Aharonov-Bohm conductance through a single-channel quantum ring

Igor Gornyi

Collaboration:

A. Dmitriev (PTI), V. Kachorovskii (PTI), D. Polyakov (KIT)

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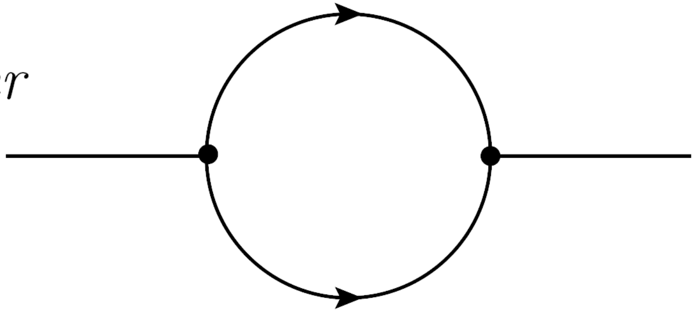
Outline:

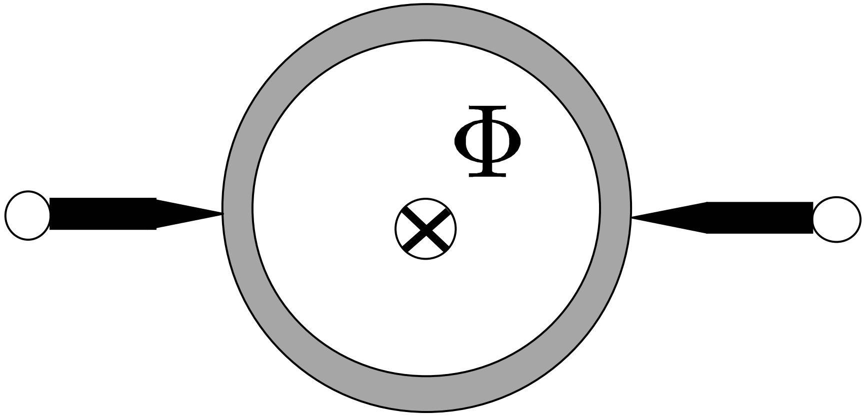
single-channel quantum-ring interferometer



- ▷ **Aharonov-Bohm resonances**
- ▷ **correlated electrons on the ring :**
  Persistent-Current Blockade
  $\longrightarrow$ novel type of Coulomb oscillations
- ▷ **dephasing vs charge quantization :**
  suppression of plasmon-induced dephasing
  Zero-Mode Dephasing

PRL '10

Aharonov-Bohm interferometer



$$I = G(\phi)V$$

Aharonov-Bohm effect:

$$\boxed{G(\phi+1) = G(\phi)}$$

$$\phi = \Phi/\Phi_0$$

$$\Phi_0 = hc/e$$

Key parameter: phase breaking rate $1/\tau_\varphi$

AB conductance: what is known

Multi-channel quantum ring (quasi-1D):
two types of conductance oscillations with the periods

$$\Delta\phi = 1, \quad \Delta\phi = 1/2$$

AB oscillations are suppressed by dephasing

In all cases

$$\frac{1}{\tau_\varphi} \propto g^n T^m \qquad n,m > 0$$

g - **interaction constant**
T - **temperature**

quasi-1D: *Ludwig, Mirlin '04*

Dephasing is due to thermal fluctuations of electric potential created by other electrons (Nyquist bath)

Altshuler, Aronov, Khmelnitskii '82

Single-channel quantum ring :

role of e-e interaction is dramatically enhanced

$G(\phi)$ for low temperature T << Δ (level spacing):

no dephasing ➔ sharp AB resonances affected by Coulomb blockade

Jagla & Balseiro '93
Kinaret, Jonson, Shekhter, Eggert ´98
Pletyukhov, Gritsev, Pauget ´06
Eroms, Mayrhofer, Grifoni ´08

We focus on the opposite limit T >> Δ:

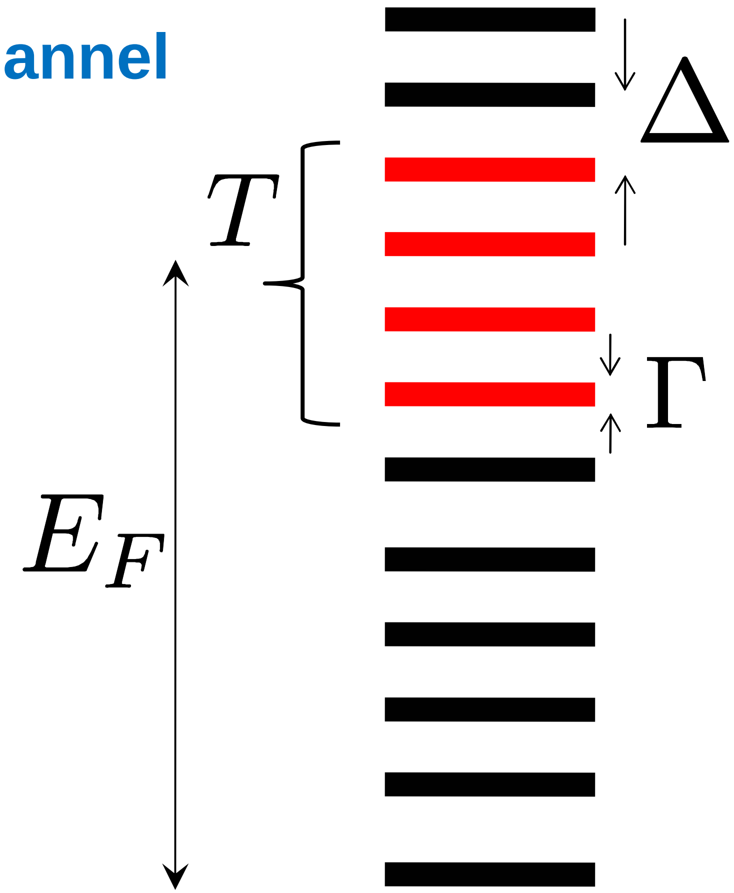
- Interference is not destroyed by thermal averaging: conductance shows sharp anti-resonances

- AB dephasing differs qualitatively from the multi-channel case

Formulation of the problem: Parameters

(spinless) electrons in a single-channel ballistic ring of length L

$$E_F \gg T \gg \Delta \gg \Gamma$$



Δ - level spacing

Γ - tunneling rate

Interaction: Luttinger liquid model

$$g \ll 1$$

Hamiltonian

$$H = H_{\mathrm{ring}} + H_{\mathrm{tun}} + H_{\mathrm{leads}}$$

$$H_{\mathrm{ring}} = \sum_{\mu=\pm} \int_0^L dx \left(-i\mu v \psi_\mu^\dagger D_x \psi_\mu + \frac{1}{2} V_0 \hat{n}_\mu \hat{n}_{-\mu} \right)$$

$$D_x = \partial_x - 2\pi i \phi / L \qquad \phi = \Phi/\Phi_0 \qquad \hat{n}_\mu = :\psi_\mu^\dagger \psi_\mu:$$

$$H_{\mathrm{tun}} = t_0 \left[\psi_L^\dagger \psi(0) + \psi_R^\dagger \psi(L/2) \right] + \text{ h.c.}$$

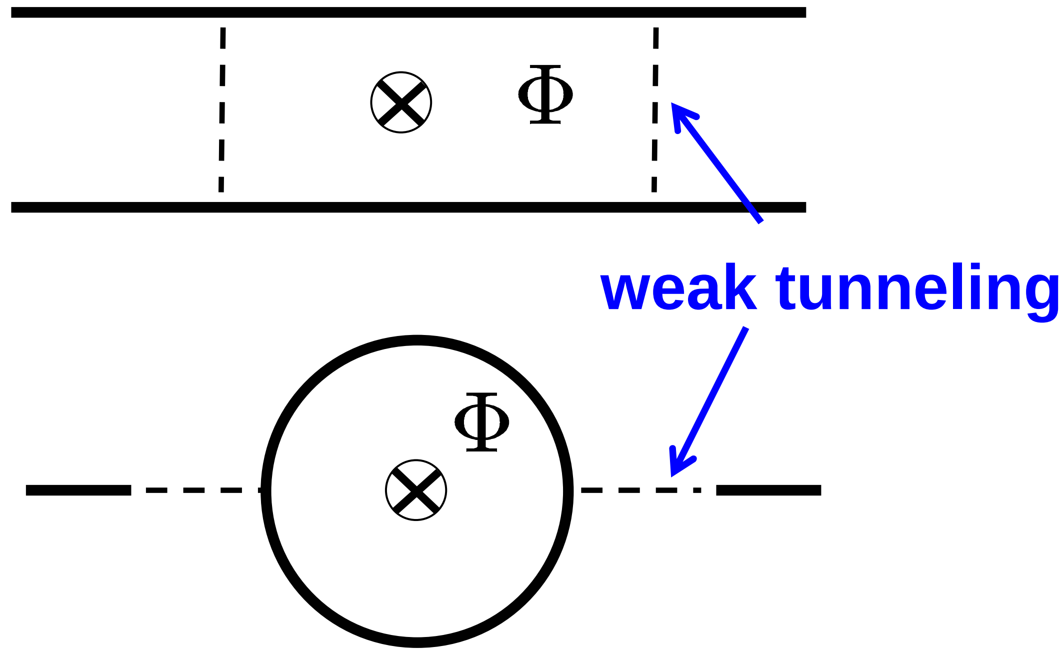
H_{leads} → **structureless density of states in the leads**

Dephasing in the almost closed geometry

naive expectation: $1/\tau_\varphi \sim g^2 T$ **is wrong!!**

Open-geometry OK !

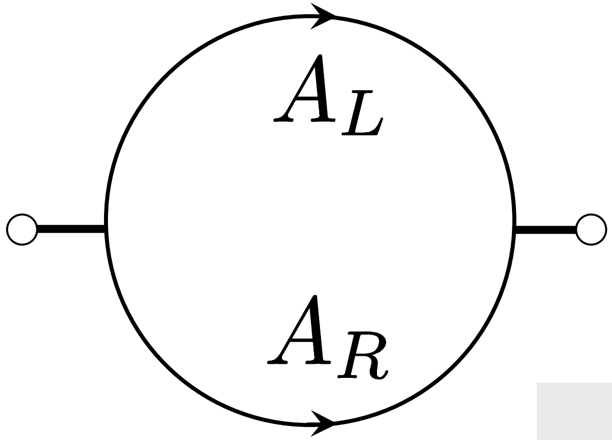
Almost closed ring



Physics: size and charge quantization

dephasing is suppressed

Noninteracting case: Landauer approach



$$G \propto \Gamma^2 \langle |A_R + A_L|^2 \rangle_T$$

$$A_R = \sum_{n=0}^{\infty} e^{i(kL+2\pi\phi)(1/2+n)}$$

lowest order in Γ

$$A_L = \sum_{n=0}^{\infty} e^{i(kL-2\pi\phi)(1/2+n)}$$

T=0 ➔ **resonances for**

$$kL + 2\pi\phi = 2\pi l$$
$$kL - 2\pi\phi = 2\pi m$$

sharp dependence on both *k* and ϕ

T>>Δ ➔ A_R, A_L oscillate rapidly with changing energy

$$A_R A_L^* = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} e^{i(kL+2\pi\phi)(1/2+n)} e^{-i(kL-2\pi\phi)(1/2+m)}$$

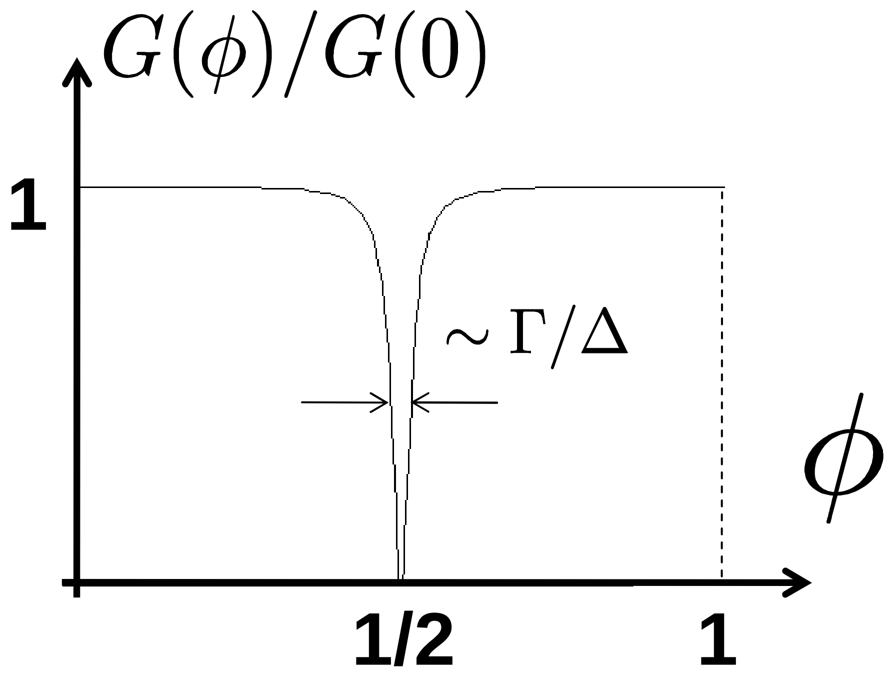
$\left\langle e^{ikL(n-m)} \right\rangle_T \to 0,$ for $n \neq m$ ➔ **only *n=m* terms contribute, no dependence on *k***

but (!) the resonant dependence on ϕ survives

$$\phi = 1/2 \quad \Rightarrow \quad A_R^n = -A_L^n \quad \Rightarrow \quad G = 0$$

destructive interference for any energy

Büttiker, Imry, Azbel '84; Gefen, Imry, Azbel '84



$$G(\phi+1) = G(\phi)$$

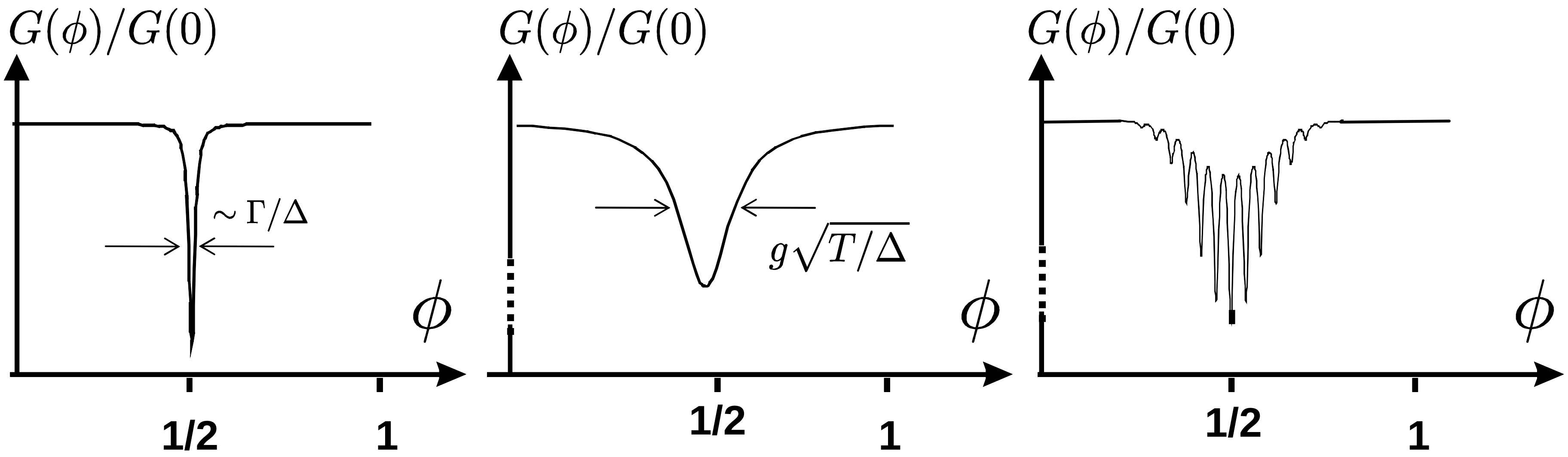
non-interacting case:
narrow AB anti-resonances
at ϕ =*1/2,3/2,…*

$$\frac{G(\phi)}{G(0)} = \frac{\cos^2(\pi\phi)[1+(\Gamma/\Delta)^2]}{\cos^2(\pi\phi)+(\Gamma/\Delta)^2} \approx \frac{(\phi-1/2)^2}{(\phi-1/2)^2+(\Gamma/\Delta)^2}$$

for $|\phi - 1/2| \ll 1$

Expectation ➔ interaction leads to the broadening of the anti-resonances: width $1/\tau_\varphi$

Evolution of AB conductance with increasing interaction strength *g*



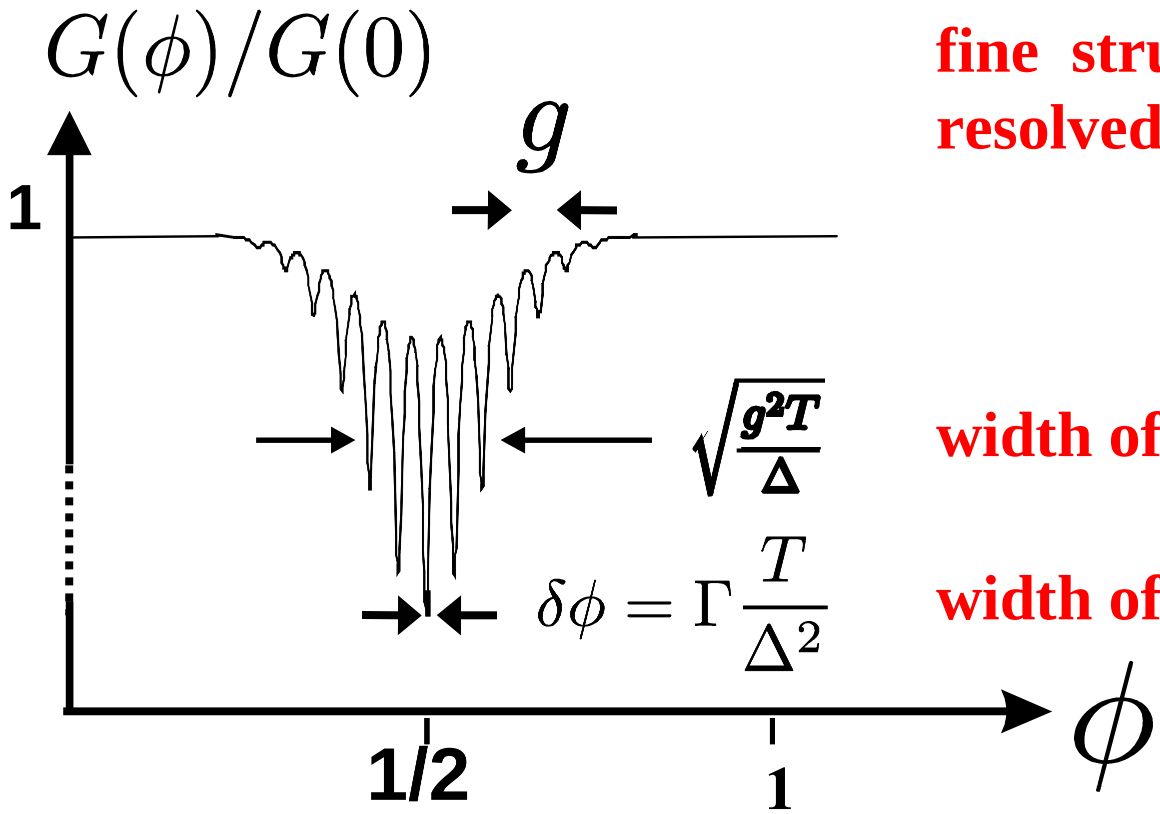
$$g \ll \frac{\Gamma}{\Delta}\left(\frac{\Delta}{T}\right)^{1/2}$$

$$\frac{\Gamma}{\Delta}\left(\frac{\Delta}{T}\right)^{1/2} \ll g \ll \frac{\Gamma T}{\Delta^2}$$

$$\frac{\Gamma T}{\Delta^2} \ll g$$

Main results:

1. Interaction splits the AB resonances



fine structure resolved for $g \gg \Gamma T/\Delta^2$

width of the "Zug"

width of a single peak

2. Interaction-induced dephasing rate does not depend on *g*

$$\frac{1}{\tau_\varphi} = \Gamma\frac{T}{\Delta}$$

Two types of excitations in a single-channel ring

1) Bosonic excitations with $q \neq 0$:

right- and left-moving plasma waves

$$\varphi_R = \varphi + \theta, \quad \varphi_L = \varphi - \theta$$

$$\rho = (1/\pi)\partial\varphi/\partial x, \quad j = (v_F/\pi)\partial\theta/\partial x.$$

2) Bosonic excitations with *q*=0 : Zero-mode excitations

$$\hat{Q} = \hat{N}_R + \hat{N}_L$$

total charge of the ring

$$\hat{J} = \hat{N}_R - \hat{N}_L$$

circular (persistent) current

$N_{R,L}$ - numbers of right- and left-movers in the ring

Zero-mode energy

$$\epsilon_{N_R,\ N_L} = \frac{\Delta}{4}\left[\frac{(N_R+N_L-2N_0)^2}{K} + K(N_R-N_L-2\phi)^2\right]$$

Splitting of AB resonances by the ZM interaction

$$G_{\text{int}}(\phi) \propto \mathrm{Re}(\mathrm{A_R A_L^*})$$

interference part of the conductance

$$A_{R,L} \propto |A_{R,L}|e^{i\alpha_{R,L}}$$

$$\alpha_R = (\epsilon_{N_R+1,N_L} - \epsilon_{N_R,N_L})L/2u$$

$$\alpha_L = (\epsilon_{N_R,N_L+1} - \epsilon_{N_R,N_L})L/2u$$

phases acquired by a tunneling electron along two interfering paths

$$\alpha_R - \alpha_L \approx g(N_R - N_L) \quad \Rightarrow \quad \delta\phi \approx g(N_R - N_L)$$

Splitting is due to the quantization of circular current $\hat{J} = \hat{N}_R - \hat{N}_L$

Persistent-Current Blockade

$$\frac{G(\phi)}{G(0)} \approx \frac{1}{Z} \sum_{N_R, N_L} e^{-\epsilon_{N_R,N_L}/T} \frac{[\phi - 1/2 - gJ]^2}{[\phi - 1/2 - gJ]^2 + (\Gamma/\Delta)^2}$$

without dephasing

$$\delta J \sim \sqrt{T/\Delta} \quad \Rightarrow \delta\phi_T = g\delta J = g\sqrt{T/\Delta}$$

width of the Zug

Absence of the plasmon mechanism of dephasing

Fluctuating potential created by the plasmon thermal bath

$$U(x,t) = U_R(x-ut) + U_L(x+ut)$$

Properties of the plasmon bath in the ring:

1) Closed geometry:

$U_{R,L}$ are periodic functions of x with the period L $\Rightarrow$ they are also periodic functions of t with the period L/u

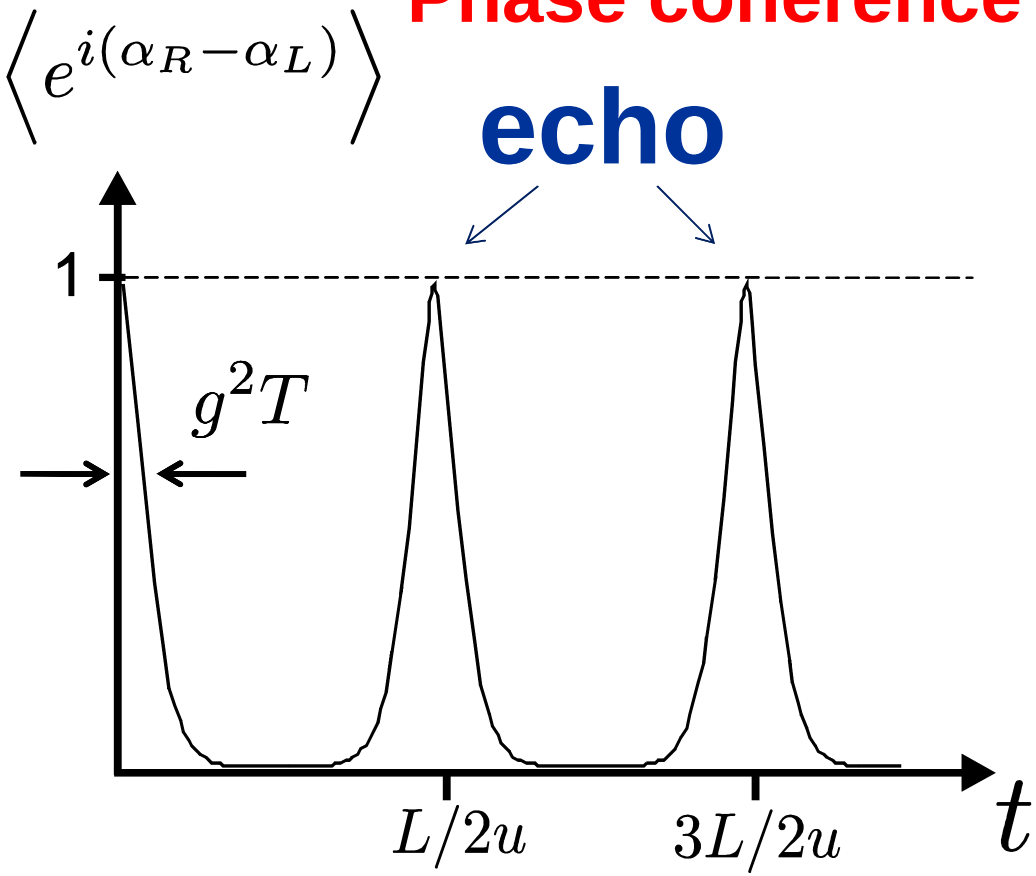
2) Hartree-Fock cancellation for a short range potential:

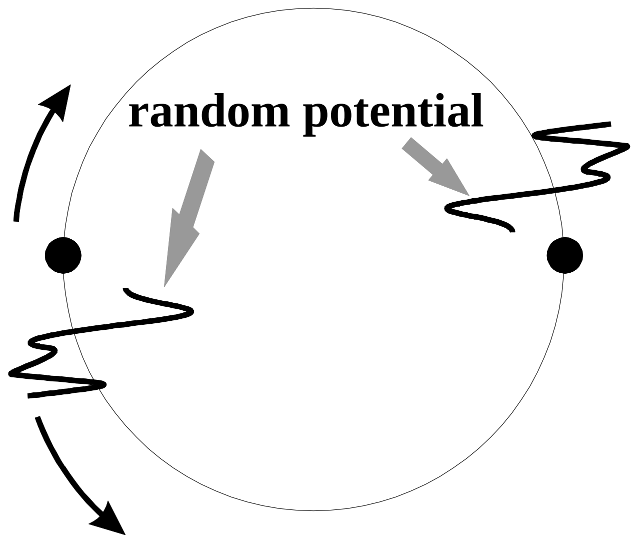
Right-movers do not interact with U_R, and
left-movers do not interact with U_L

$$\alpha_R = \int_0^t dt U_L[x(t) + ut] = \int_0^t dt\; U_L(2ut)$$
$$\alpha_L = \int_0^t dt U_R[x(t) - ut] = \int_0^t dt\; U_R(-2ut)$$

$U_{R,L}$ are random on the short time scale but repeat themselves periodically with the period $L/2u$

Phase coherence is restored periodically





Dephasing is due to circular-current fluctuations

$$\phi_{N_R,N_L} = \frac{1}{2} + g(N_R - N_L)$$

positions of the resonances

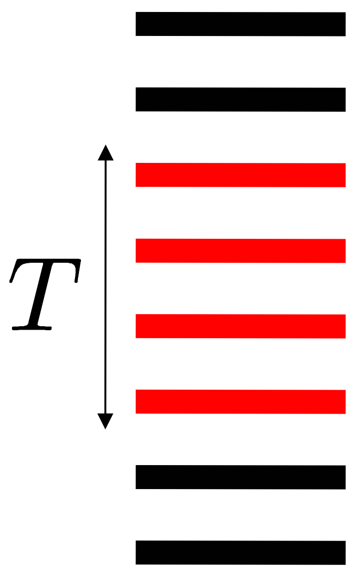
$$N_{R,L} \to N_{R,L}(t), \qquad J(t) = N_R(t) - N_L(t)$$

$$V_{eff}(t) = gJ(t)\Delta$$

dephasing potential

$$e^{i(\alpha_R - \alpha_L)} = e^{i\int_0^t V_{eff}dt} = e^{ig\Delta\int_0^t Jdt}$$

Time scale of the current fluctuations



$$J(t) = N_R(t) - N_L(t)$$

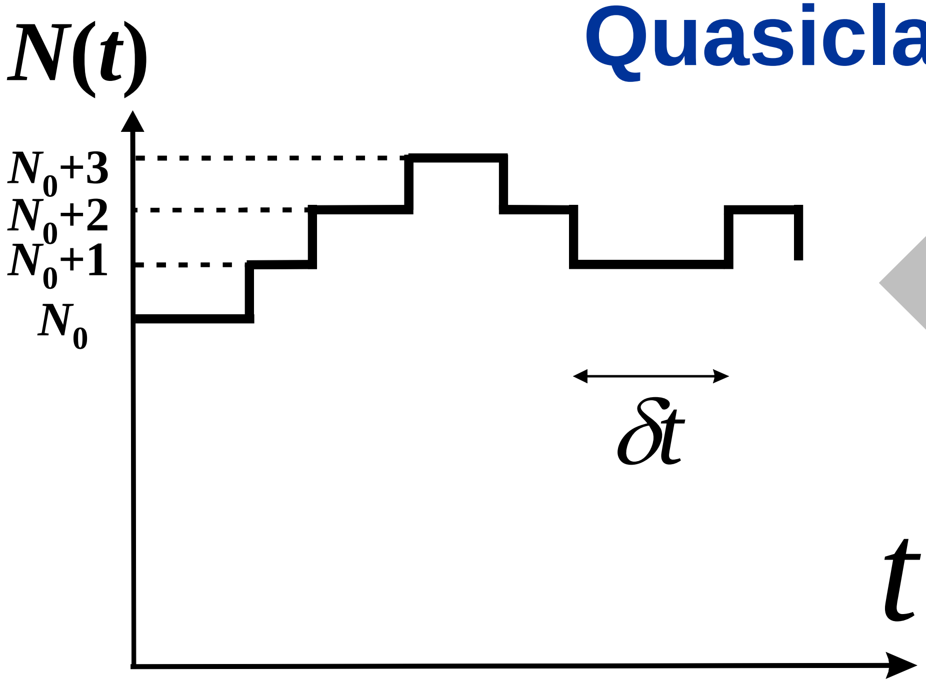
T/Δ **- number of active levels >>1**

population of one level fluctuates on a time scale $1/\Gamma$

$N_{R,L} \rightarrow N_{R,L}+1$ ⬅ **shorter time scale :**

$$\delta t = \frac{1}{\Gamma}\frac{\Delta}{T}$$

Quasiclassical approach



telegraph noise

$$\delta t = \frac{1}{\Gamma}\frac{\Delta}{T}$$

$$e^S = \left\langle e^{ig\Delta\int_0^t dt[N(t)-N_0]}\right\rangle = \exp\left\{\frac{\left[\frac{\sin(g\Delta t)}{g\Delta} - t\right]}{\delta t}\right\} \approx \exp(-t/\delta t)$$

for $t \gg 1/g\Delta$

$$\Gamma_\varphi = \Gamma\frac{T}{\Delta}$$

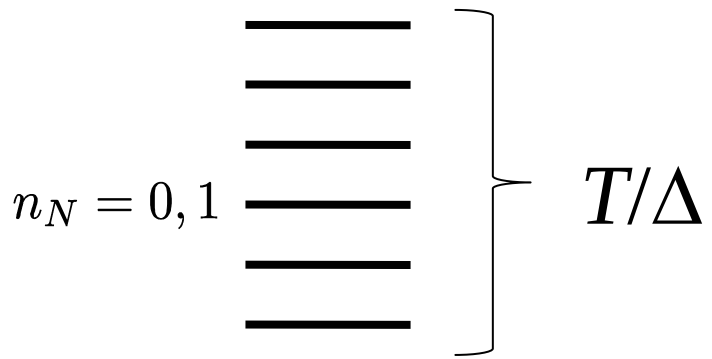
- **much faster than the tunneling**
- **does not depend on the interaction**

Dephasing by telegraph noise

$$e^S = \left\langle e^{ig\Delta \int_0^t dt (N_R - N_L)} \right\rangle = \left\langle e^{ig\Delta \int_0^t dt N_R} e^{-ig\Delta \int_0^t dt N_L} \right\rangle$$

$$= \left\langle e^{ig\Delta \int_0^t dt N} \right\rangle \times \langle \ \text{c.c.} \ \rangle$$

$$N_R = n_1 + n_2 + \cdots + n_N + \cdots$$



$$e^S = \prod_N \left\langle e^{ig\Delta n_N t} \right\rangle \prod_N \langle c.c. \rangle$$

$n_N = 0, 1$ ➡ **telegraph noise** ➡ **dephasing**

Paladino, Faoro, Falci, Fazio '02
Grishin, Yurkevich, Lerner '05
Galperin, Altshuler, Bergli, Shantsev '06
Schriefl, Makhlin, Shnirman,Schoen '06
Neder, Marquardt '07
Neuenhahn, Kubala, Abel, Marquardt '08
Abel, Marquardt '08

Dephasing action

$$e^S = \prod_N \left[(1-f_N)e^{-\Gamma t f_N} + e^{ig\Delta t}e^{-\Gamma t(1-f_N)}\right]$$
$$\times \prod_N \left[(1-f_N)e^{-\Gamma t f_N} + e^{-ig\Delta t}e^{-\Gamma t(1-f_N)}\right]$$

$$e^S = \underbrace{\exp\left[-\frac{2T}{\Delta}(1-\cos g\Delta t)\right]}_{e^{S_0}} \underbrace{\exp\left\{-\Gamma_\varphi\left[t(1+\cos g\Delta t) - \frac{2\sin g\Delta t}{g\Delta}\right]\right\}}_{e^{S_{\mathrm{deph}}}}$$

here: linear in Γ terms only

$$\Gamma_\varphi = \Gamma\frac{T}{\Delta}$$

zero mode dephasing

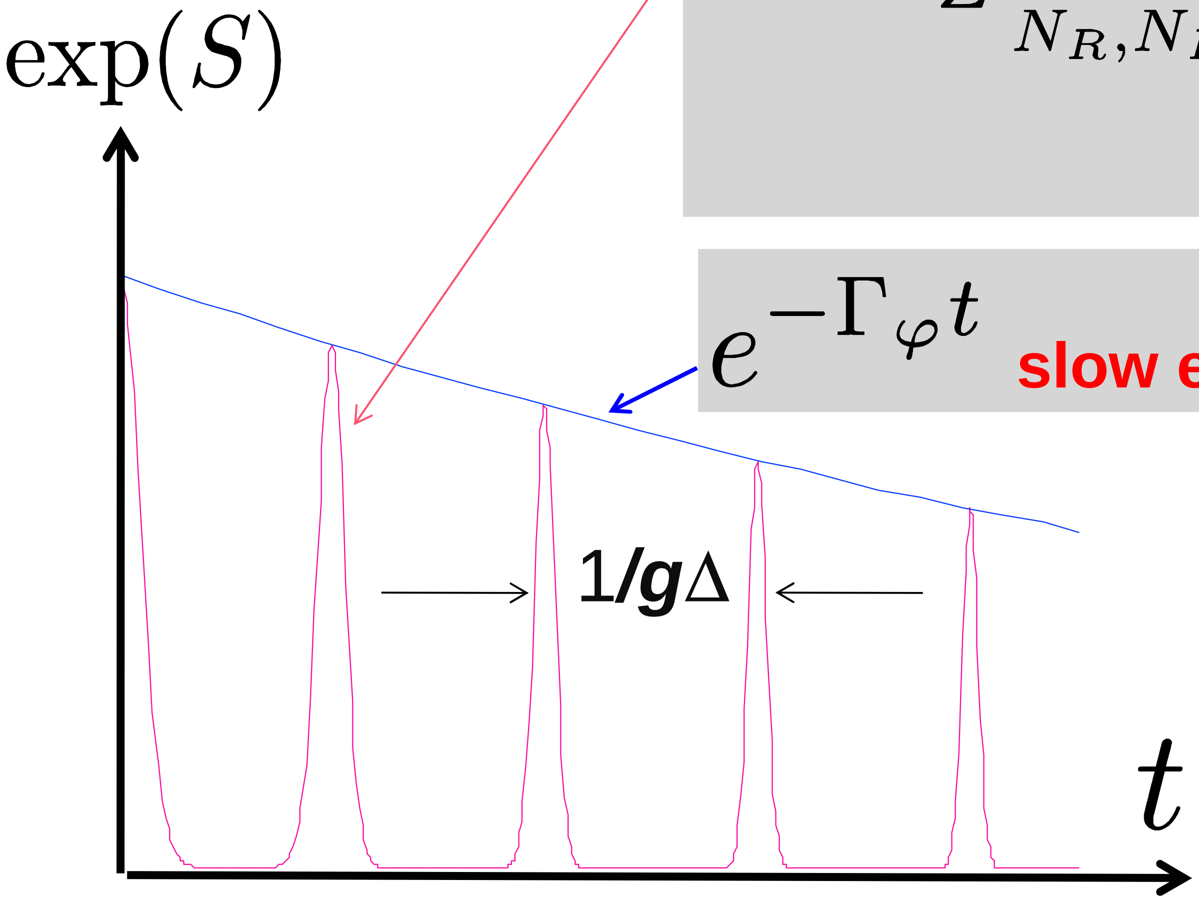
Quantum beats in the dephasing action

$$e^{S_0} = \frac{1}{Z} \sum_{N_R,N_L} e^{ig\Delta Jt} e^{-\epsilon_{N_R,N_L}/T}$$

coherent oscillations
in a closed ring

$$e^{-\Gamma_\varphi t}$$

slow envelope



Persistent-current blockade

Sum over the winding number: $t_n = 2\pi(n+1/2)/\Delta$

$$\frac{G_{\text{int}}(\phi)}{G(0)} \simeq -\frac{2\pi\Gamma}{\Delta}\sum_{n=0}^{\infty}\cos[2\Delta(\phi-1/2)t_n]\, e^{-\Gamma t_n - S(t_n)}$$

$$\frac{G(\phi)}{G(0)} \approx \frac{1}{Z}\sum_{N_R,N_L} e^{-\epsilon_{N_R,N_L}/T}\left[1-\frac{\Gamma\Gamma_\varphi}{\Delta^2(\phi-1/2-gJ)^2+\Gamma_\varphi^2}\right]$$

$$\Gamma_\varphi = \Gamma\frac{T}{\Delta}$$

Quantum properties of Luttinger liquid

$$\Gamma \to \Gamma\left(\frac{T}{E_F}\right)^{g^2/2}$$

Renormalization of the tunneling rate

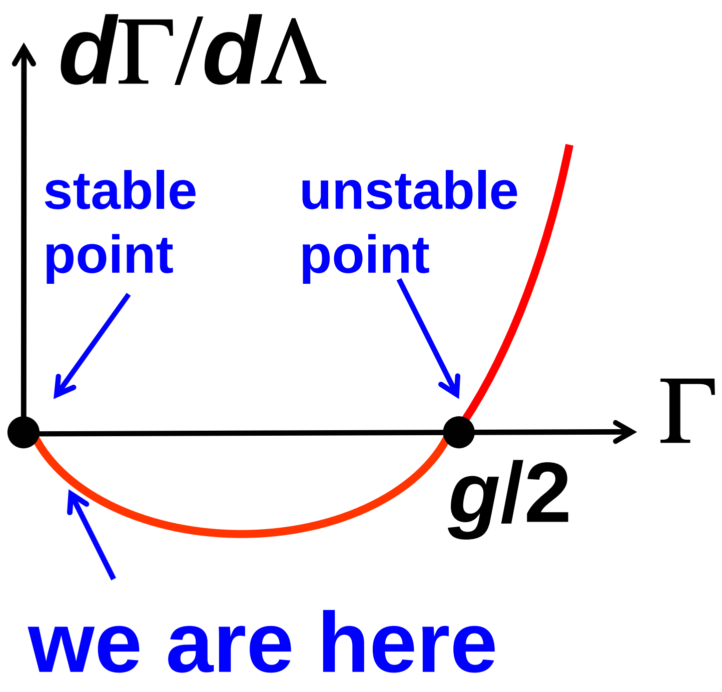
$$\frac{d\Gamma}{d\Lambda} = g\Gamma^2 - \frac{g^2\Gamma}{2}$$

non-trivial geometry, junction of 3 wires (Das, Rao, Sen ’04)

renormalization of the bulk density of states

Γ in units of Δ (tunneling transparency of the contact)

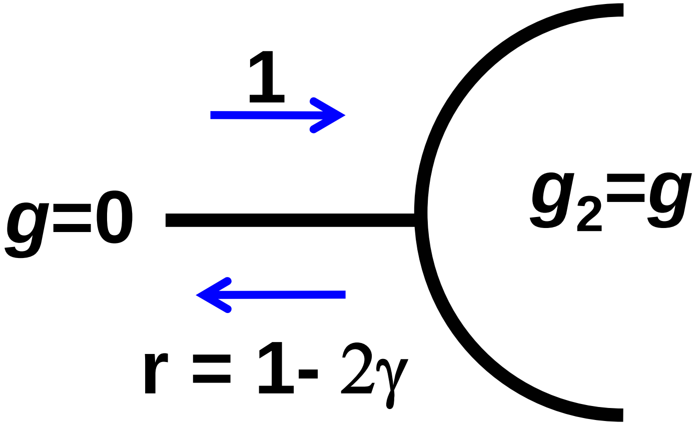
$$\Lambda = \ln\left(\frac{E_F}{T}\right)$$

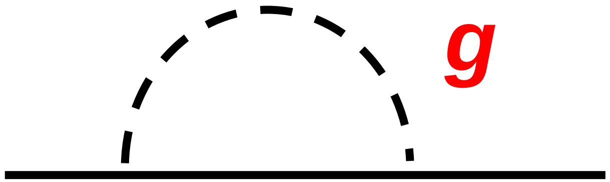


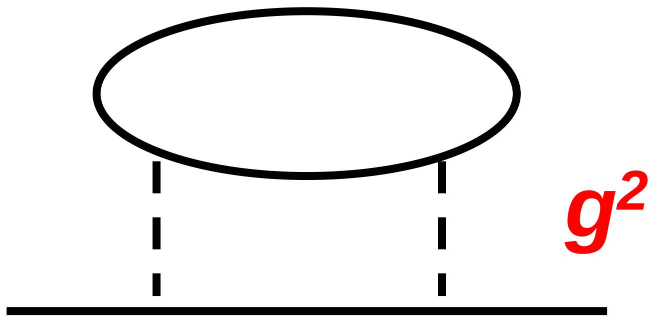
$\Gamma_0 << g$ ➔ renormalization of the density of states dominates

Tunneling Hamiltonian (point contact): small γ

Perturbative renormalization
(g <<1, γ << 1):

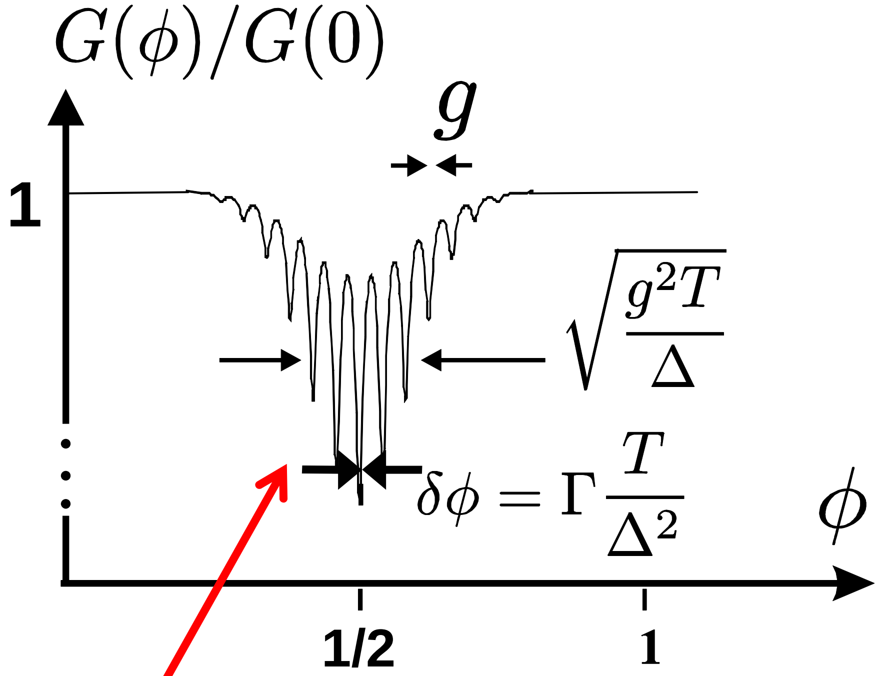






fermion loop ➔ opposite sign

Summary of the main results



height of n-th resonance:

$$\left(\frac{G_{int}}{G_{cl}}\right)_n = \exp(-n^2\Delta/T)\left(\frac{\Delta}{T}\right)^{3/2}$$

width of the resonances:

$$\delta\phi = \frac{\Gamma_\varphi}{\Delta}$$

persistent current blocks tunneling current

interference part of the conductance:

$$\frac{G_{int}(\Phi)}{G(0)} \simeq -\left\langle \frac{\Gamma\Gamma_\varphi}{[\phi - 1/2 - \alpha J]^2\Delta^2 + \Gamma_\varphi^2}\right\rangle_{\text{Gibbs}}$$

Zero-mode dephasing: $\Gamma_\varphi = \Gamma T/\Delta$

Conclusions

- Electron-electron interactions $\rightarrow$ profound effect on transport through a single-channel ring
- Main phenomenon: persistent-current blockade
- PCB-induced splitting of AB antiresonances
- Zero-mode dephasing due to fluctuations of PCB
- PCB is robust (spin, curvature, asymmetry...)