero, PRB '01; . . .

- transport in Kondo regime

  Clerk, Ambegaokar, PRB '00; Avishai, Golub, Zaikin, PRB '03; Siano, Egger, PRL '04; Sellier, Kopp, Kroha, Barash, PRB '05; Nussinov, Shnirman, Arovas, Balatsky, Zhu, PRB '05; Bergeret, Levy Yeyati, Martin-Rodero, PRB '06; Lopez, Choi, Aguado, PRB '07; Karrasch, Oguri, Meden, PRB '08; Karrasch, Meden, PRB '09; . . .

- Josephson current through QD with spin-orbit coupling

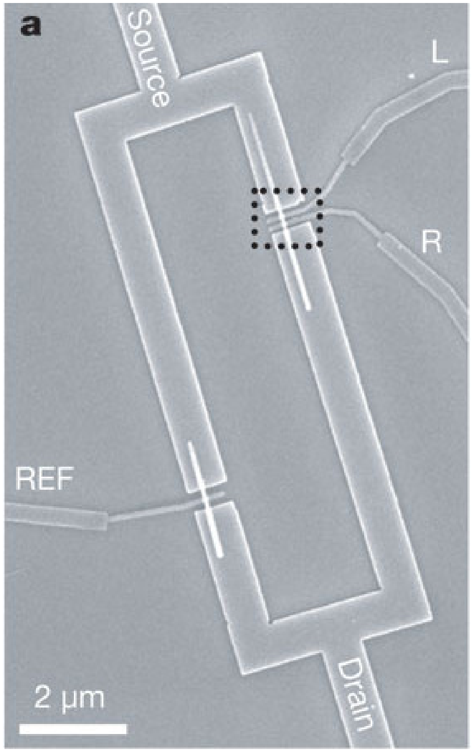
  Dell'Anna, Zazunov, Egger, Martin, PRB '07; Zazunov, Egger, Jonckheere, Martin, PRL '09; . . .

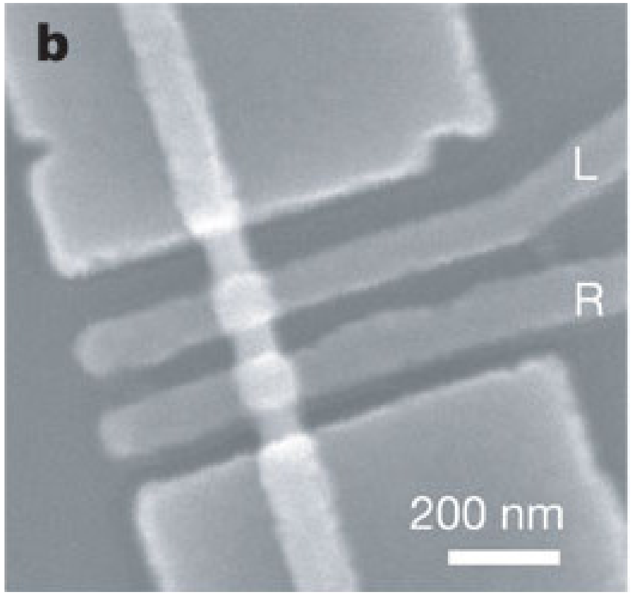
Supercurrent reversal in quantum dots

Jorden A. van Dam[1], Yuli V. Nazarov[1], Erik P. A. M. Bakkers[2], Silvano De Franceschi[1,3] & Leo P. Kouwenhoven[1]

Nature **442**, 667 (2006)

- quantum dot in InAs nanowire





Supercurrent reversal in quantum dots

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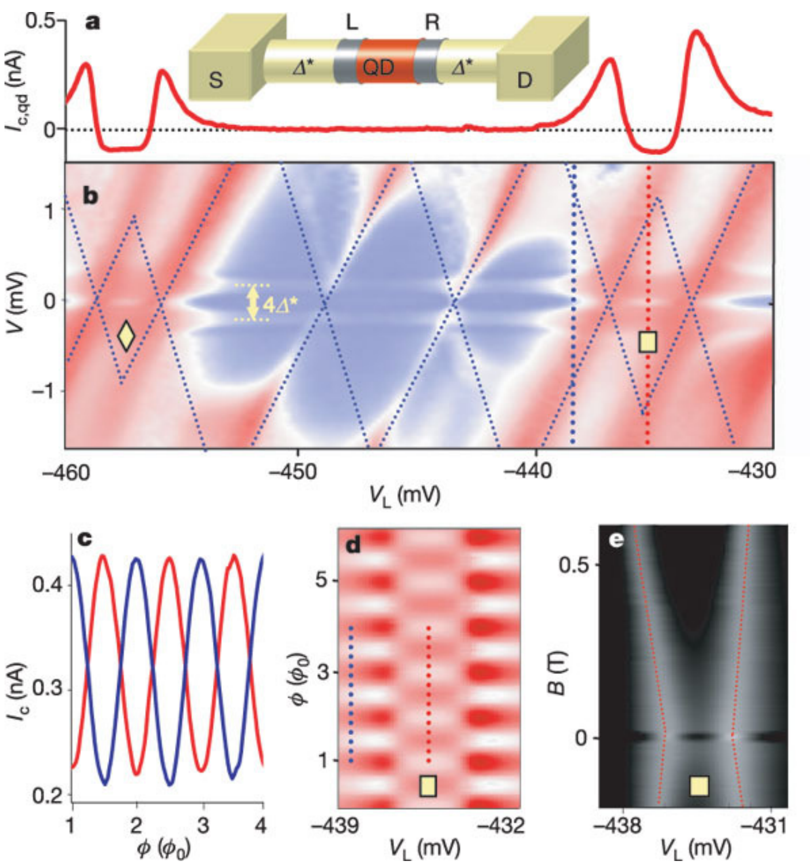
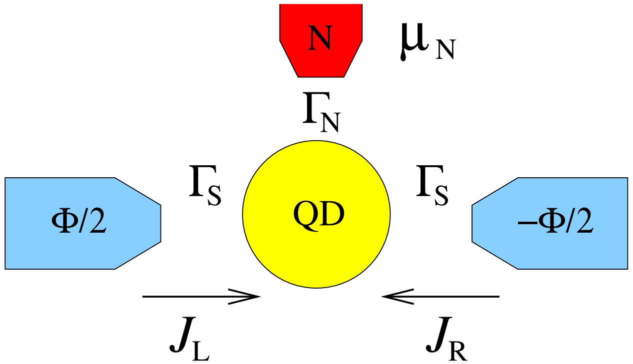


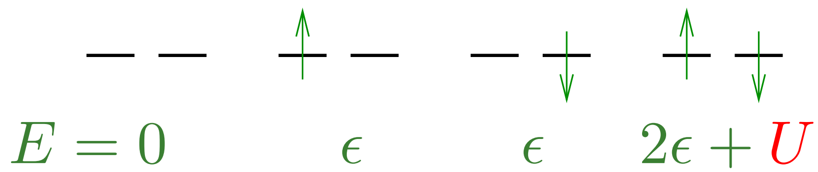
- Josephson coupling due to cotunneling
- $J_{\text{jos}} \propto \Gamma^2$
- π-state for odd occupation
- theory:
  van Dam et al., Nature '06
  Glazman, Matveev, JETP Lett. '89
  Rozhkov, Arovas, Guinea, PRB '01

How to Establish a Josephson Coupling?

- higher-order tunneling (cotunneling)
  - $J_{\text{jos}} \propto \Gamma^2$
- (equilibrium) proximity effect in single-level QD
  - finite pair amplitude: $\langle d_\downarrow d_\uparrow \rangle \neq 0$
  - requirement: $E_0 \approx E_{\uparrow\downarrow} < E_\uparrow = E_\downarrow$
  - impossible for large charging energy, $U \gg k_\mathrm{B}T, \Gamma$
- nonequilibrium proximity effect in single-level QD
  - finite pair amplitude: $\langle d_\downarrow d_\uparrow \rangle \neq 0$
  - third, normal electrode drives dot out of equilibrium

Model: Anderson Hamiltonian





- quantum dot: $H_{\mathrm{D}} = \sum_\sigma \epsilon d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$
- leads: $H_\eta = \sum_{k\sigma} \epsilon_k c_{\eta k\sigma}^\dagger c_{\eta k\sigma} - \sum_k \left(\Delta_\eta c_{\eta k\uparrow}^\dagger c_{\eta -k\downarrow}^\dagger + \mathrm{H.c.}\right)$
  - superconducting leads: phase biased with $\pm i\Phi/2$
  - normal lead: voltage biased with μ_{N}
- tunneling: $H_{\mathrm{tunn},\eta} = V_\eta \sum_{k\sigma} \left(c_{\eta k\sigma}^\dagger d_\sigma + \mathrm{H.c.}\right)$

$\Rightarrow$ charging energy, superconductivity, and nonequilibrium

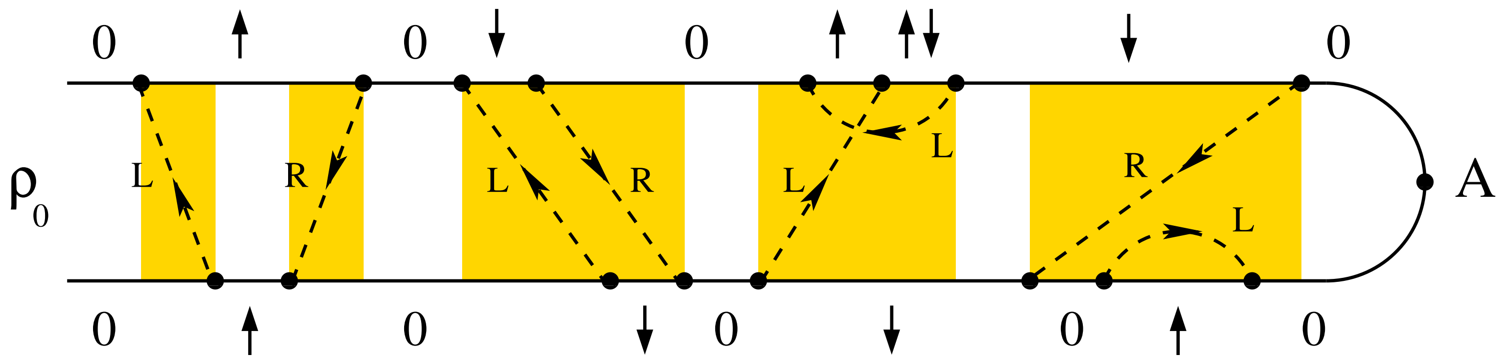
Diagrammatic Transport Theory

Pala, Governale, J.K., NJP '08; Governale, Pala, J.K., PRB '08

general idea: reduced density matrix for dot

– integrate out leads: contractions $\langle c^\dagger_{\eta k\sigma} c_{\eta k\sigma} \rangle$ and $\langle c_{\eta k\sigma} c^\dagger_{\eta k\sigma} \rangle$

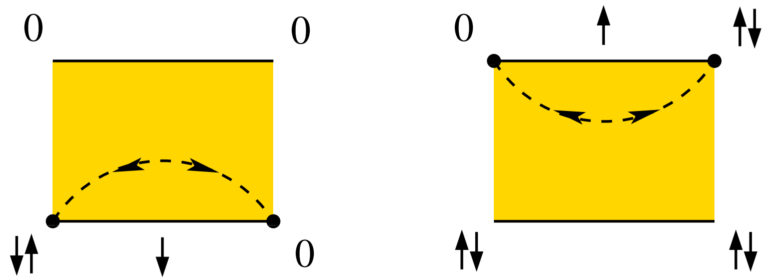
– expand in tunnel coupling, treat interaction exactly



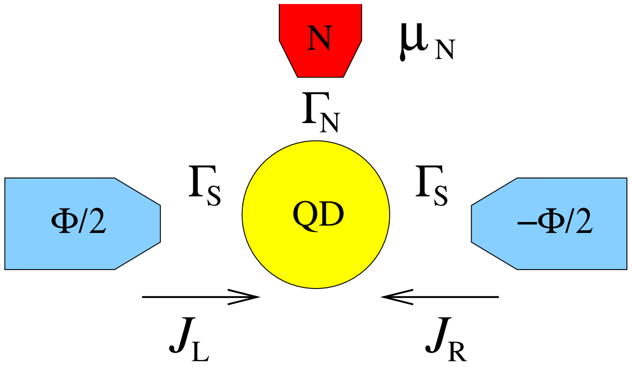
changes due to superconductivity:

– anomalous contractions: $\langle c^\dagger_{\eta k\sigma} c^\dagger_{\eta -k -\sigma} \rangle$ and $\langle c_{\eta -k -\sigma} c_{\eta k\sigma} \rangle$

– finite pair amplitude on dot: $\langle d_\downarrow d_\uparrow \rangle \neq 0$



2S-dot-N in Nonequilibrium



Governale, Pala, J.K., PRB '08

- $|\Delta| \to \infty$: all orders in Γ_S
- first order in Γ_N

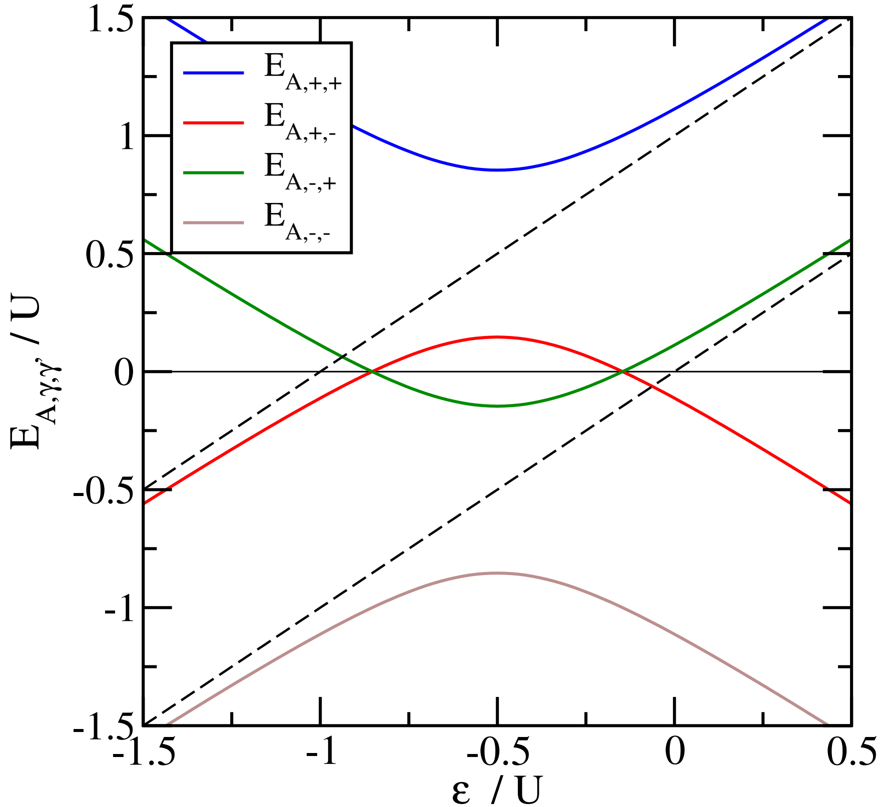
isospin representation

$$I_\mathrm{x} = \mathrm{Re}\langle d_\downarrow d_\uparrow \rangle; \quad I_\mathrm{y} = \mathrm{Im}\langle d_\downarrow d_\uparrow \rangle; \quad I_\mathrm{z} = \frac{\langle d_\uparrow^\dagger d_\uparrow + d_\downarrow^\dagger d_\downarrow \rangle - 1}{2}$$

- kinetic equation for isospin: $\frac{d\mathbf{I}}{dt} = \mathbf{A} - \mathbf{R} \cdot \mathbf{I} + \mathbf{I} \times \mathbf{B}$
- how to proximize the dot:
  - combined S-dot-N Andreev reflection: $\mathbf{A}^{(1)}$
  - Andreev reflection at S-dot interface: $\mathbf{I} \times \mathbf{B}^{(0)}$

Andreev bound states

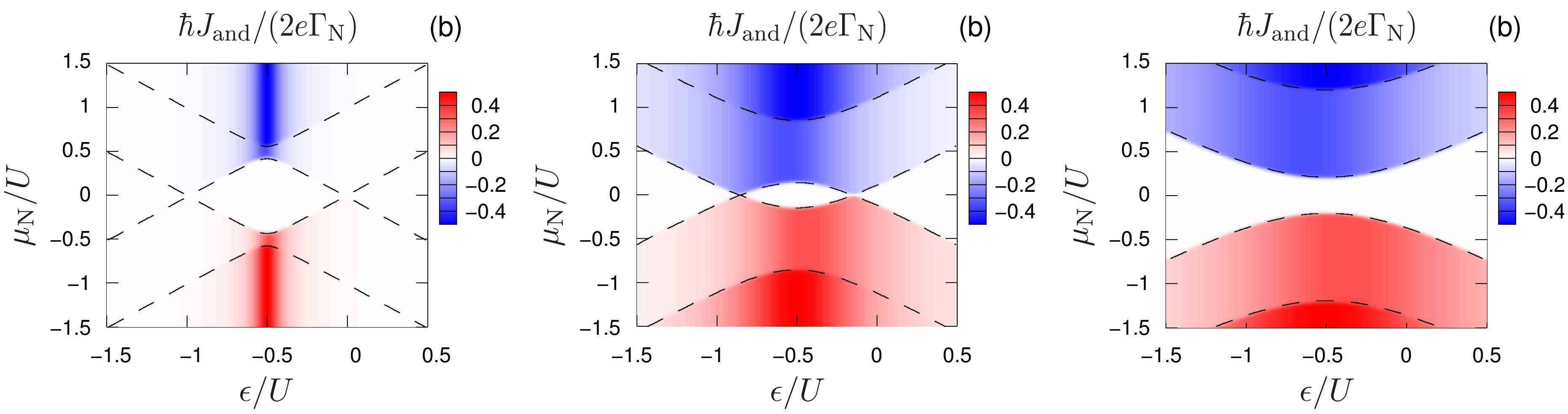
- without SC: resonances at ϵ and $\epsilon + U$
- with SC: Andreev bound-state energies:
  $E_{A,\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_S^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$



- four resonances
- avoided crossing
  - at $\delta = 2\epsilon + U = 0$
  - with gap $2\Gamma_S |\cos(\Phi/2)|$

Andreev Current $J_L + J_R$

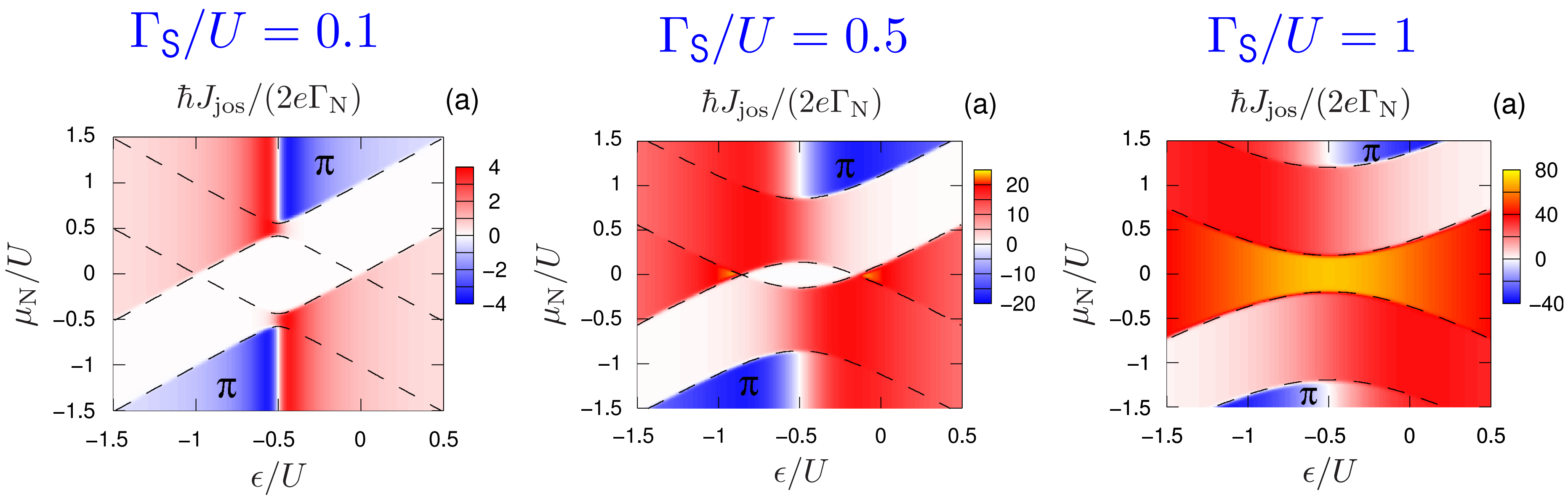
$\Gamma_S/U = 0.1$ $\Gamma_S/U = 0.5$ $\Gamma_S/U = 1$



- Andreev current probes nonequilibrium proximity effect
- maximal around $\varepsilon = -U/2$
- Andreev bound-state energies:

$E_{A,\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_S^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$

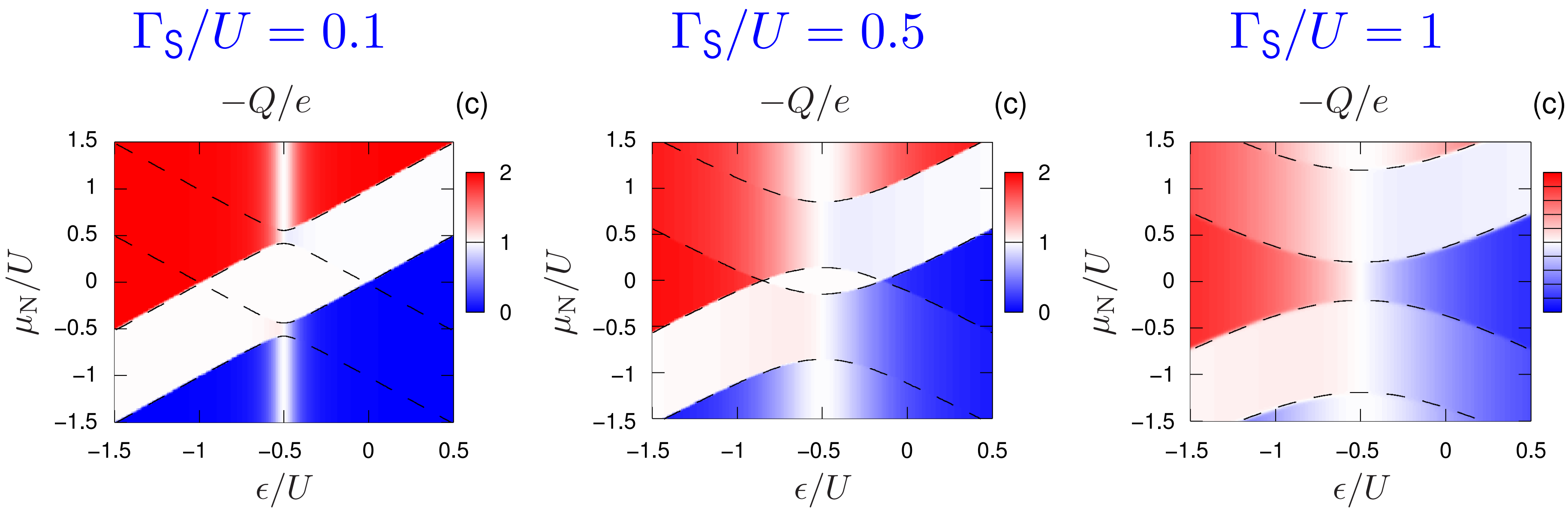
Josephson Current $J_L - J_R$



- equilibrium Josephson current suppressed by charging
- nonequilibrium Josephson current for finite μ_N
- π-transition driven by ϵ or μ_N
- Andreev bound-state energies:

  $E_{A,\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_S^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$

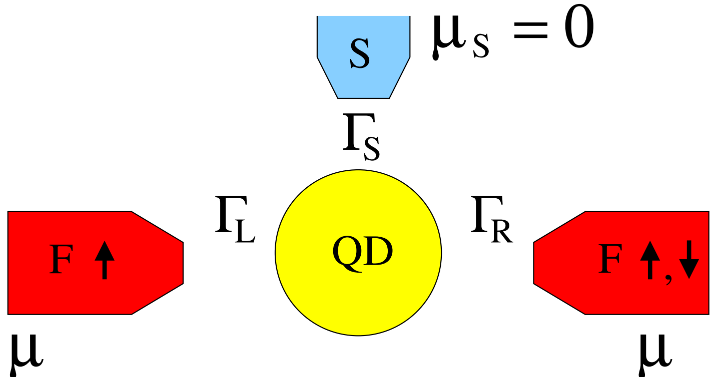
Dot Charge



- dot charge probes nonequilibrium proximity effect
- nonmonotonicity around $\varepsilon = -U/2$
- Andreev bound-state energies:
  $E_{A,\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma\sqrt{(\epsilon + U/2)^2 + \Gamma_S^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$

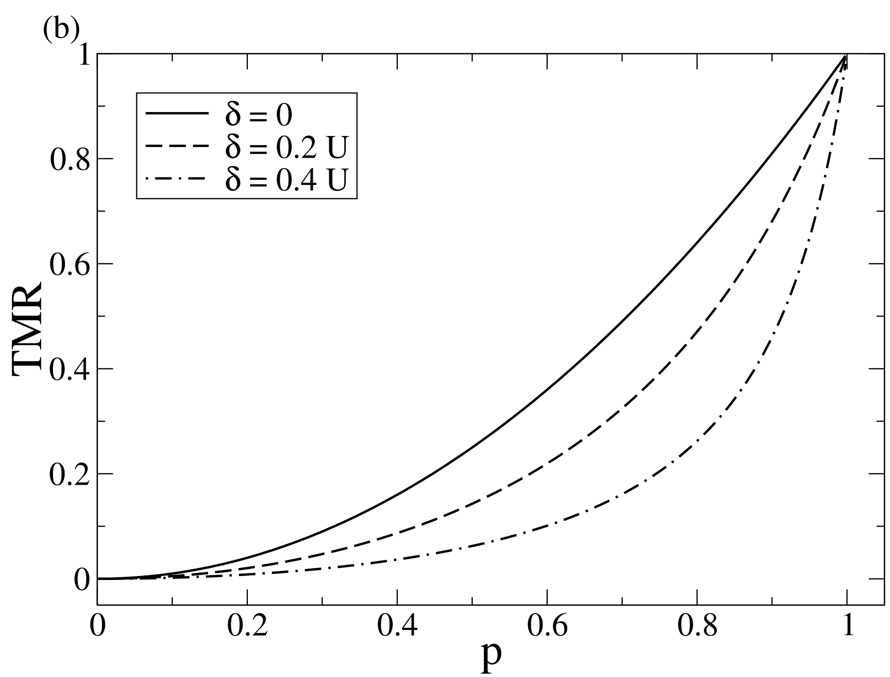
Crossed Andreev Reflection

Futterer, Governale, Pala, J.K., PRB '09

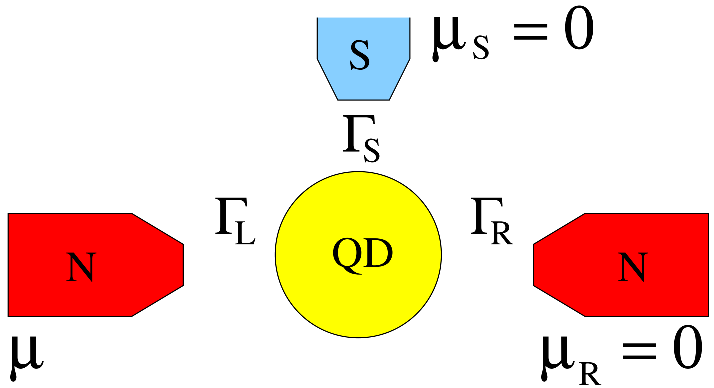


- $|\Delta| \to \infty$: all orders in Γ_S
- first order in $\Gamma_{L,R}$
- spin polarization p in leads
- parallel/antiparallel leads

- $\mu_S = 0,\ \mu_L = \mu_R = \mu$
- $\mathrm{TMR} = \dfrac{J_S^{\mathrm{anti}} - J_S^{\mathrm{para}}}{J_S^{\mathrm{anti}}}$
- crossed Andreev reflection (Cooper-pair splitter)



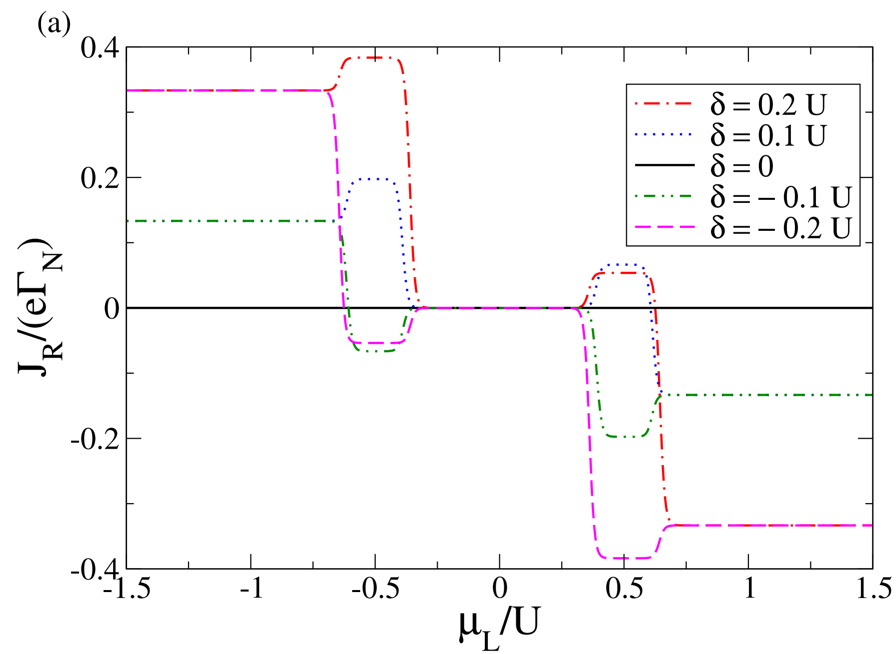
Negative Nonlocal Conductance



Futterer, Governale, Pala, J.K., PRB '09

- $|\Delta| \to \infty$: all orders in Γ_S
- first order in $\Gamma_{L,R}$

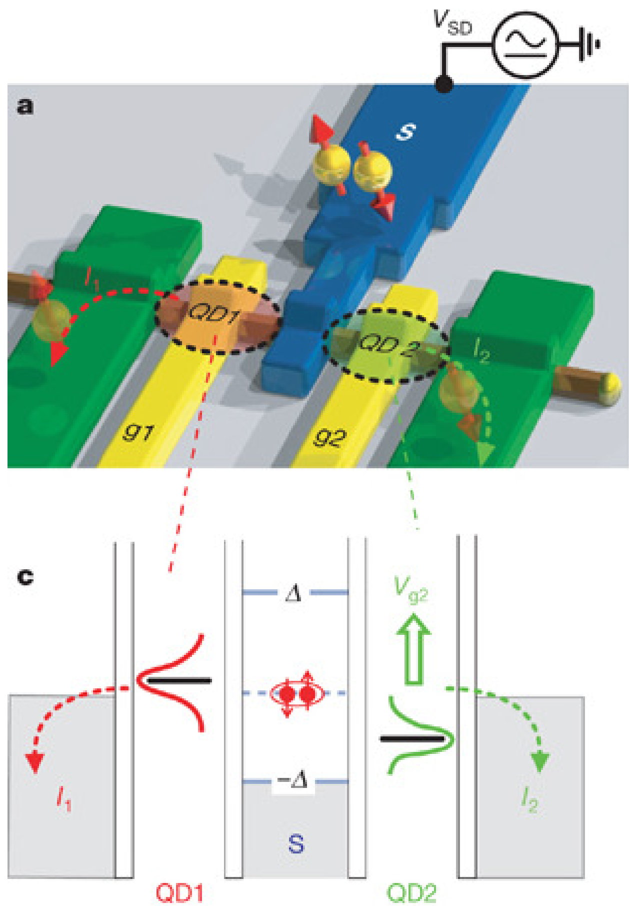
- $\mu_S = \mu_R = 0$, $\mu_L = \mu$
- nonlocal conduct. $G = -\frac{J_R}{\mu_L}$
- four steps
- sensitive to detuning δ
- negative nonlocal conduct.

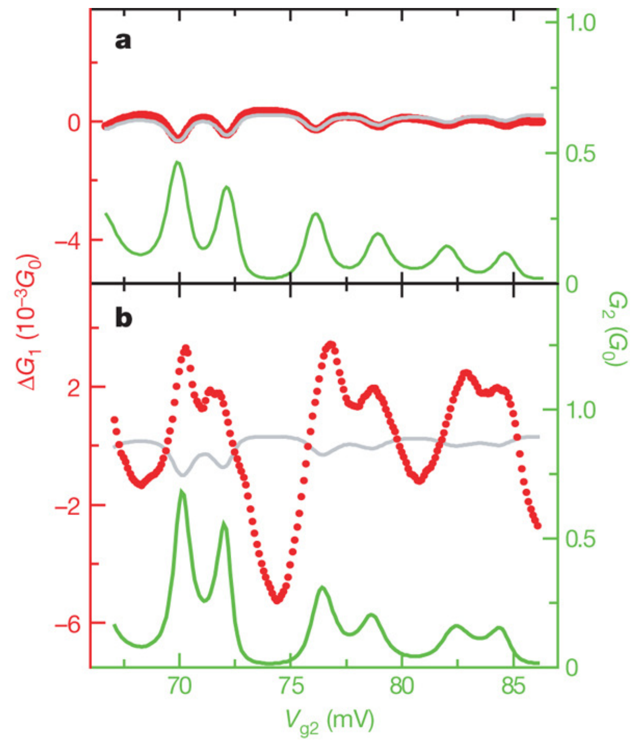


Cooper pair splitter realized in a two-quantum-dot Y-junction

L. Hofstetter[1]*, S. Csonka[1,2]*, J. Nygård[3] & C. Schönenberger[1]

Nature **461**, 960 (2009)





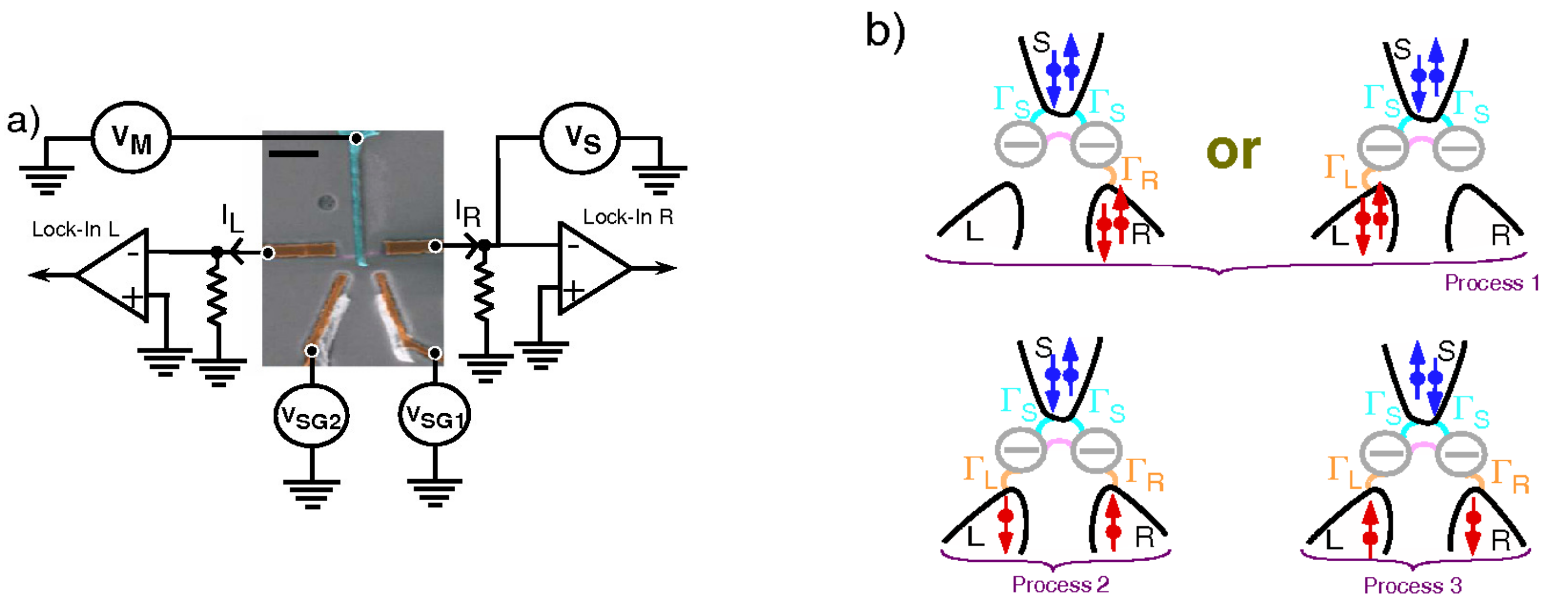
→ spin-entangled electron pair in normal leads

PRL 104, 026801 (2010)
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS
week ending 15 JANUARY 2010

Carbon Nanotubes as Cooper-Pair Beam Splitters

L. G. Herrmann,[1,2,5] F. Portier,[3] P. Roche,[3] A. Levy Yeyati,[4] T. Kontos,[1,2,*] and C. Strunk[5]

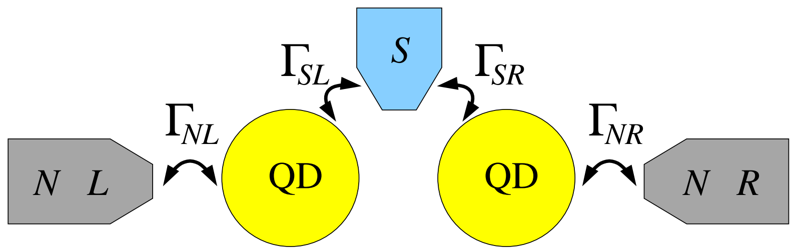
- double quantum dot in carbon nanotube



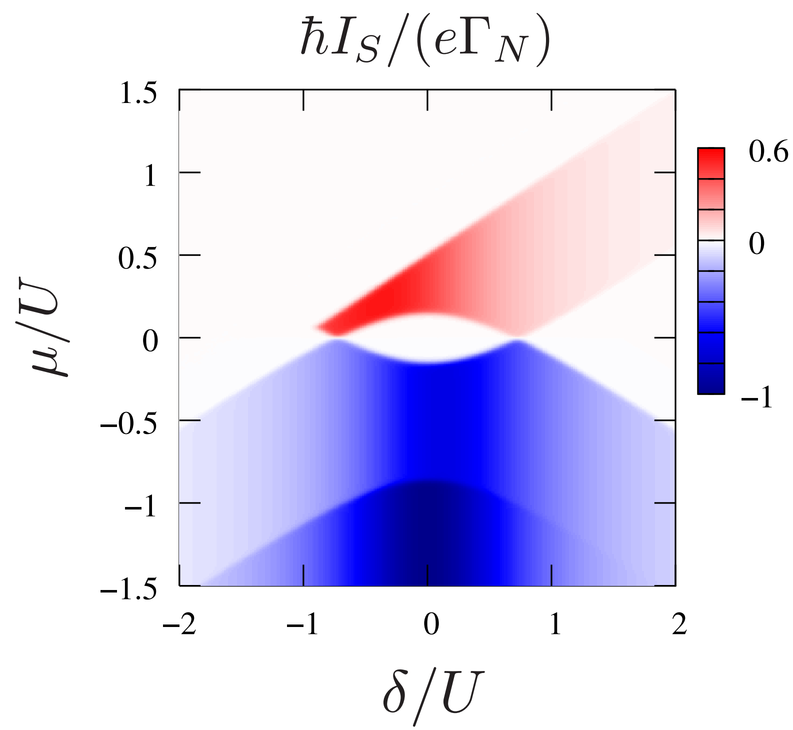
→ spin-entangled electron pair in normal leads

Proximity Effect in Double-Dot Systems

Eldridge, Pala, Governale, J.K., preprint '10

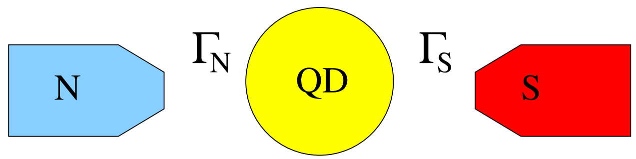


- non-local pair amplitude: $\langle d_{L\uparrow} d_{R\downarrow} - d_{L\downarrow} d_{R\uparrow} \rangle \neq 0$
- negative bias $\mu_S < 0$:
  - transport through singlet
  - Cooper pair splitter
- positive bias $\mu_S > 0$:
  - spin triplet occupied
  - triplet blockade

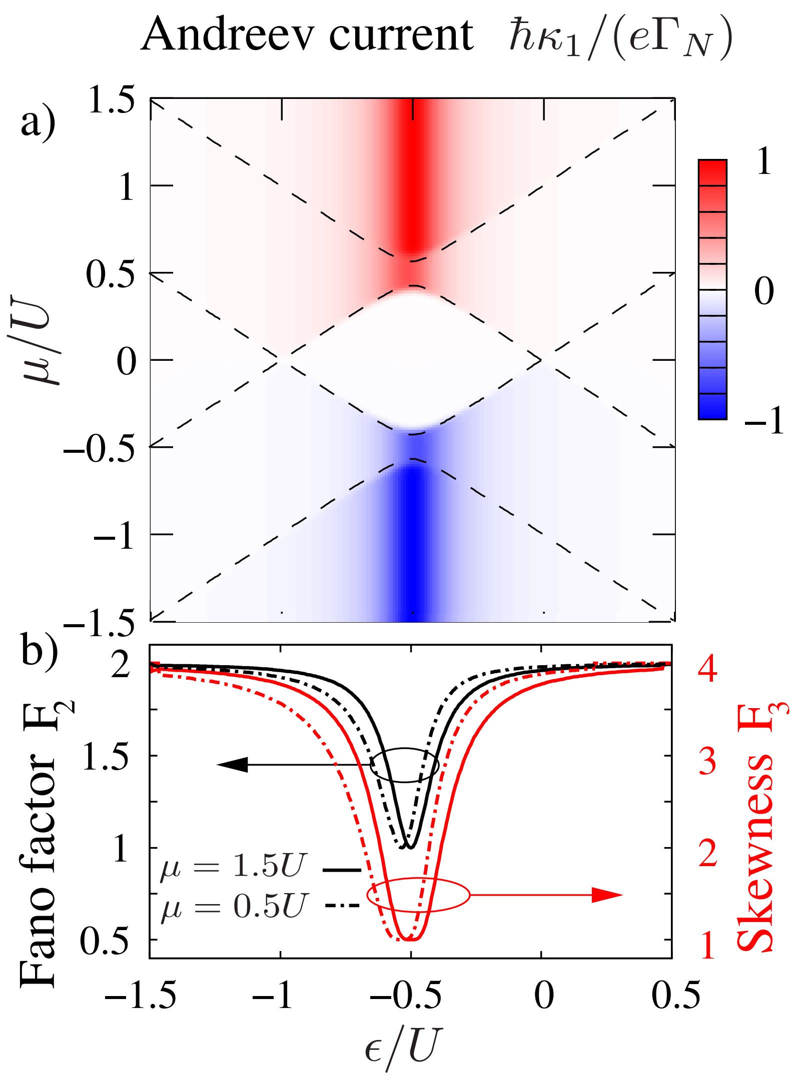


Shot Noise Reveals Proximity Effect

Braggio, Governale, Pala, J.K., preprint ’10

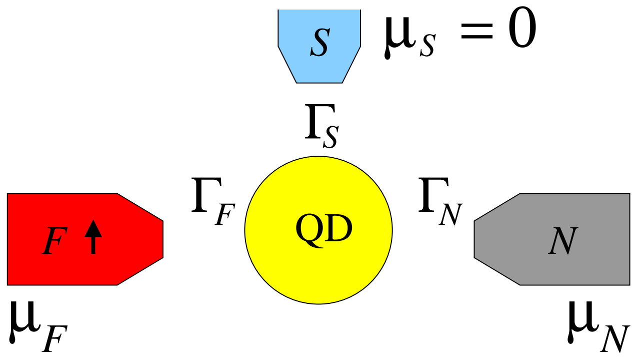


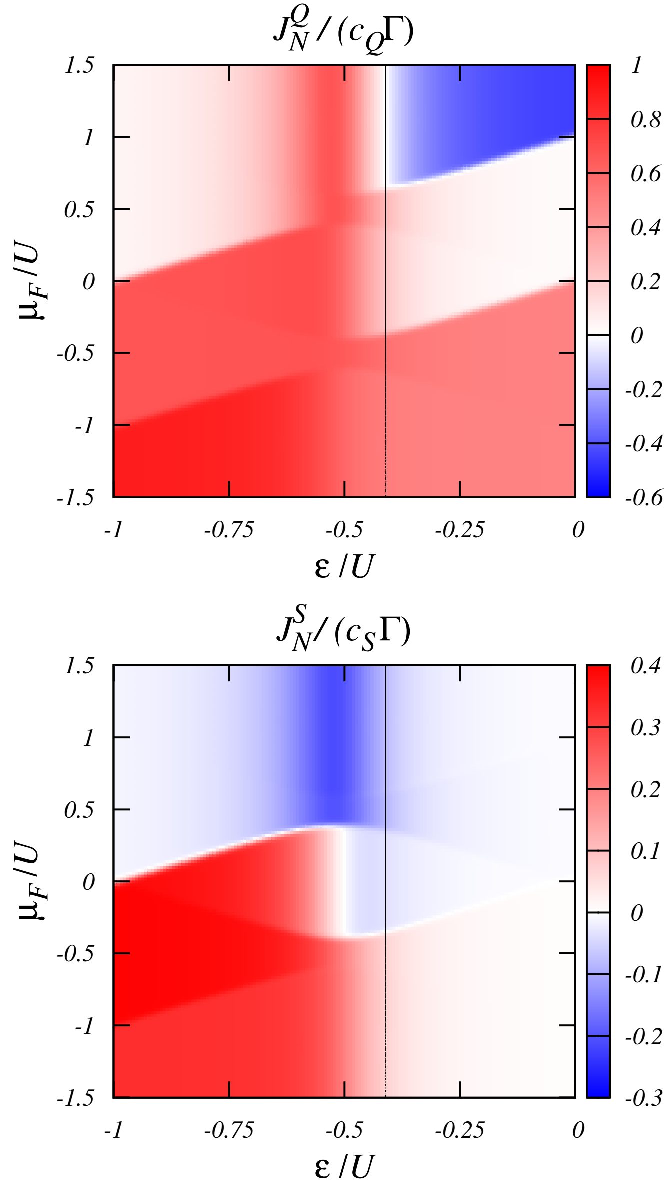
- off resonance ($|\delta| \gg \Gamma_S$)
  - Cooper pair cotunneling
  - Poissonian transfer of $2e$
  - Fano factor $F = 2$
- on resonance ($|\delta| \ll \Gamma_S$)
  - Cooper pair oscillations, interrupted by SET
  - Poissonian transfer of e
  - Fano factor $F = 1$



Generation of Pure Spin Currents

Futterer, Governale, J.K., EPL '10





- $\mu_S = 0 < E_{A,+,-} < E_{A,+,-}$
- $E_{A,+,-} < \mu_N < E_{A,+,-}$
- $E_{A,+,-} < E_{A,+,-} < \mu_F$
- $J_N^Q = 0$ and $J_N^S \neq 0$
- pure spin current in normal lead

Conclusions

- nonequilibrium proximity effect in single-level QDs
  - 0-π-transitions for Josephson current
  - local and non-local Andreev transport
  - tunable by gate and bias voltage
- proximity effect in double dots
  - Cooper pair splitter
  - triplet blockade
- proximity effect revealed by shot noise
- generation of pure spin currents